C. I.: The part common to any class whatever and to the null class is the null class; the sum of any class whatever and of the whole is the whole. The sum of the null class and of any class whatever is equal to the latter; the part common to the whole and any class whatever is equal to the latter.

P. I.: The simultaneous affirmation of any proposition whatever and of a false proposition is equivalent to the latter (i. e., it is false); while their alternative affirmation is equal to the former. The simultaneous affirmation of any proposition whatever and of a true proposition is equivalent to the former; while their alternative affirmation is equivalent to the latter (i. e., it is true).

Remark.—If we accept the four preceding formulas as axioms, because of the proof afforded by the double interpretation, we may deduce from them the paradoxical formulas

$$0 < x, \quad \text{and} \quad x < 1,$$

by means of the equivalences established above,

$$(a = ab) = (a < b) = (a + b = b).$$

14. The Law of Duality.—We have proved that a perfect symmetry exists between the formulas relating to multiplication and those relating to addition. We can pass from one class to the other by interchanging the signs of addition and multiplication, on condition that we also interchange the terms 0 and 1 and reverse the meaning of the sign $<$ (or transpose the two members of an inclusion). This symmetry, or *duality* as it is called, which exists in principles and definitions, must also exist in all the formulas deduced from them as long as no principle or definition is introduced which would overthrow them. Hence a true formula may be deduced from another true formula by transforming it by the principle of duality; that is, by following the rule given above. In its application the *law of duality* makes it possible to replace two demonstrations by one. It is well to note that this law is derived from the definitions of addition and multiplication (the formulas for which are reciprocal by duality) and not, as is often thought[1], from the laws of negation which have not yet been stated. We shall see that these laws possess the same property and consequently preserve the duality, but they do not originate it; and duality would exist even if the idea of negation were not introduced. For instance, the equality (§ 12)

$$ab + ac + bc = (a + b)\ (a + c)\ (b + c)$$

is its own reciprocal by duality, for its two members are transformed into each other by duality.

It is worth remarking that the law of duality is only applicable to primary propositions. We call [after BOOLE] those propositions *primary* which contain but one copula ($<$ or $=$). We call those propositions *secondary* of which both members (connected by the copula $<$ or $=$) are primary propositions, and so on. For instance, the principle of identity and the principle of simplification are primary propositions, while the principle of the syllogism and the principle of composition are secondary propositions.

15. Definition of Negation.—The introduction of the terms 0 and 1 makes it possible for us to define *negation*. This is a "uni-nary" operation which transforms a single term into another term called its *negative*.[2] The negative of a is called not-a and is written a'.[3] Its formal definition implies the following postulate of existence[4]:

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1 [BOOLE thus derives it (*Laws of Thought*, London 1854, Chap. III, Prop. IV).]

2 [In French] the same word *negation* denotes both the operation and its result, which becomes equivocal. The result ought to be denoted by another word, like [the English] "negative". Some authors say, "supplementary" or "supplement", [e. g. BOOLE and HUNTINGTON]. Classical logic makes use of the term "contradictory" especially for propositions.

3 We adopt here the notation of MACCOLL; SCHRÖDER indicates not-a by a_1 which prevents the use of indices and obliges us to express them as exponents. The notation a' has the advantage of excluding neither indices nor exponents. The notation $\bar{a}$ employed by many authors is inconvenient for typographical reasons. When the negative affects a proposition written in an explicit form (with a copula) it is applied to the copula ($<$ or $=$) by a vertical bar ($\not<$ or $\neq$). The accent can be considered as the indication of a vertical bar applied to letters.

4 [BOOLE follows Aristotle in usually calling the law of duality the