For, if we develop with respect to x, we have

$$ax + bx' + abx + abx' = (a + ab)x + (b + ab)x' = ax + bx'.$$

Cor. 2. We have the equivalence

$$ax + bx' + c = (a + c)x + (b + c)x'.$$

For if we develop the term c with respect to x, we find

$$ax + bx' + cx + cx' = (a + c)x + (b + c)x'.$$

Thus, when a function contains terms (whose sum is represented by c) independent of x, we can always reduce it to the developed form $ax + bx'$ by adding c to the coefficients of both x and x'. Therefore we can always consider a function to be reduced to this form.

In practice, we perform the development by multiplying each term which does not contain a certain letter (x for instance) by $(x + x')$ and by developing the product according to the distributive law. Then, when desired, like terms may be reduced to a single term.

25. The Formulas of De Morgan.—*In any development of I, the sum of a certain number of constituents is the negative of the sum of all the others.*

For, by hypothesis, the sum of these two sums is equal to 1, and their product is equal to 0, since the product of two different constituents is zero.

From this proposition may be deduced the formulas of DE MORGAN:

$$(a + b)' = a'b', \quad (ab)' = a' + b'.$$

Demonstration.—Let us develop the sum $(a + b)$:

$$a + b = ab + ab' + ab + a'b = ab + ab' + a'b.$$

Now the development of 1 with respect to a and b contains the three terms of this development plus a fourth term $a'b'$. This fourth term, therefore, is the negative of the sum of the other three.

We can demonstrate the second formula either by a correlative argument (*i. e.*, considering the development of 0 by factors) or by observing that the development of $(a' + b')$,

$$a'b + ab' + a'b',$$

differs from the development of 1 only by the summand ab.

How DE MORGAN's formulas may be generalized is now clear; for instance we have for a sum of three terms,

$$a + b + c = abc + abc' + ab'c + ab'c' + a'bc + a'bc' + a'b'c.$$

This development differs from the development of 1 only by the term $a'b'c'$. Thus we can demonstrate the formulas

$$(a + b + c)' = a'b'c', \quad (abc)' = a' + b' + c',$$

which are generalizations of DE MORGAN's formulas.

The formulas of DE MORGAN are in very frequent use in calculation, for they make it possible to perform the negation of a sum or a product by transferring the negation to the simple terms: the negative of a sum is the product of the negatives of its summands; the negative of a product is the sum of the negatives of its factors.

These formulas, again, make it possible to pass from a primary proposition to its correlative proposition by duality, and to demonstrate their equivalence. For this purpose it is only necessary to apply the law of contraposition to the given proposition, and then to perform the negation of both members.

Example:

$$ab + ac + bc = (a + b)(a + c)(b + c).$$

Demonstration:

$$(ab + ac + bc)' = [(a + b)(a + c)(b + c)],$$
$$(ab)'(ac)'(bc)' = (a + b)' + (a + c)' + (b + c)',$$
$$(a' + b')(a' + c')(b' + c') = a'b' + a'c' + b'c'.$$

Since the simple terms, a, b, c, may be any terms, we may suppress the sign of negation by which they are affected, and obtain the given formula.

Thus DE MORGAN's formulas furnish a means by which to find or to demonstrate the formula correlative to another; but, as we have said above (§ 14), they are not the basis of this correlation.