The solution is

$$t = (A + a)\ (B + b)\ (C + c) \dots + u\,(A'a + B'b + C'c + \dots).$$

The resultant is verified by hypothesis since it is

$$ABC \dots = 0,$$

which is the resultant of the given equation

$$f(x, y, z, \dots) = 0.$$

We can see how this equation contributes to restrict the variability of t. Since t was defined only by the function φ, it was determined by the double inclusion

$$abc \dots < t < a + b + c + \dots.$$

Now that we take into account the condition $f = 0$, t is determined by the double inclusion

$$(A + a)\ (B + b)\ (C + c) \dots < t < (A'a + B'b + C'c + \dots).$$ [1]

The inferior limit can only have increased and the superior limit diminished, for

$$abc \dots < (A + a)\ (B + b)\ (C + c) \dots$$

and

$$A'a + B'b + C'c \dots < a + b + c \dots.$$

The limits do not change if $A = B = C = \dots = 0$, that is, if the equation $f = 0$ is reduced to an identity, and this was evident *a priori*.

42. The Method of Poretsky.—The method of BOOLE and SCHRÖDER which we have heretofore discussed is clearly inspired by the example of ordinary algebra, and it is summed up in two processes analogous to those of algebra, namely the solution of equations with reference to unknown quantities and elimination of the unknowns. Of these processes the second is much the more important from a logical point of view, and BOOLE was even on the point of considering deduction as essentially consisting in the *elimination of middle*

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[1] WHITEHEAD, *Universal Algebra*, p. 63.

terms. This notion, which is too restricted, was suggested by the example of the syllogism, in which the conclusion results from the elimination of the middle term, and which for a long time was wrongly considered as the only type of mediate deduction.[1]

However this may be, BOOLE and SCHRÖDER have exaggerated the analogy between the algebra of logic and ordinary algebra. In logic, the distinction of known and unknown terms is artificial and almost useless. All the terms are—in principle at least—known, and it is simply a question, certain relations between them being given, of deducing new relations (unknown or not explicitly known) from these known relations. This is the purpose of PORETSKY's method which we shall now expound. It may be summed up in three laws, the *law of forms*, the *law of consequences* and the *law of causes*.

43. **The Law of Forms.**—This law answers the following problem: An equality being given, to find for any term (simple or complex) a determination equivalent to this equality. In other words, the question is to find all the *forms* equivalent to this equality, any term at all being given as its first member.

We know that any equality can be reduced to a form in which the second member is 0 or 1; *i. e.*, to one of the two equivalent forms

$$N = 0, \qquad N' = 1.$$

The function N is what PORETSKY calls the *logical zero* of the given equality; N' is its logical *whole*.[2]

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[1] In fact, the fundamental formula of elimination

$$(ax + bx' = 0) < (ab = 0)$$

is, as we have seen, only another form and a consequence of the principle of the syllogism

$$(b < x < a') < (b < a').$$

[2] They are called "logical" to distinguish them from the identical *zero* and *whole*, *i. e.*, to indicate that these two terms are not equal to 0 and 1 respectively except by virtue of the data of the problem.