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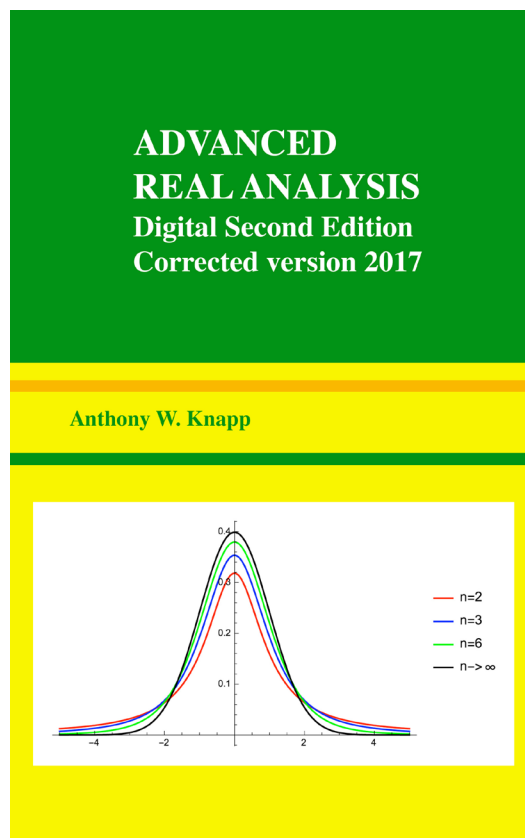
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Anthony W. Knapp

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Anthony W. Knapp
81 Upper Sheep Pasture Road
East Setauket, N.Y. 11733–1729, U.S.A.
Email to: aknapp@math.stonybrook.edu
Homepage: www.math.stonybrook.edu/~aknapp

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INDEX OF NOTATION

See also the list of Notation and Terminology on pages xix–xxii. In the list below, items are alphabetized according to their key symbols. For letters the order is lower case, italic upper case, Roman upper case, script upper case, blackboard bold, and Gothic. Next come items whose key symbol is Greek, and then come items whose key symbol is a nonletter. The last of these are grouped by type.

- | | |
|--|--|
| \widehat{a} , 153
\mathcal{A}_m^* , 153
$\mathbb{A}_{\mathbb{Q}}$, 271
$(\cdot)^c$, 266
c_k , 96
$C^\infty(E, \mathbb{C})$, $C^\infty(E, \mathbb{R})$, 324
$C_{\text{com}}^\infty(U)$, 131
$C_{\text{com}}^+(G)$, 226
C_K^∞ , 180
$\mathcal{C}_p(M)$, 327
\mathcal{C}_p^0 , \mathcal{C}_p^1 , 345
d , 368
$d^{(\alpha)}$, 250
dx , 237
$d_l x$, $d_r x$, 230
$(dF)_p$, dF_p , $dF(p)$, 330
D_j , 55
D^α , 55
$P(D)$, 284
$P(x, D)$, 185
$Q(D)$, 55
$\mathcal{D}'(M)$, 352
$\mathcal{D}'(U)$, 180
E_τ , 260
$E(x)$, 379
$\mathcal{E}'(M)$, 352
$\mathcal{E}'(U)$, 114
F_x , 380 | \mathcal{A}_F , P_F , 393
\mathcal{F} , 436
gf , fg , 222
gx , xg , 222
$g\mu$, μg , 222
$g_{\kappa'/\kappa}(\cdot)$, 338
$g_\kappa(x)$, 351
G^{tr} , 355
G_κ , 362
G/H , 214
$GL(N, \mathbb{C})$, 213
$GL(N, \mathbb{F})$, 338
$GL(N, \mathbb{R})$, 213, 223, 371
$GL_{\mathbb{C}}(V)$, 241
\mathcal{G} , 310, 357, 361
H^s , 63, 100
H_K^s , 366
$H_{\text{com}}^s(M)$, $H_{\text{loc}}^s(M)$, 366
aH , 214
G/H , 214
$H_n(x)$, 32
$H(f, \varphi)$, 226
$H^p(\mathbb{R}_+^2)$, 101
\mathcal{H} , 236
$\mathcal{H}^p(\mathbb{R}_+^{N+1})$, 81
J_m , 242, 490
$J_n(r)$, 12
l , 256 |
|--|--|

- ℓ_φ , 227
 L , 501
 $L(u)$, 19
 $L^*(v)$, 20
 $L^p([0, 1])$, $0 < p < 1$, 109
 $L^p(G)$, 237
 $L_k^2(T^N)$, 103
 $L_k^p(U)$, 63
 $L_{\text{com}}^1(M, \mu_g)$, $L_{\text{loc}}^1(M, \mu_g)$, 352
 $LAP(G)$, 272
 \mathcal{L} , 500
 m_0 , 455
 m_1 , 456
 M_κ , \tilde{M}_κ , 322
 $\mathcal{M}(L^2(S, \mu))$, 160
 $N(\mu, \sigma^2)$, 413
 $O(N)$, 218
 $p'(x, dy)$, 392
 P , 377
 $P(D)$, 284
 $P(x, D)$, 185
 $P(x, t)$, $P_t(x)$, 80
 $P(x, 2\pi i\xi)$, 287
 P_j , 470
 $P_m(x, 2\pi i\xi)$, 287
 $P_m(t)$, 16
 $P_r(\theta)$, 15
 P_N , 501
 $Q(D)$, 55
 \mathbb{Q}_p , 270
 r , 257
 $r(a)$, 150
 $R(\lambda)$, 150
 \mathbb{R}_+^{N+1} , 80
 $s(\omega)$, 420
 $\text{sgn } \sigma$, 242
 $\text{sing supp } u$, 303
 $\text{support}(\cdot)$, 192
 $S_{1,0}^m(U)$, $S_{1,0}^{-\infty}(U)$, 307
 $S_{1,0,0}^m(U \times U)$, $S_{1,0,0}^{-\infty}(U \times U)$, 356
 $SL(2, \mathbb{R})$, 236, 268
 $SL(N, \mathbb{F})$, 218
 $SO(N)$, 218, 264
 $Sp(N, \mathbb{F})$, 218
 $SU(2)$, 268
 $\mathcal{S}'(\mathbb{R}^N)$, 59
 \mathfrak{S}_n , 242
 $t(\omega)$, 420
 $(\cdot)^{\text{tr}}$, 188
 T^N , 102
 T_f , 187, 352
 $T(M)$, 331, 344
 $T(M, \mathbb{R})$, $T(M, \mathbb{C})$, 344
 $T^*(M)$, 344, 345
 $T^*(M, \mathbb{R})$, 355
 $T^*(M, \mathbb{R})^\times$, 364
 $T_p(M)$, 328
 $T_p^*(M)$, 345
 $\text{Tr } L$, 51, 249
 u_{xt} , 2
 aU , Ub , 213
 UV , U^{-1} , 213
 $U(N)$, 218
 V_0 , 454
 $\{V_j\}_{j=-\infty}^\infty$, 454
 $\bar{x}(\omega)$, 420
 $x_j(q)$, 323
 dx , 237
 φ_x , 190
 \mathbb{Z}^N , 292
 \mathbb{Z}_p , 270
Greek
 $|\alpha|$, 55
 $\alpha!$, 55
 D^α , 55
 ${}^1\gamma$, 477
 ${}^m\gamma$, 488
 ${}^m\gamma_{0,k}$, 482
 $\Gamma(x)$, 420
 δ , 206
 δg , 489

Δ , 2, 276, 480
 $\Delta(g)$, 230
 Δ_{2m} , 493
 $\Delta_G(t)$, 232
 εh , 489
 $\kappa : M_\kappa \rightarrow \tilde{M}_\kappa$, 322
 $d\mu(gH)$, 235
 μ_g , 351
 μ_x , 379
 μ_{x_1, \dots, x_N} , 381
 ν , 472
 π , 338
 $[-\pi, \pi]^N$, 292
 $\sigma(a)$, 150
 $\sigma_L(p, \xi)$, 355
 $\sigma_\kappa(x, \xi)$, 364
 φ , 454
 $\varphi_{j,k}$, 454
 φ_x , 190, 409
 ϕ_κ , 338
 Φ , 413
 f^Φ , 221
 μ_Φ , 221
 $\Phi(f)$, 257
 $\chi^2(k)$, 424
 ψ , 455, 456
 Ω , 377

Arrows

$\xrightarrow{\text{a.s.}}$, 403
 $\xrightarrow{\mathcal{P}}$, 403
 $\xrightarrow{\mathcal{L}}$, 403

Norms

$\|\cdot\|_{\text{HS}}$, 47
 $\|\cdot\|_{\mathcal{H}_p}$, 81
 $\|\cdot\|_{H^s}$, 100
 $\|\cdot\|_{K,p}$, 113
 $\|\cdot\|_{L_k^p}$, 63
 $\|\cdot\|_{L_k^2(T^N)}$, 103
 $|\cdot|_p$, 269
 $\|\cdot\|_{p,\alpha}$, 180
 $\|\cdot\|_{P,Q}$, 56
 $\|\cdot\|_{\alpha,\beta}$, 180

Other unary operations

∂U , 61
 \hat{a} , 153
 f^\vee , T^\vee , 189
 $f^\#$, $f^{\#\#}$, 234

Binary operations

$\langle T, \varphi \rangle$, 59, 181
 $K_1 - K_2$, xix, 129
 $f * h$, 237
 \oplus , 247
 $U \sqcup V$, $\bigsqcup_i W_i$, 335
 \sim , 357
 $\varphi_1 \otimes \varphi_2$, 361

Other symbols

1 , 214
 $\bar{\partial}$, 369
 $\frac{\partial}{\partial \bar{z}}$, 276
 $\left[\frac{\partial}{\partial x_j}\right]_p$, 328

INDEX

- about a point, 322
- absolutely uniform convergence, 6
- adele, 271
- adjoint, formal, 20
- Alaoglu's Theorem, **120**, 146
 - preliminary form, 81, 82
- algebra
 - associative, 121
 - Banach, 122, 146
 - commuting, 161
 - multiplication, 160
- almost periodic, **272**, 273
- amplitude, 356
- Approximation Theorem, **255**, 267, 274
- Artin product formula, 270
- Ascoli's Theorem, 145
- associated Legendre equation, 16
- associative algebra, 121
- Atiyah–Singer Index Theorem, 369
- atlas, 322
 - B*-spline, 478, **493**
 - Baire Category Theorem, 114, 264
 - Baire set, xxvi
 - Banach algebra, **122**, 146
 - band-limited, 440
 - base space, 338
 - basic separation theorem, 127
 - Battle–Lemarié degree, 483, **495**
 - Battle–Lemarié scaling function, 495
 - Battle–Lemarié wavelet, 483, **495**
 - Beltrami equation, 94
 - Bernstein polynomial, 430
 - Bessel function, 12, 31
 - Bessel potential, **100**, 543
 - Bessel's equation, 12
 - binomial distribution, 432
 - biorthogonal wavelet, 536
 - blocking, 442
 - Bohr compactification, 272
 - Borel set, xxvi
 - Borel–Cantelli Lemma, 398
 - boundary data, 2
 - boundary of open set, 61
 - boundary values, 2
 - boundary-value problem, 2, 277
 - bounding point, 126
 - Brouwer Fixed-point Theorem, 145
 - Brownian motion, **391**, 431
 - bundle
 - coordinate vector, 338
 - cotangent, 344
 - space, 338
 - tangent, 344
 - vector, 341
 - C^* algebra, 157
 - Calderón–Zygmund decomposition, 86
 - Calderón–Zygmund Theorem, **83**, 102, 370
 - Cantor diagonal process, 225
 - Cantor measure, 431
 - Cauchy data, 279, 280
 - Cauchy problem, **284**, 320
 - Cauchy–Kovalevskaya Theorem, **278**, 279, 281, 282, 318, 320
 - Cauchy–Peano Existence Theorem, 145
 - Cauchy–Riemann equations, 92, 276
 - Cauchy–Riemann operator, **287**, 318, 349
 - Central Limit Theorem, 413
 - chain rule, 331
 - character, 250
 - group, 250
 - multiplicative, 242
 - sign, 242
 - characteristic, 283
 - characteristic function, 409
 - chart, 322
 - atlas of, 322

- compatible, 322
- Chebyshev's inequality, 397
- chi-square distribution, 424
- circled set, 132
- closed convex hull, 140
- closed subgroup, 217
- Cohen–Daubechies–Feauveau family, 536
- coin tossing, 376
- commuting algebra, 161
- compact group, 217
- compact operator, 34
- compact ring, 270
- compactification
 - Bohr, 272
 - one-point, 124
 - Stone–Čech, 125
- compatible chart, 322
- completely continuous, 34
- complex tangent bundle, 344
- compression, 442, 535
- conditional probability, 383
- cone condition, 67
- constant coefficients, 279, 282, 292, 300
- continuity
 - complete, 34
 - left uniform, 273
 - strong, 256
 - uniform, 219
 - weak, 267
- continuous dual, 114
- contragredient representation, **245**, 266
- convergence
 - absolutely uniform, 6
 - almost surely, 403
 - in distribution, 404
 - in law, 404
 - in probability, 398, **403**
 - uniform on compact sets, 75
 - with probability 1, 403
- convex set, 125
- convolution
 - of distributions, 192, 195
 - of functions, 186, 237
 - of measures, 189
- coordinate function, 338
- coordinate transformation, 338
- coordinate vector bundle, 338
 - equivalence, 373
- cotangent bundle, 344
- countable, xxiii
- critical point, 372
- critical value, 372
- cumulative distribution function, 38
- curve, 333
 - integral, 333
- cyclic vector, 162
- Daubechies scaling function, **513**, 544
- Daubechies wavelet, 498, **513**, 544
 - order, 498, 513, **516**
- de Moivre and Laplace, 432
- de Rham's Theorem, 368
- decomposition algorithm, 533
- degree of B -spline, 483
- degree of Battle–Lemarié wavelet, 483, **495**
- degree of homogeneity, **xxiv**, 83, 355
- degree of representation, 250
- degree of spline, 476
- degrees of freedom, 420, 424
- density, 381
- derivation, 328
- derivative
 - transverse, 209
 - weak, **62**, 103, 290
- diffeomorphism, 326
- differential, 330
- differential 1-form, 348
 - smooth, 348
- differential operator, linear, 19, **353**
 - transpose of, 20, **353**
- differentiation of distribution, 188
- dimension of smooth manifold, 322
- Dirac distribution, 201
- Dirac operator, 369
- direct limit, **139**, 177
- direct sum, 247
- directed system, 177
- Dirichlet problem, **13**, 288
- discrete cosine transform, 537
- discrete group, 213
- discrete wavelet transform, 529
- disjoint union, 335
- distribution, **179**, 290, 352
 - arbitrary, 180
 - binomial, 432
 - chi-square, 424
 - convolution with, 192, 195
 - differentiation of, 188

- Dirac, 201
- equal to a locally integrable function, 183
- equal to a smooth function, 183
- Fourier series of, 209
- Fourier transform of, **60**, 202, 203
- gamma, 423
- given by function, 187
- Gosset's t , 420
- Heaviside, 318
- kernel, **310**, 357, 361
- localization of, 186
- normal, 413
- of compact support, **114**, 352
- of random variable, 379
- operation on, 187
- periodic, 209
- Poisson, 432
- probability, 379
- product with smooth function, 188
- Student's t , 420
- support of, 181
- supported at $\{0\}$, 208
- supported on vector subspace, 208
- tempered, 58
- distribution function, 380
- Divergence Theorem, 70

- eigenfunction, 19
- eigenspace, 36
- eigenvalue, 19, 36
- eigenvector, 36
- elliptic
 - differential equation, 288
 - operator, index, 368
 - pseudodifferential operator, **315**, 366
- equal to a function, distribution, 183
- equivalent coordinate vector bundles, 373
- equivalent representations, **244**, 259
 - unitarily, 266
- equivalent vector bundles, 373
- ergodic measure, **143**, 176
- essential image, 166
- event, 376, **378**
- exhausting sequence, **64**, 113, 350
- expectation, 379
- expected value, 379
- experiment, 418
- exponential type, 509
- extreme point, **140**, 175

- F. and M. Riesz Theorem, 102
- face, 140
- Fatou's Theorem, 81
- Fejér and F. Riesz Lemma, 504
- filter, 450, 498
- finite-dimensional representation, 241
- finite-dimensional topological vector space, 111
- formal adjoint, 20
- formally self adjoint, 20
- Fourier inversion formula for compact group, 267
- Fourier series
 - multiple, **96**, 98, 192
 - of distribution, 209
 - use in local solvability, 292
 - use in separation of variables, 5
- Fourier transform, 436
 - of distribution of compact support, 203
 - of tempered distribution, **60**, 202
- Franklin wavelet, 483
- Fréchet limit, 139
- Fréchet space, 139
- freezing principle, 285
- functional calculus, 167
- fundamental solution, 206, 290, 300, 302, 318

- gamma distribution, 423
- gamma function, 420
- Gårding's inequality, 286
- Gelfand transform, 153
- Gelfand–Mazur Theorem, 151
- general linear group, 213, 371
- generalized pseudodifferential operator, 356
 - transpose of, 356
- generating function, 455, 456
- germ, 327
- Gibbs phenomenon, 438
- Gosset, 418
- Gosset's t distribution, 420
- Green's formula, 20, 31, 72, 73, 206
- Green's function, **25**, 290
- group action, 222
- group character, 250

- Haar covering function, 226
- Haar measure, **223**, 232
- Haar multiresolution analysis, 454

- Haar scaling function, 444
- Haar series expansion
 - one-sided, 450
 - two-sided, 449
- Haar system, **443**, 444, 541
- Haar wavelet, 444
- Hahn–Banach Theorem, 126
- half space
 - Poisson integral formula, 80
 - Poisson kernel, 80
- Hardy space, 101
- harmonic, 2, **69**
- harmonic measure, 317
- harmonic polynomial, **244**, 263
- heat equation, 2, **6**
- heat operator, 287
- Heaviside distribution, 318
- Heaviside function, 318
- Helmert matrix, 428
- Hermite polynomial, 32
- Hermite’s equation, 32
- Hermitian matrix, 36
- high-pass filter, 450
- Hilbert transform, **xxvi**, 83, 92, 101, 211
- Hilbert–Schmidt norm, 47
- Hilbert–Schmidt operator, 47
- Hilbert–Schmidt Theorem, 22, 25, **42**, 43
- Hirzebruch–Riemann–Roch Theorem, 369
- Hodge theory, 368
- Hölder condition, 523
- holomorphic polynomial, 243
- homogeneous function, **xxiv**, 83, 355
- homogeneous partial differential equation,
 - 1, 276
- homogeneous space, 214
- homomorphism of topological groups, 213
- Hopf maximum principle, 297
- hyperbolic, 289

- idele, 272
- identically distributed, 381
- identification space, 335
- identity component, 264
- identity element, 214
- independent events, 384
- independent random variables, 385
- index
 - Meyer, 472
 - of elliptic operator, 368
- indicator function, xxv
- inductive limit topology, **139**, 174
- initial data, 2
- initial-value problem, 277
- integrable locally, 62
- integral curve, 333
- integral operator, 41
 - trace of, 98
- Interior Mapping Theorem, 173
- internal point, 126
- invariant subspace, 243, **259**
- inverse, 149
- invertible, 149
- involution, 157
- irreducible representation, 245, **259**
- isomorphism of topological groups, 213
- isomorphism of topological vector spaces, 106
- isotypic subspace, 261

- joint probability density, 382
- joint probability distribution, 381
- JPEG, 442, **536**
- JPEG 2000, 539

- kernel
 - distribution, **310**, 357, 361
 - of integral operator, 41
 - Poisson, 15, 80
- knot of spline, 477
- Kolmogorov Extension Theorem, **394**, 430
- Kolmogorov’s inequality, 399
- Krein–Milman Theorem, 140

- Laplace equation, 2, 13
- Laplacian, **2**, 206, 276, 287
- Law of Large Numbers
 - Strong, 398
 - Weak, 398
- left almost periodic, 272
- left coset, 214
- left Haar measure, 223
- left inverse, 149
- left parametrix, 310
- left uniform continuity, 273
- left-invariant vector field, 371
- left-regular representation, 256
- Legendre equation
 - associated, 16
 - ordinary, 16

- Legendre polynomial, **16**, 31
- Leibniz rule, 63
- Lévy Continuity Theorem, 411
- Lévy example, **286**, 349
- LF topology, 139
- Lie group, 349
- line segment, 140
- linear
 - differential operator, 19, **353**
 - functional, multiplicative, **122**, 148, 152
 - homogeneous partial differential equation, 276
 - operator (*see* operator)
 - partial differential equation, 1, 276
 - partial differential operator, **185**, 188
 - topological space, 106
- Liouville, 78
- local coordinate system, **323**, 372
- local neighborhood base, 132
- local solvability, 292
- localization of distribution, 186
- locally compact abelian group, 270
- locally compact field, 270
- locally compact group, 217
- locally compact ring, 271
- locally compact topological vector space, **111**, 265
- locally convex, 128
- locally integrable, 62
- low-pass filter, **450**, 498

- magnification, 449
- manifold, 322
 - Riemannian, 349
 - smooth, 322
- mapping function, 373
- Marcinkiewicz Interpolation Theorem, 95
- margin of error, 417
- Markov–Kakutani Theorem, 143
- matrix
 - Hermitian, 36
 - orthogonal, **218**, 426
 - rotation, 218
 - trace of, 249
 - unitary, 37, **218**
- matrix coefficient, 250
- matrix representation, 241
- maximal abelian self-adjoint subalgebra, 161
- maximal ideal, 152

- maximum principle, 297
- mean, 379
 - sample, 420
- mean-value property, **72**, 73
- measurable set of type F , 393
- measure
 - Cantor, 431
 - harmonic, 317
 - smooth, 351
- measure 0, 372
- metric, Riemannian, 349
- Meyer index, 472
- Meyer multiresolution analysis, 472
- Meyer scaling function, 473
- Meyer system, **473**, 542
- Meyer wavelet, **474**, 542
- modular function, 230
- modulus of ellipticity, 296
- Monotone Class Lemma, 170
- multi-index, 55
- multiple Fourier series, **96**, 98, 102
 - of distribution, 209
 - use in local solvability, 292
- multiplication algebra, 160
- multiplication of smooth function
 - and distribution, 188
- multiplicative character, 242
- multiplicative linear functional, **122**, 148, 152
- multiplicity, 251
- multiresolution analysis, 454
 - Daubechies system, 512
 - Haar system, 454
 - Meyer, 472
 - Shannon, 466
- negative, xxiii
- noncharacteristic, 283
- norm, Hilbert–Schmidt, 47
- normal distribution, 413
- normal operator, 52, **165**
- normed linear space, 107

- one-dimensional wavelet, 435
- one-point compactification, 124
- one-sided wavelet expansion, 529
- operation on distribution, 187
 - by -1 in domain, 189
 - by convolution, 192, 195
 - by linear partial differential operator, 188

- of differentiation, 188
 - of Fourier transform, 202
 - of multiplication, 188
 - of transpose, 187
- operator
 - compact, 34
 - completely continuous, 34
 - differential, linear, 353
 - Dirac, 369
 - elliptic, 288
 - elliptic pseudodifferential, **315**, 366
 - generalized pseudodifferential, 356
 - Hilbert–Schmidt, 47
 - hyperbolic, 289
 - integral, 41
 - normal, 52, **165**
 - orthogonal, 45
 - positive semidefinite, 165
 - pseudodifferential, 306, **308**, 362
 - smoothing, 291, 301, 308, 357
 - trace of, **51**, 98
 - trace-class, 49
 - transpose of, 185
 - unitary, **45**, 163, 165
- order of Daubechies wavelet, 498, 513, **516**
- order of differential equation, 276
- order of differential operator, **185**, 353
- orthogonal group, 218
- orthogonal matrix, **218**, 426
- orthogonal operator, 45
- orthogonal wavelet, 435
- orthonormalization trick, 483
- p -adic integer, 270
- p -adic norm, 269
- Paley–Wiener Theorem, 512, 517, **518**
- parametrix, 292, **301**, 307, 310, 315
- partial differential equation, 276
 - elliptic, 288
 - homogeneous, 1, 276
 - hyperbolic, 289
 - linear, 1, 276
 - linear homogeneous, 276
 - order, 276
 - system, 276
- partial differential operator
 - elliptic, 288
 - hyperbolic, 289
 - linear **185**, 188
 - transpose of, **185**, 353
- partition of unity, **65**, 113, 174, 351, 371
- periodic distribution, 209
- Peter–Weyl Theorem, 252
- Phragmén–Lindelöf Theorem, 517
- Picard–Lindelöf Existence Theorem, 145
- Plancherel formula for compact group, **254**, 257
- Poisson distribution, 432
- Poisson integral formula for half space, 80
- Poisson integral formula for unit disk, 15
- Poisson kernel for half space, 80
- Poisson kernel for unit disk, 15
- Poisson’s equation, 291
- polynomial
 - Bernstein, 430
 - harmonic, **244**, 263
 - holomorphic, 243
 - trigonometric, 254, 499
- Portmanteau Lemma, 406
- positive, xxiii
- positive definite function, **142**, 176
- positive semidefinite operator, 165
- Principal Axis Theorem, 289
- principal symbol, **287**, 355, 364
- probability, 376, **378**
 - conditional, 383
 - joint, 382
- probability distribution, 379
 - joint, 381
- probability model, 376
- probability space, 378
- product of topological groups, 213
- projection, 338
- projective space, 371
- properly supported, 313, 357, 361
- pseudodifferential operator, 306, **308**, 362
 - elliptic, **315**, 366
 - generalized, 356
 - transpose of, 308, **356**
- pseudolocal, **311**, 357, 362
- pseudonorm, 56
- pull back, 221
- push forward, 221
- quotient of topological vector space, 110
- quotient space, 214
 - of Lie group, 349
- Radon–Nikodym Theorem, 224

- random number, 378
- random variable, 378
- rank, 338
- real analytic, 528
- real tangent bundle, 344
- reconstruction algorithm, 534
- regularizing, 61
- Rellich's Lemma, 368
- representation
 - contragredient, **245**, 266
 - finite-dimensional, 241
 - irreducible, 245, **259**
 - left-regular, 256
 - matrix, 241
 - right-regular, 257
 - standard, 242
 - trivial, 242
 - unitary, 245, 256
- resolution, 449
- resolvent, 150
- resolvent set, 150
- restricted direct product, **178**, 271
- Riemann–Roch Theorem, 369
- Riemannian manifold, 349
- Riemannian metric, 349
- Riesz Convexity Theorem, 95
- Riesz Representation Theorem, 118, 164, 220
- Riesz transform, 93
- right almost periodic, 272
- right group action, 223
- right Haar measure, 223
- right inverse, 149
- right parametrix, 307
- right-regular representation, 257
- rotation group, 218
- rotation matrix, 218

- sample, 418
- sample mean, 420
- sample variance, 420
- sampling, 439
- Sard's Theorem, 372
- satisfy cone condition, 67
- scaling equation, 455
- scaling filter, 498
- scaling function, 454
 - Battle-Lemarié, 495
 - Daubechies, **513**, 544
 - Haar, 444
 - Meyer, 473
 - Shannon, **465**, 510
- Schauder–Tychonoff Theorem, 144
- Schrödinger's equation, 32
- Schur orthogonality, 249
- Schur's Lemma, 247
- Schwartz Kernel Theorem, 209, **361**
- Schwartz space, 55
- Schwarz Reflection Principle, 78
- section, 347
 - smooth, 347
- self adjoint, formally, 20
- self-adjoint subalgebra, 161
- seminorm, 56
- separable, xxv
- separating family of seminorms, 57, **107**
- separation of variables, 1, **3**
- Shannon multiresolution analysis, 466
- Shannon Sampling Theorem, **440**, 535
- Shannon scaling function, **465**, 510
- Shannon system, 465
- Shannon wavelet, **466**, 468
- sign character, 242
- signal, 435
- singular support, **183**, 303, 362
- smooth curve, 333
- smooth differential 1-form, 348
- smooth function, 323, **326**
- smooth manifold, 322
 - dimension, 322
- smooth measure, 351
- smooth section, 347
- smooth structure, 322
- smooth vector field, 331, 332, 348
- smoothing operator, 291, 301, 308, 357
- Sobolev space, **63**, 100, 103, 290, 308, 366
- Sobolev's Theorem, 67, 69, 100, 103, 368
- space-boundary data, 2
- special linear group, 218
- Spectral Mapping Theorem, 155
- spectral radius, 150
- spectral radius formula, 155
- Spectral Theorem
 - finite-dimensional, 37
 - for bounded normal operator, 166
 - for bounded self-adjoint operator, 165
 - for compact self-adjoint operator, 39
 - for unbounded self-adjoint operator, 172
- spectrum, 150, 167

- spline, 476
 - of compact support, 477
- standard representation, 242
- standard symbol, 307
- statistical inference, 417
- Stieltjes integral, 380
- stochastic process, 393
- Stone Representation Theorem, **121**, 147, 176
- Stone–Čech compactification, 125
- Stone–Weierstrass Theorem, 31, 124, 169, 263
- strict equivalence, 341
- strict inductive limit topology, 139
- strong continuity, 256
- Strong Law of Large Numbers, 398
- Student’s t distribution, 420
- Sturm–Liouville eigenvalue problem, 20
- Sturm–Liouville theory, 5, **19**, 172
- Sturm’s Theorem, 5, 21
- superposition principle, 1
- support function of convex set, 126
- support of distribution, 115, **181**, 352
 - singular, 183, 303, 362
- support of function, **xxv**, 324
- supported properly, 313, 357, 361
- symbol, **287**, 306
 - principal, **287**, 355, 364
 - standard, 307
- symplectic group, 218
- system of partial differential equations, 276

- t distribution, 420
- tangent bundle, **332**, 344
- tangent space, 328
- tangent vector, 328
- tempered distribution, 58
- Theorem of de Moivre and Laplace, 432
- topological field, 269
- topological group, 213
 - isomorphism for, 213
- topological ring, 270
- topological vector space, 106
 - defined by seminorms, 107
 - finite-dimensional, 111
 - isomorphism for, 106
 - locally compact, **111**, 265
 - locally convex, 128
 - quotient of, 110
- trace of integral operator, 98
- trace of linear map, 249
- trace of matrix, 249
- trace of operator, 51
- trace-class operator, 49
- transition function, 338
- transition matrix, 338
- translate, 190
- transpose, 187, 352
 - of generalized pseudodifferential operator, 356
 - of operator, 185
 - of ordinary differential operator, 20
 - of partial differential operator, **185**, 353
 - of pseudodifferential operator, 308, **356**
- transverse derivative, 209
- trigonometric polynomial, 254, 499
- trivial representation, 242
- two-sided parametrix, 315
- Tychonoff Product Theorem, 120, 225

- ultrametric inequality, 269
- Uncertainty Principle, 436
- uniform continuity, 219
 - left, 273
- uniform convergence on compact sets, 75
- unimodular group, 232
- unit disk
 - Poisson integral formula, 15
 - Poisson kernel, 15
- unit sphere, 370
- unitarily equivalent, 266
- unitary group, 218
- unitary matrix, 37, **218**
- unitary operator, **45**, 163, 165
- unitary representation, 245, 256
- Urysohn Metrization Theorem, 323

- Vandermonde convolution, 542
- variance, 397
 - sample, 420
- vector bundle, 341
 - coordinate, 338
 - equivalence, 373
- vector field, 331, 348
 - left-invariant, 371
 - smooth, 331, 332, 348
- vibrating drum, 18
- vibrating string, 17

- wave operator, 287

- wave equation, 3, **17**
- wavelet, **435**, 455
 - Battle–Lemarié, 483, **495**
 - biorthogonal, 536
 - Cohen–Daubechies–Feauveau, 536
 - Daubechies, 498, **513**, 544
 - Franklin, 483
 - Haar, 444
 - Meyer, **474**, 542
 - one-dimensional, 435
 - orthogonal, 435
 - Shannon, **466**, 468
- wavelet equation, 456
 - one-sided, 529
- wavelet expansion, 435
- weak continuity, 267
- weak derivative, **62**, 103, 290
- Weak Law of Large Numbers, 398
- weak topology on normed linear space, 108,
116
- weak-star topology on dual of normed linear
space, 109, **116**
- weakly analytic, 150
- Weierstrass Approximation Theorem, 430
- Weyl integration formula, 269
- Wiener Covering Lemma, 87

