0. Introduction.

In 1872, Klein proposed group theory as a means of formulating and understanding geometrical constructions. The resulting programme has been termed the "Erlangen programme". Since that time the two subjects have been closely linked. The subject of geometric group theory might be viewed as Klein's programme in reverse — geometrical ideas are used to give new insights into group theory. Although largely a creation of the last twenty years or so, its antecedents can be traced back to the early 20th century. For example, Dehn used hyperbolic geometry to solve the word problem in a surface group. His ideas were subsequently formalised in terms of "small cancellation theory", in some sense a forerunner of modern geometric group theory (while remaining an active field in itself). The observation, due to Efremovich, Schwarz and Milnor, that a group acting discretely cocompactly on a proper space resembles, on a large scale, the space on which it acts, is key to the development of the subject.

The subject draws on ideas from across mathematics, though one can identify two particular sources of inspiration. One is low-dimensional topology, in particular 3-manifold theory. Another is hyperbolic geometry. The work of Thurston in the late 1970s showed that these two subjects were intimately linked. The resulting flurry of activity might be seen as the birth of geometric group theory as a subject in its own right. The work of Gromov in the 1980s was particularly influential. We note especially his papers on hyperbolic groups and asymptotic invariants.

The subject has now grown into a major field. It would be impossible to give even a representative overview in notes such as these. I have directed attention towards giving a basic introduction to hyperbolic groups and spaces. These are of fundamental importance, though of course, many other directions would have been possible. (We say almost nothing, for example, about the vast subject of non-positively curved spaces.) Similarly, many of the diversions are informed by my own intersts, and I offer my apologies if these have been over-represented.