(that is to say, the part common to a' and x). The solution is generally indeterminate (between the limits a' and b); it is determinate only when the limits are equal,

$$a' = b,$$

for then

$$x = b + a'x = b + bx = b = a'.$$

Then the equation assumes the form

$$(ax + a'x' = 0) = (a' = x)$$

and is equivalent to the double inclusion

$$(a' < x < a') = (x = a').$$

31. The Resultant of Elimination.—When ab is not zero, the equation is impossible (always false), because it has a false consequence. It is for this reason that SCHRÖDER considers the resultant of the elimination as a *condition* of the equation. But we must not be misled by this equivocal word. The resultant of the elimination of x is not a *cause* of the equation, it is a *consequence* of it; it is not a *sufficient* but a *necessary* condition.

The same conclusion may be reached by observing that ab is the inferior limit of the function $ax + bx'$, and that consequently the function can not vanish unless this limit is 0.

$$(ab < ax + bx')\,(ax + bx' = 0) < (ab = 0).$$

We can express the resultant of elimination in other equivalent forms; for instance, if we write the equation in the form

$$(a + x')\,(b + x) = 0,$$

we observe that the resultant

$$ab = 0$$

is obtained simply by dropping the unknown quantity (by suppressing the terms x and x'). Again the equation may be written:

$$a'x + b'x' = 1$$

and the resultant of elimination:

$$a' + b' = 1.$$

Here again it is obtained simply by dropping the unknown quantity.[1]

Remark. If in the equation

$$ax + bx' = 0$$

we substitute for the unknown quantity x its value derived from the equations,

$$x = a'x + bx', \quad x' = ax + b'x',$$

we find

$$(abx + abx' = 0) = (ab = 0),$$

that is to say, the resultant of the elimination of x which, as we have seen, is a consequence of the equation itself. Thus we are assured that the value of x verifies this equation. Therefore we can, with VOIGT, define the solution of an equation as that value which, when substituted for x in the equation, reduces it to the resultant of the elimination of x.

Special Case.— When the equation contains a term independent of x, *i. e.*, when it is of the form

$$ax + bx' + c = 0$$

it is equivalent to

$$(a + c)x + (b + c)x' = 0,$$

and the resultant of elimination is

$$(a + c)(b + c) = ab + c = 0,$$

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[1] This is the method of elimination of Mrs. LADD-FRANKLIN and Mr. MITCHELL, but this rule is deceptive in its apparent simplicity, for it cannot be applied to the same equation when put in either of the forms

$$ax + bx' = 0, \quad (a' + x')(b' + x) = 1.$$

Now, on the other hand, as we shall see (§ 54), for inequalities it may be applied to the forms

$$ax + bx' \neq 0, \quad (a' + x')(b' + x) \neq 1.$$

and not to the equivalent forms

$$(a + x')(b + x) \neq 0, \quad a'x + b'x' \neq 1.$$

Consequently, it has not the mnemonic property attributed to it, for, to use it correctly, it is necessary to recall to which forms it is applicable.

whence we derive this practical rule: To obtain the resultant of the elimination of x in this case, it is sufficient to equate to zero the product of the coefficients of x and x', and add to them the term independent of x.

32. The Case of Indetermination.—Just as the resultant

$$ab = 0$$

corresponds to the case when the equation is possible, so the equality

$$a + b = 0$$

corresponds to the case of *absolute indetermination.* For in this case the equation both of whose coefficients are zero $(a = 0)$, $(b = 0)$, is reduced to an identity $(0 = 0)$, and therefore is "identically" verified, whatever the value of x may be; it does not determine the value of x at all, since the double inclusion

$$b < x < a'$$

then becomes

$$0 < x < 1,$$

which does not limit in any way the variability of x. In this case we say that the equation is *indeterminate.*

We shall reach the same conclusion if we observe that $(a + b)$ is the superior limit of the function $ax + bx'$ and that, if this limit is 0, the function is necessarily zero for all values of x,

$$(ax + bx' < a + b)\ (a + b = 0) < (ax + bx' = 0).$$

Special Case.—When the equation contains a term independent of x,

$$ax + bx' + c = 0,$$

the condition of absolute indetermination takes the form

$$a + b + c = 0.$$

For

$$ax + bx' + c = (a + c)x + (b + c)x',$$

$$(a + c) + (b + c) = a + b + c = 0.$$