Thirteenth International Conference on
Geometry, Integrability and Quantization
June 3–8, 2011, Varna, Bulgaria
Ivaïlo M. Mladenov, Andrei Ludu
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2012, pp 159–169
doi: 10.7546/giq-12-2011-159-169

THE NEWMAN JANIS ALGORITHM: A REVIEW OF SOME RESULTS

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Abstract. In this paper we review some interesting results obtained through the Newman-Janis algorithm, a solution generating technique employed in General Relativity. We also describe the use of this algorithm in different theories, namely f(R), Einstein-Maxwell-Dilaton gravity, Braneworld, Born-Infeld Monopole and we focus on the validity of the results.

1. Introduction

In order to find new solutions of Einstein field equations, several methods were introduced, such as the Newman-Penrose formalism and a technique founded by Newman and Janis [15]. In the recent years, many papers have appeared focusing on the Newman-Janis algorithm, considered as a solution generating technique which provides metrics of reduced symmetries from symmetric ones. Our aim is to give a summary of the use of this method and to review the most interesting applications in different gravity theories.

The outline of this paper is as follows: in Section 2, we present the general procedure of the Newman-Janis algorithm (henceforth NJA). In Section 3, we review the interesting attempts made in General Relativity, focusing on the most intriguing results. In Section 4, we have analyzed how to apply this technique to the f(R) modified theories of gravity, by following the procedure used in GR. Then, in Section 5, an application of NJA in the Einstein-Maxwell-dilaton-axion Gravity is discussed. Finally we draw some conclusions.

2. A Brief Description of the Method

Following [15], we show how it is possible to derive the Kerr solution from the Schwarzschild one through the NJA. Let us start by writing the **Schwarzschild metric**, considered as a static spherically symmetric seed metric, in advanced Eddington-Finkelstein coordinates (i.e., the g_{rr} component is eliminated by a change of coordinates and a crossterm is introduced)

$$\mathrm{d}s^2 = \left(1 - \frac{2m}{r}\right)\mathrm{du}^2 + 2\mathrm{dudr} - \mathrm{r}^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2).$$

By introducing the formalism of null tetrad, the contravariant metric components may be written as

$$g^{\mu\nu} = l^\mu n^\nu + l^\nu n^\mu - m^\mu \bar{m}^\nu - m^\nu \bar{m}^\mu \tag{1}$$

where

$$\begin{aligned} l_\mu l^\mu &= m_\mu m^\mu = n_\mu n^\mu = 0 \\ l_\mu n^\mu &= -m_\mu \bar{m}^\mu = 1 \\ l_\mu m^\mu &= n_\mu m^\mu = 0. \end{aligned}$$

For the Schwarzschild spacetime the **null tetrad vectors** $(l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$ are

$$\begin{aligned} l^\mu &= \delta_1^\mu \\ n^\mu &= \delta_0^\mu - \frac{1}{2}\left(1 - \frac{2m}{r}\right)\delta_1^\mu \\ m^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu + \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right) \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right). \end{aligned}$$

Now, let the coordinate r to take complex values so the complex conjugate of r appears

$$\begin{aligned} l^\mu &= \delta_1^\mu \\ n^\mu &= \delta_0^\mu - \frac{1}{2}\left(1 - m\left[\frac{1}{r} + \frac{1}{\bar{r}}\right]\right)\delta_1^\mu \\ m^\mu &= \frac{1}{\sqrt{2}\bar{r}}\left(\delta_2^\mu + \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right) \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right) \end{aligned}$$

then it is possible to perform the following complex coordinate transformation on the null vectors

$$r' = r + \mathrm{ia}\cos\vartheta, \qquad \mathrm{u}' = \mathrm{u} - \mathrm{ia}\cos\vartheta, \qquad \vartheta' = \vartheta, \qquad \varphi' = \varphi$$

where a is a real parameter. By requiring that r' and u' are real (that is considering the transformations as a complex rotation of the ϑ, φ plane), one obtains the new tetrad

$$\begin{aligned}
l^\mu &= \delta_1^\mu \\
n^\mu &= \delta_0^\mu - \frac{1}{2}\left(1 - \frac{2mr'}{r'^2 + a^2\cos^2\vartheta}\right)\delta_1^\mu \\
m^\mu &= \frac{1}{\sqrt{2}(r' + \mathrm{ia}\cos\vartheta)}\left[\mathrm{ia}\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right] \\
\bar{m}^\mu &= \frac{1}{\sqrt{2}(r' - \mathrm{ia}\cos\vartheta)}\left[-\mathrm{ia}\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu - \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right].
\end{aligned}$$

The contravariant components of a new metric can be defined from the above null vectors according to equation (1). This gives the promised Kerr solution in advanced null coordinates. By performing a transformation on the null coordinate u and the angle coordinate φ, one obtains the usual representation of the Kerr metric in Boyer-Lindquist coordinates.

The same procedure can be used to get the Kerr-Newman metric from the Reissner-Nordström one [14]. The first two exact solutions obtained through the Newman-Janis algorithm are the vacuum solution (Kerr) and the electro-vacuum solution (Kerr-Newman).

3. The NJA in General Relativity

After the introduction of the NJA which allows to generate the Kerr solution from Schwarzschild metric and the Kerr-Newman solution from Reissner-Nordström metric, this method has been applied with the aim to generate new solutions. In this section we discuss some interesting results obtained in different cases.

Generally the NJA has been treated as an useful procedure for generating new solutions of Einstein's equations from known static spherically symmetric ones, thus the method turns out to be suitable for studying rotating systems in General Relativity.

Precisely in [5], in order to show that new metrics can be obtained, the Newman-Janis technique with a different complex coordinate transformation is applied to the Schwarzschild solution in null polar coordinates, as follows

$$r' = r + \mathrm{i}(\mathrm{a}\cos\vartheta + \mathrm{b}), \quad \mathrm{u}' = \mathrm{u} - \mathrm{i}(\mathrm{a}\cos\vartheta + 2\mathrm{b}\ln(\sin\vartheta)) + 2\mathrm{ib}\ln(\tan\vartheta/2).$$

The final result is a solution of Einstein's equations in vacuum and appears as a combination of Kerr metric and the NUT space metric. It depends, in fact, on three arbitrary parameters m, a and b. When $a = b = 0$ the Schwarzschild solution is obtained; if $a = 0$ it becomes the NUT space and if $b = 0$ it becomes the Kerr solution. Subsequently, it was demonstrated that by performing a more general complex coordinate transformation, see [6], it is possible to find the most general solution of Einstein field equations obtainable in this way and in which a non vanishing cosmological constant is allowed. With this result by setting $\Lambda = 0$ one recovers the standard form of the NUT space. However, in this way it is impossible to find the Kerr solution with the cosmological constant and, in particular, this shows that the Carter's Kerr de Sitter metric cannot be obtained with the NJA.

An explanation concerning the success of this "trick" is shown in [10]. Here, is pointed out that, as ensues from [16] where the Kerr-Schild metric is obtained by performing an imaginary displacement ($\mathrm{i}a$) of the coordinates, a complex translation of coordinates is allowed in GR when a coordinates system is found in which the pseudo energy-momentum tensor vanishes or the Einstein equations are linear. This works only in an algebraically special Kerr-Schild geometry.

Several attempts have been made by using the NJA to generate the interior Kerr solutions ([8], [11], [13], [18]) but these were unsuccessful in finding a solution that is both physically reasonable and can be matched smoothly to the Kerr metric. In particular, in [11] the algorithm was indeed applied to an interior spherically symmetric metric to describe an internal source model for the Kerr exterior solution. The resulting metric was then matched to the vacuum Kerr solution on an oblate spheroid. In [8], in order to obtain new possible sources for Kerr metric, the NJA was applied to a generic static spherically symmetric seed metric. Then, to join any two stationary and axially symmetric metrics, the Darmois-Israel junction conditions were imposed on a suitable separating hypersurface, thus having a vanishing surface stress-energy tensor.

For this reason, these results were considered as starting point to perform a generalization of the algorithm and to demonstrate why this method is successful. To do this, is necessary to remove some of the ambiguities appearing in the original derivation, as shown in [7], where it was also considered the problems arising when the rotation is included.

Subsequently, the Newman-Janis method, combined with the Wang-Wu technique, (see [13]), was used to generate new embedded solutions describing more complicated systems, like Kerr-Newman-de Sitter. Furthermore it was shown that all rotating embedded solutions can be written in Kerr-Schild form which seems the most suitable form for the validity of NJA.

In a more recent paper [18], in order to obtain Kerr interior solutions, starting from the Schwarzschild solution, the NJA is performed. Then a discussion of the Slowly Rotating Limit is also presented.

By starting with these results, a more deep analysis, concerning the ambiguities which arise in dealing with the NJA and the problems appearing when the cosmological constant is introduced, has been given in [1].

4. The NJA in f(R)-gravity

Fourth order theories appear as a quite natural modification of GR theory. They consist in a straightforward generalization of the Lagrangian in the Einstein-Hilbert action by choosing a generic function of the Ricci scalar, $f(R)$. The field equations following from this modified Lagrangian are of fourth order, i.e., they contain derivatives up to the fourth order of the components of the metric with respect to the spacetime coordinates. Recently, the interest in $f(R)$ theories, in particular in spherically symmetric solutions of $f(R)$ is increased. This should be the starting point to test the validity of the NJA in $f(R)$ gravity, see [2].

4.1. The Standard Procedure

Let us consider the spherically symmetric metric as

$$\mathrm{d}s^2 = (\alpha + \beta r)\mathrm{d}t^2 - \frac{1}{2}(\frac{\beta r}{\alpha + \beta r})\mathrm{d}r^2 - r^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2).$$

By following the standard procedure as shown in Section 2, the metric is written in Eddington-Finkelstein coordinates $(u, r, \vartheta, \varphi)$ and its null tetrad is

$$\begin{aligned} l^\mu &= \delta_1^\mu \\ n^\mu &= \left(\sqrt{\frac{2}{\beta r}}\right)\delta_0^\mu - \left(-1 - \frac{2\alpha}{r\beta}\right)\delta_1^\mu \\ m^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu + \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right) \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right). \end{aligned}$$

After the complexification of the radial coordinate r, it is possible to apply the NJA as usual

$$r' = r + \mathrm{ia}\cos\vartheta, \qquad \mathrm{u}' = \mathrm{u} - \mathrm{ia}\cos\vartheta.$$

The resulting null tetrad appears in the following from

$$
\begin{aligned}
l^\mu &= \delta^\mu_1\\
n^\mu &= -\left[1+\frac{\alpha}{\beta}\frac{\Re(r)}{\Sigma^2}\right]\delta^\mu_1-\left(\sqrt{\frac{2}{\beta\Sigma}}\right)\delta^\mu_0\\
m^\mu &= \frac{1}{\sqrt{2}(r+\mathrm{i}a\cos\vartheta)}\left[\mathrm{i}a(\delta^\mu_0-\delta^\mu_1)\sin\vartheta+\delta^\mu_2+\frac{\mathrm{i}}{\sin\vartheta}\delta^\mu_3\right]\\
\bar{m}^\mu &= \frac{1}{\sqrt{2}(r-\mathrm{i}a\cos\vartheta)}\left[-\mathrm{i}a(\delta^\mu_0-\delta^\mu_1)\sin\vartheta+\delta^\mu_2-\frac{\mathrm{i}}{\sin\vartheta}\delta^\mu_3\right]
\end{aligned}
$$

where $\Sigma = \sqrt{r^2+a^2\cos^2\vartheta}$. Now, making a *gauge transformation* (see also [8]), in order that the only off-diagonal term is $g_{\varphi t}$, one obtains an axially symmetric metric as expected

$$
g_{\mu\nu} = \begin{pmatrix}
\frac{r(\alpha+\beta r)+a^2\beta\cos^2\vartheta}{\Sigma} & 0 & 0 & \frac{a(-2\Xi+\Gamma\Sigma^{3/2})\sin^2\vartheta}{2\Sigma}\\
\cdot & \frac{\beta\Sigma^2}{2\alpha r+\Lambda} & 0 & 0\\
\cdot & \cdot & -\Sigma^2 & 0\\
\cdot & \cdot & \cdot & -\left[\Sigma^2-\frac{a^2(\Xi-\Gamma\Sigma^{3/2})\sin^2\vartheta}{\Sigma}\right]\sin^2\vartheta
\end{pmatrix}
$$

where

$$
\Lambda = \beta(a^2+r^2+\Sigma^2), \qquad \Xi = \alpha r+\beta\Sigma^2, \qquad \Gamma=\sqrt{2\beta}.
$$

The method can be also applied to any spherically solution derived in f(R)-gravity.

5. The NJA and “Rotating Dilaton-axion Black Hole"

In this section we discuss the application of the Newman-Janis method in Einstein-Maxwell-dilaton-axion gravity, which is an interesting generalization of Einstein-Maxwell theory obtained in the low energy limit of the heterotic string theory. Precisely, in [20] is shown how the NJA can be used to derive the rotating dilaton-axion black hole solution from the static spherically symmetric charged dilaton black hole solution, found by Gibbons and independently by Garfinkle, Horowitz and Strominger. Since Sen (see [17]), was able to generate the rotating charged black hole solution by starting from the Kerr solution, it seems natural to verify if the Sen's solutions can be generated via NJA from the GGHS solutions.

The first step is to write the metric describing the dilaton black hole solution, namely GGHS, in the suitable form directly generated from the Schwarzschild solution [21]

$$
\mathrm{d}s^2 = \left(\frac{1-\frac{r_1}{r}}{1+\frac{r_2}{r}}\right)\mathrm{d}t^2-\left(\frac{1-\frac{r_1}{r}}{1+\frac{r_2}{r}}\right)^{-1}\mathrm{d}r^2-r^2\left(1+\frac{r_2}{r}\right)(\mathrm{d}\vartheta^2+\sin^2\vartheta\mathrm{d}\varphi^2)
$$

where $r_1 + r_2 = 2M, \quad r_2 = \frac{Q^2}{M}$ and M and Q are the mass and the charge of the dilaton black hole. After expressing the metric in advanced coordinates with

$$\mathrm{d}t = \mathrm{d}u + \left(\frac{1 - \frac{r_1}{r}}{1 + \frac{r_2}{r}}\right)^{-1} \mathrm{d}r$$

is it possible to introduce the null tetrad

$$\begin{aligned}
l^\mu &= \delta_1^\mu \\
n^\mu &= \delta_0^\mu - \frac{1}{2}\left(\frac{1 - \frac{r_1}{r}}{1 + \frac{r_2}{r}}\right) \\
m^\mu &= \frac{1}{\sqrt{2}r\sqrt{1 + \frac{r_2}{r}}}\left(\delta_2^\mu + \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right) \\
\bar{m}^\mu &= \frac{1}{\sqrt{2}r\sqrt{1 + \frac{r_2}{r}}}\left(\delta_2^\mu - \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right).
\end{aligned}$$

Following the standard procedure, one obtains the new null tetrad

$$\begin{aligned}
l^\mu &= \delta_1^\mu \\
n^\mu &= \delta_0^\mu - \frac{1}{2}\left(\frac{1 - \frac{r_1 r}{\Sigma}}{1 + \frac{r_2 r}{\Sigma}}\right)\delta_1^\mu \\
m^\mu &= \frac{1}{\sqrt{2}(r + \mathrm{i}a\cos\vartheta)}\frac{1}{\sqrt{1 + \frac{r_2 r}{\Sigma}}}\left(\delta_2^\mu + \mathrm{i}a\cos\vartheta(\delta_0^\mu - \delta_1^\mu) + \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right) \\
\bar{m}^\mu &= \frac{1}{\sqrt{2}(r - \mathrm{i}a\cos\vartheta)}\frac{1}{\sqrt{1 + \frac{r_2 r}{\Sigma}}}\left(\delta_2^\mu - \mathrm{i}a\cos\vartheta(\delta_0^\mu - \delta_1^\mu) - \frac{\mathrm{i}}{\sin\vartheta}\delta_3^\mu\right)
\end{aligned}$$

where $\Sigma = r^2 + \cos^2\vartheta$. After further simplifications and a suitable choice of coordinates, the rotating dilaton-axion black hole metric reads

$$\begin{aligned}
\mathrm{d}s^2 = \left(1 - \frac{2Mr}{\tilde{\Sigma}}\right)\mathrm{d}t^2 - \tilde{\Sigma}\left(\frac{\mathrm{d}r^2}{\Delta} + \mathrm{d}\vartheta^2\right) + \frac{4Mra\sin^2\vartheta}{\tilde{\Sigma}}\mathrm{d}t\mathrm{d}\varphi \\
- \left(r(r + r_2) + a^2 + \frac{2Mra^2\sin^2\vartheta}{\tilde{\Sigma}}\right)\sin^2\vartheta\mathrm{d}\varphi^2
\end{aligned}$$

where

$$\tilde{\Sigma} = r(r + r_2) + a^2\cos^2\vartheta, \qquad \Delta = r(r - r_1) + a^2.$$

The final result coincides with what is expected. However it is known that the static spherically symmetric charged dilaton black hole is also a solution to the truncated theory without axion field (i.e., Einstein-Maxwell-dilaton gravity) but, in this case, the Newman-Janis method does not work. The reason should be that the full theory has a larger symmetry group than the truncated one, [20].

6. Other Theories: Braneworld and Born Infeld Monopole

The Newman-Janis algorithm has been also applied in Braneworld theory and in Born-Infeld theory in an attempt to verify if this method is successful in generating rotating solutions also in other theories.

6.1. The NJA in Braneworld

In the framework of Braneworld, the NJA is applied to three static, spherically symmetric seed metrics in the following form [19]

$$\mathrm{d}s^2 = -\mathrm{e}^{2\varphi(r)}\mathrm{d}t^2 + \mathrm{e}^{2\lambda(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

with three different choices of the functions $\mathrm{e}^{2\varphi(r)}$ and $\mathrm{e}^{2\lambda(r)}$, which are respectively

$$\begin{aligned}
\mathrm{e}^{2\varphi(r)} &= \left[(1+\epsilon)\sqrt{1-\frac{2M}{r}}-\epsilon\right]^2, & \mathrm{e}^{2\lambda(r)} &= \left(1-\frac{2M}{r}\right)^{-1}\\
\mathrm{e}^{2\varphi(r)} &= \left(1-\frac{2M}{r}\right), & \mathrm{e}^{2\lambda(r)} &= \frac{\left(1-\frac{3M}{2r}\right)}{\left(1-\frac{2M}{r}\right)\left(1-\frac{r_o}{r}\right)}\\
\mathrm{e}^{2\varphi(r)} &= \left(1-\frac{2M}{r}-\frac{4}{3}\frac{Ml^2}{r^3}\right), & \mathrm{e}^{2\lambda(r)} &= \left(1-\frac{2M}{r}-\frac{2Ml^2}{r^3}\right)^{-1}.
\end{aligned}$$

It is noticed that all these three metrics reduce to the Schwarzschild solution in the appropriate limits which are respectively

$$\epsilon \to 0, \qquad r_0 \to 3m/2, \qquad l \to 0$$

where l is the curvature lenght. After the usual complexification and the introduction of Boyer-Lindquist coordinates, the resulting metric is

$$\begin{aligned}
\mathrm{d}s^2 = &-\mathrm{e}^2\varphi\mathrm{d}t^2 - 2a\sin^2\vartheta\mathrm{e}^{\varphi}(\mathrm{e}^{\lambda}-\mathrm{e}^{\varphi})\mathrm{d}t\mathrm{d}\psi + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\mathrm{d}\vartheta^2\\
&+\sin^2\vartheta[\Sigma + a^2\sin^2\vartheta\mathrm{e}^{\varphi}(2\mathrm{e}^{\lambda}-\mathrm{e}^{\varphi})]\mathrm{d}\psi^2
\end{aligned}$$

where the exponential functions become

$$\mathrm{e}^{\varphi(r)} = \left[(1+\epsilon)\sqrt{1-\frac{2Mr}{\Sigma}} - \epsilon\right], \qquad \mathrm{e}^{\lambda(r)} = \left(1-\frac{2Mr}{\Sigma}\right)^{-1/2}$$

$$\mathrm{e}^{\varphi(r)} = \left(1-\frac{2Mr}{\Sigma}\right)^{1/2}, \qquad \mathrm{e}^{\lambda(r)} = \frac{\left(1-\frac{3Mr}{2\Sigma}\right)^{1/2}}{\left(1-\frac{2Mr}{\Sigma}\right)^{1/2}\left(1-\frac{r_o r}{\Sigma}\right)^{1/2}}$$

$$\mathrm{e}^{\varphi(r)} = \left(1-\frac{2Mr}{\Sigma}-\frac{4}{3}\frac{Ml^2 r}{\Sigma^2}\right)^{1/2}, \qquad \mathrm{e}^{\lambda(r)} = \left(1-\frac{2Mr}{\Sigma}-\frac{2Ml^2 r}{\Sigma^2}\right)^{-1/2}$$

and

$$\Sigma = r^2 + a^2\cos^2\vartheta, \qquad \Delta = \Sigma\mathrm{e}^{-2\lambda} + a^2\sin^2\vartheta.$$

It is straightforward to see that all three metrics obtained do not satisfy the condition to be a valid braneworld solution, i.e., $R = 0$, so that it appears that the NJA in this form is not useful to get more general rotating Braneworld Black Holes solutions from the static, spherically symmetric ones, even though it partially works in order to generate the metric for a rotating source on the brane and for the tidal Kerr-Newman black hole.

6.2. The NJA in Born Infeld Monopole

The Born-Infeld theory is a non linear generalization of Maxwell electrodynamics, considered as the only *completely exceptional* regular non linear electrodynamics. With the advent of new developments of the string and brane theories, the BI electrodynamics has undergone a revival interest.

The application of the NJA to the static spherically symmetric metric of a Born-Infeld monopole, firstly investigated by Hoffmann [12], is discussed in [4]. The aim is to determine if the metric obtained via NJA coincides with the metric obtained from the Born-Infeld monopole with rotation. By following the original steps given by Newman and Janis, the starting metric in Eddington-Finkelstein coordinates is

$$\mathrm{d}s^2 = -\left(\frac{\Delta}{r^2}\right)\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + r^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2)$$

where

$$u = t - r - f(r), \qquad \Delta = r^2 - 2GMr + Q^2(r)$$

and $Q^2(r)$ is a complicated function of the Born-Infeld radius. Performing the usual complex transformation, the final result, in the suitable Boyer-Lindquist coordinates, is

$$g_{\mu\nu} = \begin{pmatrix} \frac{a\sin^2\vartheta - \Delta}{\rho^2} & 0 & 0 & \frac{a\sin^2\vartheta[\Delta-(r^2+a^2)]}{\rho^2} \\ 0 & \frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ \frac{a\sin^2\vartheta[\Delta-(r^2+a^2)]}{\rho^2} & 0 & 0 & -\left[\frac{sin^2\vartheta[(r^2+a^2)^2-\Delta a^2\sin^2\vartheta]}{\rho^2}\right] \end{pmatrix}$$

where $\Delta \simeq r^2 - 2GMr + Q^2(r) + a^2$.

This metric corresponds to the Kerr metric in Boyer-Lindquist coordinates when $Q(r) = 0$ and to the static Born-Infeld monopole when $a = 0$. It results that, even though this new metric reproduce the behavior of the metric for a rotating spherical charged source, it cannot be associated with the source of the rotating Born-Infeld monopole. This aspect is deeply analyzed in [4], where this problem is pointed out comparing the structure of the energy-momentum tensors (considered on the same basis vectors) for both metrics. From this study it comes out that the interpretation given by Newmann and Janis to the complex coordinates transformations works only for a linear theory.

7. Conclusion

In this paper we have summarized the principal results obtained through the application of the NJA. We have taken into account two class of theories in which the method is successful: the f(R)-theories and the Einstein-Maxwell-Dilaton gravity. Is is worth noting some interesting aspect which arise from the application of method in these theories. In f(R)-theories it should be verified if the resulting metric is a solution of Einstein modified equations. As pointed out in Section 5, in the truncated theory without axion field the NJA does not generate the expected rotating solution.

Then we have considered two classes of theories in which the NJA does not return the expected results: the Braneworld scenario and the Born-Infeld theory. In both cases it is still unclear why this method fails and it seems useful to perform a modification of the algorithm in order to extend its application to other theories.

Acknowledgements

Partially supported by Agenzia Spaziale Italiana (ASI) through the WP5000 project, by Italian Ministero Istruzione Università e Ricerca (MIUR) through the PRIN2008 grant and by the INFN-MICINN collaboration.

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