

Just as the product of several equalities is reduced to one single equality, the sum (the alternative) of several inequalities may be reduced to a single inequality. But neither several alternative equalities nor several simultaneous inequalities can be reduced to one.

55. System of an Equation and an Inequation.—We shall limit our study to the case of a simultaneous equality and inequality. For instance, let the two premises be

$$(ax + bx' = 0) (cx + dx' \neq 0).$$

To satisfy the former (the equation) its resultant $ab = 0$ must be verified. The solution of this equation is

$$x = a'x + bx'.$$

Substituting this expression (which is equivalent to the equation) in the inequation, the latter becomes

$$(a'c + ad)x + (bc + b'd)x' \neq 0.$$

Its resultant (the condition of its solvability) is

$$(a'c + ad + bc + b'd \neq 0) = [(a' + b)c + (a + b')d \neq 0],$$

which, taking into account the resultant of the equality,

$$(ab = 0) = (a' + b = a') = (a + b' = b')$$

may be reduced to

$$a'c + b'd \neq 0.$$

The same result may be reached by observing that the equality is equivalent to the two inclusions

$$(x < a') (x' < b'),$$

and by multiplying both members of each by the same term

$$(cx < a'c) (dx' < b'd) < (cx + dx' < a'c + b'd)$$

$$(cx + dx' \neq 0) < (a'c + b'd \neq 0).$$

This resultant implies the resultant of the inequality taken alone

$$c + d \neq 0,$$

so that we do not need to take the latter into account. It

is therefore sufficient to add to it the resultant of the equality to have the complete resultant of the proposed system

$$(ab = 0) (a'c + b'd \neq 0).$$

The solution of the transformed inequality (which consequently involves the solution of the equality) is

$$x \neq (a'c' + ad')x + (bc + b'd)x'.$$

56. Formulas Peculiar to the Calculus of Propositions.

—All the formulas which we have hitherto noted are valid alike for propositions and for concepts. We shall now establish a series of formulas which are valid only for propositions, because all of them are derived from an axiom peculiar to the calculus of propositions, which may be called the *principle of assertion*.

This axiom is as follows:

$$(Ax. X.) \quad (a = 1) = a.$$

P. I.: To say that a proposition a is true is to state the proposition itself. In other words, to state a proposition is to affirm the truth of that proposition.¹

Corollary:

$$a' = (a' = 1) = (a = 0).$$

P. I.: The negative of a proposition a is equivalent to the affirmation that this proposition is false.

By Ax. IX (§ 20), we already have

$$(a = 1) (a = 0) = 0,$$

“A proposition cannot be both true and false at the same time”, for

$$(Syll.) \quad (a = 1) (a = 0) < (1 = 0) = 0.$$

¹ We can see at once that this formula is not susceptible of a conceptual interpretation (C. I.); for, if a is a concept, $(a = 1)$ is a proposition, and we would then have a logical equality (identity) between a concept and a proposition, which is absurd.