2. The condition is necessary, for if

$$\prod_x (ax + bx') = 0,$$

the equation is true, in particular, for the value $x = a$; hence

$$a + b = 0.$$

Therefore the equivalence

$$\prod_x (ax + bx' = 0) = (a + b = 0)$$

is proved.[1] In this instance, the equation reduces to an identity: its first member is "identically" null.

34. The Expression of an Inclusion by Means of an Indeterminate.—The foregoing notation is indispensable in almost every case where variables or indeterminates occur in one member of an equivalence, which are not present in the other. For instance, certain authors predicate the two following equivalences

$$(a < b) = (a = bu) = (a + v = b),$$

in which u, v are two "indeterminates". Now, each of the two equalities has the inclusion $(a < b)$ as its consequence, as we may assure ourselves by eliminating u and v respectively from the following equalities:

1. $[a(b' + u') + a'bu = 0] = [(ab' + a'b)u + au' = 0].$

Resultant:

$$[(ab' + a'b)\,a = 0] = (ab' = 0) = (a < b).$$

2. $[(a + v)b' + a'bv = 0] = [b'v + (ab' + a'b)v' = 0].$

Resultant:

$$[b'(ab' + a'b) = 0] = (ab' = 0) + (a < b).$$

But we cannot say, conversely, that the inclusion implies the two equalities for *any values* of u and v; and, in fact, we restrict ourselves to the proof that this implication holds for some value of u and v, namely for the particular values

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[1] EUGEN MÜLLER, *op. cit.*

$$u = a, \quad b = v;$$

for we have

$$(a = ab) = (a < b) = (a + b = b).$$

But we cannot conclude, from the fact that the implication (and therefore also the equivalence) is true for *some* value of the indeterminates, that it is true for *all*; in particular, it is not true for the values

$$u = 1, \quad v = 0,$$

for then $(a = bu)$ and $(a + v = b)$ become $(a = b)$, which obviously asserts more than the given inclusion $(a < b)$.[1]

Therefore we can write only the equivalences

$$(a < b) = \sum_u (a = bu) = \sum_v (a + v = b),$$

but the three expressions

$$(a < b), \quad \prod_u (a = bu), \quad \prod_v (a + v = b)$$

are not equivalent.[2]

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[1] Likewise if we make

$$u = 0, \quad v = 1,$$

we obtain the equalities

$$(a = 0), \quad (b = 1),$$

which assert still more than the given inclusion.

[2] According to the remark in the preceding note, it is clear that we have

$$\prod_v (a = bu) = (a = b = 0), \quad \prod_v (a + v = b) = (a = b = 1),$$

since the equalities affected by the sign $\prod$ may be likewise verified by the values

$$u = 0, \quad u = 1 \quad \text{and} \quad v = 0, \quad v = 1.$$

If we wish to know within what limits the indeterminates u and v are variable, it is sufficient to solve with respect to them the equations

$$(a < b) = (a = bu), \quad (a < b) = (a + v = b),$$

or

$$ab' = a'bu + ab' + au', \quad ab' = ab' + b'v + a'bv',$$

or

$$a'bu + abu' = 0, \quad a'b'v + a'bv' = 0,$$