In fact, we have the following formal implications:

$$(N + X = 0) < (N = 0) < (NX = 0),$$
$$(N'X' = 1) < (N' = 1) = (N' + X' = 1).$$

Applying the law of forms, the formula of the consequences becomes

$$U = (N' + X')\, U + NXU',$$

and the formula of the causes

$$U = N'X'\,U + (N + X)\, U';$$

or, more generally, since X and X' are indeterminate terms, and consequently are not necessarily the negatives of each other, the formula of the consequences will be

$$U = (N' + X)\, U + NYU',$$

and the formula of the causes

$$U = N'XU + (N + Y)\, U'.$$

The first denotes that U is contained in $(N' + X)$ and contains NY; which indeed results, *a fortiori*, from the hypothesis that U is contained in N' and contains N.

The second formula denotes that U is contained in $N'X$ and contains $N' + Y$ whence results, *a fortiori*, that U is contained in N' and contains N.

We can express this rule verbally if we agree to call every class contained in another a *sub-class*, and every class that contains another a *super-class*. We then say: To obtain all the consequences of an equality (put in the form $U = N'U + NU'$), it is sufficient to substitute for its logical whole N' all its super-classes, and, for its logical zero N, all its sub-classes. Conversely, to obtain all the causes of the same equality, it is sufficient to substitute for its logical whole all its sub-classes, and for its logical zero, all its super-classes.

47. Example: Venn's Problem.—*The members of the administrative council of a financial society are either bondholders or shareholders, but not both. Now, all the bond-*

holders form a part of the council. What conclusion must we draw?

Let a be the class of the members of the council; let b be the class of the bondholders and c that of the shareholders. The data of the problem may be expressed as follows:

$$a < bc' + b'c, \qquad b < a.$$

Reducing to a single developed equality,

$$a\,(bc = b'c') = 0, \qquad a'b = 0,$$

(1) $$abc + ab'c' + a'bc + a'bc' = 0.$$

This equality, which contains 4 of the constituents, is equivalent to the following, which contains the four others,

(2) $$abc' + ab'c + a'b'c + a'b'c' = 1.$$

This equality may be expressed in as many different forms as there are classes in the universe of the three terms a, b, c.

Ex. 1. $$a = abc' + ab'c + a'bc + a'bc',$$

that is,

$$b < a < bc' + b'c,$$

Ex. 2. $$b = abc' + ab'c' = ac';$$

Ex. 3. $$c = ab'c + a'b'c + ab'c' + a'bc'$$

that is,

$$ab' + a'b < c < b'.$$

These are the solutions obtained by solving equation (1) with respect to a, b, and c.

From equality (1) we can derive 16 consequences as follows:

1. $abc = 0;$
2. $(ab'c' = 0) = (a < b + c);$
3. $(a'bc = 0) = (bc < a);$
4. $(a'bc' = 0) = (b < a + c);$

5. $(abc + ab'c' = 0) = (a < bc' + b'c)$ [1st premise];

6. $(abc + a'bc = 0) = (bc = 0)$;

7. $(abc + a'bc' = 0) = (b < ac' + a'c)$;

8. $(ab'c' + a'bc = 0) = (bc < a < b + c)$;

9. $(ab'c' + a'bc' = 0) = (ab' + a'b < c)$;

10. $(a'bc + a'bc' = 0) = (a'b = 0)$ [2d premise];

11. $(abc + ab'c' + a'bc = 0) = (bc + ab'c' = 0)$;

12. $abc + ab'c' + a'bc' = 0$;

13. $(abc + a'bc + a'bc' = 0) = (bc + a'bc') = 0$;

14. $ab'c' + a'bc + a'bc' = 0$.

The last two consequences, as we know, are the identity $(0 = 0)$ and the equality (1) itself. Among the preceding consequences will be especially noted the 6th $(bc = 0)$, the resultant of the elimination of a, and the 10th $(a'b = 0)$, the resultant of the elimination of c. When b is eliminated the resultant is the identity

$$[(a' + c)\ ac' = 0] = (0 = 0).$$

Finally, we can deduce from the equality (1) or its equivalent (2) the following 16 causes:

1. $(abc' = 1) = (a = 1)\ (b = 1)\ (c = 0)$;

2. $(ab'c = 1) = (a = 1)\ (b = 0)\ (c = 1)$;

3. $(a'b'c = 1) = (a = 0)\ (b = 0)\ (c = 1)$;

4. $(a'b'c' = 1) = (a = 0)\ (b = 0)\ (c = 0)$;

5. $(abc' + ab'c = 1) = (a = 1)\ (b' = c)$;

6. $(abc' + a'b'c = 1) = (a = b = c')$;

7. $(abc' + a'b'c' = 1) = (c = 0)\ (a = b)$;

8. $(ab'c + a'b'c = 1) = (b = 0)\ (c = 1)$;

9. $(ab'c + a'b'c' = 1) = (b = 0)\ (a = c)$;

10. $(a'b'c + a'b'c' = 1) = (a = 0)\ (b = 0)$;

11. $(abc' + ab'c + a'b'c = 1) = (b = c')\ (c' < a);$

12. $(abc' + ab'c + a'b'c' = 1) = (bc = 0)\ (a = b + c);$

13. $(abc' + a'b'c + a'b'c' = 1) = (ac = 0)\ (a = b);$

14. $(ab'c + a'b'c + a'b'c' = 1) = (b = 0)\ (a < c).$

The last two causes, as we know, are the equality (1) itself and the absurdity $(1 = 0)$. It is evident that the cause independent of a is the 8th $(b = 0)\ (c = 1)$, and the cause independent of c is the 10th $(a = 0)\ (b = 0)$. There is no cause, properly speaking, independent of b. The most "natural" cause, the one which may be at once divined simply by the exercise of common sense, is the 12th:

$$(bc = 0)\ (a = b + c).$$

But other causes are just as possible; for instance the 9th $(b = 0)\ (a = c)$, the 7th $(c = 0)\ (a = b)$, or the 13th $(ac = 0)\ (a = b)$.

We see that this method furnishes the complete enumeration of all possible cases. In particular, it comprises, among the *forms* of an equality, the solutions deducible therefrom with respect to such and such an "unknown quantity", and, among the *consequences* of an equality, the resultants of the elimination of such and such a term.

48. The Geometrical Diagrams of Venn.—PORETSKY'S method may be looked upon as the perfection of the methods of STANLEY JEVONS and VENN.

Conversely, it finds in them a geometrical and mechanical illustration, for VENN'S method is translated in geometrical diagrams which represent all the constituents, so that, in order to obtain the result, we need only strike out (by shading) those which are made to vanish by the data of the problem. For instance, the universe of three terms a, b, c, represented by the unbounded plane, is divided by three simple closed contours into eight regions which represent the eight constituents (Fig. 1).