A COLLECTION OF PAPERS IN MATHEMATICS AND RELATED SCIENCES,
A FESTSCHRIFT IN HONOUR OF THE LATE GALAYE DIA
EDITORS : PROFS HAMET SEYDI, GANE SAMB LO, ABOUBAKARY DIAKHABY
SPAS-EDS (SAINT-LOUIS, CALGARY, 2018), WWW.STATPAS.ORG/SPASEDS/
IN EUCLID (WWW.PROJECTEUCLID.ORG)
DOI : 10.16929/SBS/2018.100

CHAPTER 9

Stochastic volatility model through MCEM: Departure from canonical SV model, *by M. M. Allaya and M. M. Kâ*

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Abstract. We analyze a discretized canonical stochastic volatility model through calibration to synthetic data as well as financial data. To achieve this end we ressort to Monte Carlo EM (Chan and Ledolter (1995)) that is, the combination of EM algorithm (Dempster *et al.* (1977), Wu (1983)) and sequential Monte Carlo methods (Gordon *et al.*(1993), Doucet *et al.* (2001), Salmond and Smith (1993)). Finally we consider a slight departure from canonical stochastic volatility model in order to assess the robustness of the MCEM procedure. Simulations studies follow.

Keywords. SMC methods; Stochastic Volatility; EM algorithm
AMS 2010 Mathematics Subject Classification. 60J05; 62M05; 65C05

Cite the chapter as :
Allaya M. M. and Kâ M. M. (2018). Stochastic volatility model through MCEM: Departure from canonical SV model. In *A Collection of Papers in Mathematics and Related Sciences, a festschrift in honour of the late Galaye Dia* (Editors : Seydi H., Lo G.S. and Diakhaby A.). Spas Editions, Euclid Series Book, pp. 141 –162. Doi : 10.16929/sbs/2018.100-02-05

©Spas Editions, Saint-Louis - Calgary 2018 H. Seydi *et al* (Eds.) A Collection of Papers in Mathematics and Related Sciences, a festschrift in honour of the late Galaye Dia. Doi : 10.16929/sbs/2018.100

1. Introduction

It is well known that financial assets prices exhibit unpredictable changes over time. This variability, measured by the variance square root of the asset price process, is commonly referred as the *volatility*. Among the approaches leading to its modeling are the ARCH processes and their generalization (Engle (1982), Bollerslev (1986)). These models try to capture the link between the innovation process at present date and its past through the conditional distribution of the innovations that is randomness of the variance process varies with the variance. Another point of view is to give volatility latent process dynamics. This is the approach we favor. This point of view has received several contributions from the scientific community. We can mention among others the work of Heston (1993), Kim *et al.* (1998), Kim and Stoffer (2006), etc. In the sequel, we calibrate a discrete stochastic volatility model to exchange rates data as well as stock market indexes. We focus on GBP/USD and YEN/USD rates. Then, we extend this study to the S&P 500, Dow Jones and NIKKEI 225 indexes. To do so, we adopted the following plan. First, we start using simulated data to gauge the estimation method. The advantage in this case is to be able to control the output parameters returned by the estimation procedure. In a second step, we consider the calibration of this model on real data sets and possibly make a comparison with estimates obtained from certain authors. Finally, we depart from the basic volatility model in order to put the estimation method to the test of robustness.

2. Framework

Consider a filtered probability space on which a risky financial asset evolves with a price dynamics following a standard geometric Brownian motion given by:

$$\frac{dS_t}{S_t} = \varpi dt + \sigma dW_t \tag{2.1}$$

where ϖ is a mean term called *drift*, σ the volatility which is constant and $(W_t)_{t\geq 0}$ a standard Brownian motion. A direct application of Itô lemma leads to the solution of the stochastic differential equation given by :

$$S_t = S_0 e^{(\varpi - \frac{1}{2}\sigma^2)t + \sigma W_t}. \tag{2.2}$$

This model has shown its limits among others the failure in capturing the so called *stylized* facts. In particular, the non-constancy of the volatility is a crucial point in financial modeling. Among the various extensions we can mention those that confer a specific dynamics to the volatility (ARCH model, GARCH model, Heston model, etc.). Singularly, the point of view pursued is the one which confers to the volatility a dynamic latent process thus unobservable or partially observable. In such case, one must associate to the volatility process an observation process which serves to quantify its realization. By adopting an exponential process Orstein Uhlenbeck for the volatility, the previous model is rewritten :

$$\begin{cases} \frac{dS_t}{S_t} = \varpi dt + \sigma_t dW_t \\ \sigma_t = \exp(U_t/2) \\ dU_t = \gamma(\delta - U_t)dt + \zeta dW_t^* \end{cases} \tag{2.3}$$

where $(W_t^*)_{t\geq 0}$ a standard brownian motion allowing correlation with $(W_t)_{t\geq 0}$, δ the long term mean, γ the speed return to the long run mean δ also ensuring stationarity of U whenever $|\gamma| < 1$, and $\zeta > 0$ a term of variance also called volatility of volatility. For the sake of simplicity, a zero correlation is imposed between the two brownian motions. In order to be able to calibrate the model to data, we discretize the model following Euler scheme given by :

$$\begin{cases} X_k = \alpha X_{k-1} + \sigma W_k \\ Y_k = \beta e^{X_k/2} V_k, \quad k \geq 1 \end{cases} \tag{2.4}$$

where (V_k) and (W_k) are independent gaussian noises independent of $X_0 \sim \mathcal{N}(0, \sigma_0^2)$ and $|\alpha| < 1$. Thus, the parameter vector is $\theta = (\alpha, \beta, \sigma)$. Note it is customary to linearize (2.4) in order to obtain a linear state space model :

$$\begin{cases} X_k = \alpha X_{k-1} + \sigma W_k \\ \log Y_k^2 = \log \beta^2 + m + X_k + \log V_k^2 - m \end{cases} \tag{2.5}$$

where $m := \mathbb{E}(\log V_k^2) = -1.27049$ et $\log V_k^2$ follows $\log \chi^2$ distribution.

3. Parameter estimation

MCEM as a combination of the Generalized Expectation Maximization algorithm (GEM) with sequential Monte Carlo methods (SMC) is a tool that

Algorithm 1 : Generalized EM algorithm

- Choose an initial guess $\boldsymbol{\theta}^{(0)}$
- **For** $m = 1, 2, \ldots$ **do**
  (1) **E-Step :** Compute $Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{(m-1)})$
  (2) **M-Step :** Find $\boldsymbol{\theta}^{(m)}$ s.t $Q(\boldsymbol{\theta}^{(m)}, \boldsymbol{\theta}^{(m-1)}) \geq Q(\boldsymbol{\theta}^{(m-1)}, \boldsymbol{\theta}^{(m-1)})$
- **EndFor.**

can be used to estimate stochastic volatility models specifically those falling in the general setting of hidden Markov models. GEM itself, can be used when classical estimation methods like Maximum likelihood estimator fails when dealing with latent variables. We do not fully detail the GEM algorithm since it is well documented (see Dempster *et al.* (1977) or McLachlan and Krishnan (2008) for a review). However, the main idea is depicted by the *Generalized EM algorithm* in Algorithm 1.
The E-step consists in computing the *intermediate quantity*, that is the conditional expectation of the logarithm of the complete data likelihood given data and the current value of the parameter vector $\theta^{(k-1)}$:

$$
\begin{aligned}
Q(\theta^{(k)}, \theta^{(k-1)}) &:= \mathbb{E}_{\theta^{(k)}}\left[\log p_{\theta^{(k-1)}}(X_{0:n}, Y_{1:n}) \mid Y_{1:n}\right] \approx -\frac{n}{2}\log[\sigma^{(k)}]^2 \\
&- \frac{1}{2}\left\{\frac{\sum_{r=1}^{n} \mathbb{E}_{\theta^{(k)}}\left[(X_r - \alpha^{(k)} X_{r-1})^2 | Y_{1:n}\right]}{[\sigma^{(k)}]^2}\right\} \\
&- \frac{1}{2}\left\{\sum_{r=1}^{n} \mathbb{E}_{\theta^{(k)}}\left[\exp(X_r - Y_r - \log[\beta^{(k)}]^2 + m) - (X_r - Y_r - \log[\beta^{(k)}]^2 + m) | Y_{1:n}\right]\right\}
\end{aligned}
$$

The M-step consists in taking the derivatives of the intermediate quantity with respect to each parameter. The mechanism for updating the parameters then obeys the following recurrence scheme:

$$
(3.1)\qquad \begin{cases} \alpha^{(k+1)} = \dfrac{\sum_{r=1}^{n} \mathbb{E}_{\theta^{(k)}}[X_{r-1}X_r|Y_{1:n}]}{\sum_{r=1}^{n} \mathbb{E}_{\theta^{(k)}}\left[X_{r-1}^2|Y_{1:n}\right]} \\ \\ \log[\beta^{(k+1)}]^2 + m = \log\left[\frac{1}{n}\sum_{r=1}^{n} \mathbb{E}_{\theta^{(k)}}[\exp(y_r - X_r + m)|Y_{1:n}]\right] \\ \\ \sigma^{(k+1)} = \sqrt{\frac{1}{n}\sum_{r=1}^{n} \left(\mathbb{E}_{\theta^{(k)}}\left[X_r|Y_{1:n}\right] - \alpha^{(k+1)}\mathbb{E}_{\theta^{(k)}}\left[X_{r-1}|Y_{1:n}\right]\right)^2} \end{cases}
$$

The conditional expectations that evolves in these computations admit rarely a close-form solution except few cases including linear and gaussian state space models. Because the underlying distributions are higher-dimensioned and complex to compute. This is why we have to use appropriate tools to approximate them. SMC are set of powerful tools inspired from genetic algorithms that are suitable whenever one is dealing with intractable random vectors. As for GEM algorithm we do not elaborate on it. We refer to Allaya (2013) for GEM algorithm and SMC methods .

4. Calibration to data

In this section, we calibrate the discretized SV model to data. We start with synthetic data as a way to control MCEM behavior in terms of outputs parameters' estimate. In a second hand we effectively calibrate SV model to real data and compare outputs to existing research on the subject using the same dataset.

4.1. Synthetic data. A first path resulting from the model (2.5) of $T = 500$ observations is produced. These data were generated under the parameter vector using $\theta^* = (0.9, \sqrt{0.1}, -0.8612)$ with $\alpha^* = 0.9$, $\sigma^* = \sqrt{0.1}$ and $\log(\beta^*)^2 = -0.8612$. The MCEM procedure is started with the initialization parameters $(\alpha^{(0)}, \log(\beta^{(0)})^2, \sigma^{(0)}) = (0.6, -0.3, \sqrt{0.3})$ At the end of the MCEM procedure, we have the evolution of the different parameters all along the 500 iterations. The joint plot of the series trajectory and the different iterations of the MCEM are given in Figure 1. One can also be interested in the average quadratic error risk for the three estimated parameters made after running MCEM procedure. Table 1 provides a summary of this magnitude.

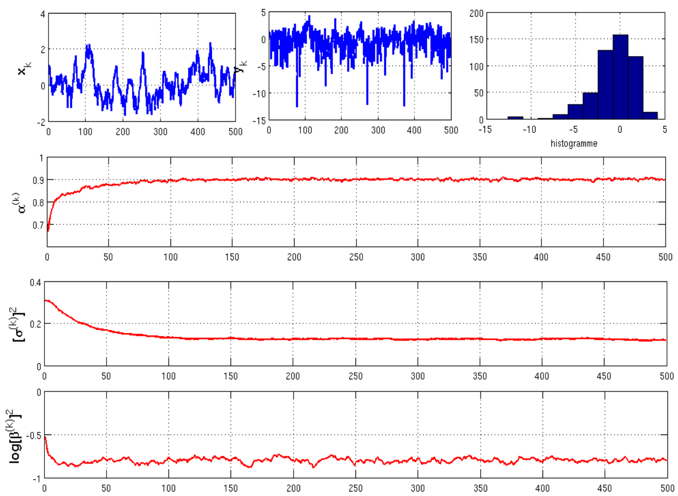

FIGURE 1. Path of (2.5) with $T = 500$ observations & MCEM iterations for $N = 200$ particles.

Estimators	$\hat{\alpha}$	$\log[\hat{\beta}]^2$	$[\hat{\sigma}]^2$
RMSE	0.0251	0.0655	0.0489

TABLE 1. Root Mean Squared Errors on 500 iterations of MCEM

We generate a second dataset with a longer time horizon $T = 4000$ with parameters $\theta^* = (0.92, \sqrt{0.4}, -0.7)$ where $\alpha^* = 0.92$, $\sigma^* = \sqrt{0.4}$ and $\log(\beta^*)^2 = -0.7$. The number of particles remains unchanged ($N = 200$). A conclusion similar to what has been already obtained above can be deduced. In Figure 2, we have a joint plot of the trajectory of the time series as well as the iterations of the MCEM.

REMARK 8. *As it can be noticed in Figure 1 and Figure 2, the MCEM iterations can be stopped earlier because of the relatively expensive computation time* [1]*. One may use a stop heuristic criterion such as the relative variation of the parameters or simply by using the likelihood ratio as in Chan and Ledolter (1995) or Kim and Stoffer (2006). Moreover, for the sake of harmony and brevity the number of iterations of the MCEM has been arbitrarily fixed at 500 for both real and synthetic data.*

[1] All the computations was done using PCs Intel i3, Core 2 Duo CPU 2.20 GHz

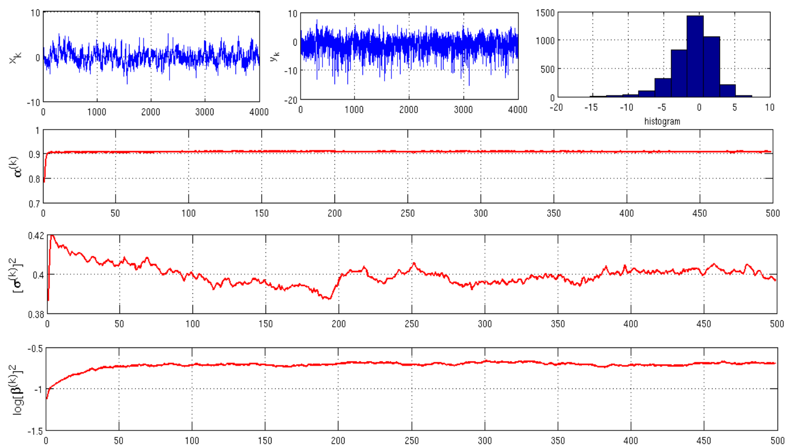

FIGURE 2. Path of (2.5) on time horizon $T = 4000$ & MCEM iterations for $N = 200$ particles.

4.2. Real data. In the following and unless otherwise stated, we maintain $N = 300$ particles and $\theta^{(0)} = (0.2, 0.1, -0.01)$ as initialization parameters of the MCEM procedure with $\alpha^{(0)} = 0.2$, $\sigma^{(0)} = \sqrt{0.1}$ and $2 \times \log(\beta^{(0)}) = -0.01$..

(1) - GBP/USD exchange rates. We have the historical GBP/USD daily exchange rates available on the *Federal Reserve System* website. We have used the period from October 1, 1981 to June 28, 1985. Indeed, this time series has been studied in Harvey and Ruiz (1994), Durban and Koopman (2000), Doucet and Tadic (2003). In Figure 3, we have represented the GBP/USD exchange rates as well as the parameters estimated after calibrating model (2.5). From (a) to (c) we have respectively the daily exchange rates p_k, the logarithm of the square returns y_k and the histogram of the latter.

(2) - USD/YEN exchange rates. We also collected the USD/YEN daily exchange rate for the period from May 31, 2005 to June 1, 2012. In Figure 4, we have summarized some of the characteristics of the latter. In plot (a), we noted by p_k the daily rate. (b) is that of the logarithm of the square of the corrected returns of their mean and y_k and in (c) the histogram of the latter. Plots (d), (e) and (f) represent the trajectories of the estimated parameters from 500 iterations of the MCEM.

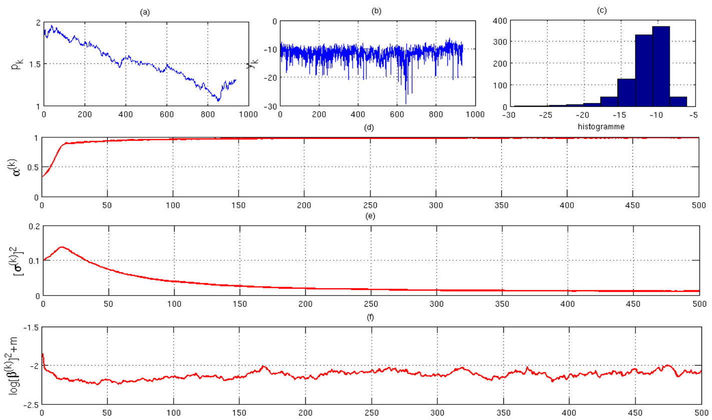

FIGURE 3. Calibrating model (2.5) to GBP/USD exchange rates.

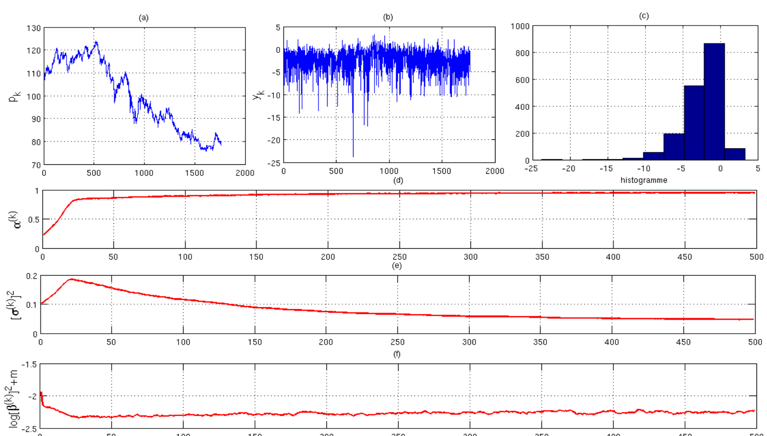

FIGURE 4. Calibrating model (2.5) to USD/YEN.

(3) - S&P 500 index. We analyze the daily data of the S & P 500 index for the period from May 20, 2008 to May 8, 2012. The daily return is formed on the quotations at the opening and at the closing. A calibration of the basic volatility model is also performed. The plot of the latter is given in Figure 5.

(4) - Dow Jones index We have daily quotations (at closing) of the Dow Jones index for the period from January 4, 1999 to September 24, 2002. An analysis similar to the previous index is performed at Figure 6.

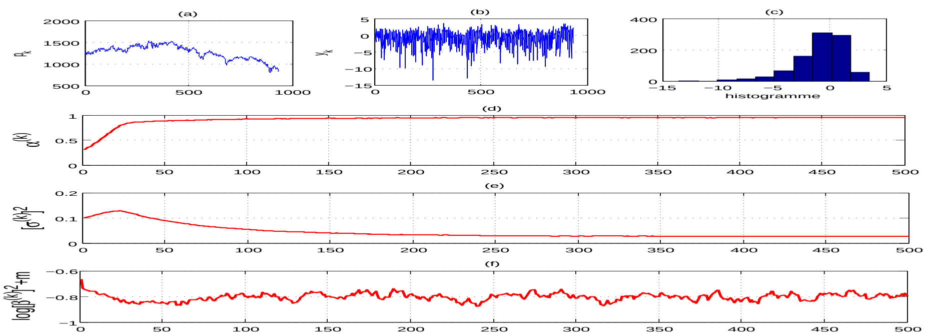

FIGURE 5. Calibrating model (2.5) to S&P 500

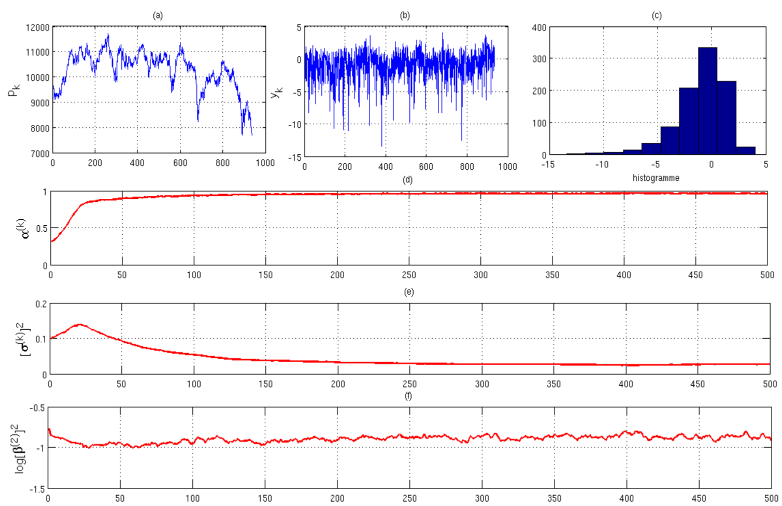

FIGURE 6. Calibrating model (2.5) to Dow Jones.

FTSE 100 index. We also examine the FTSE 100 index for the period from January 04, 1999 to September 24, 2002. The results are shown in Figure 7.

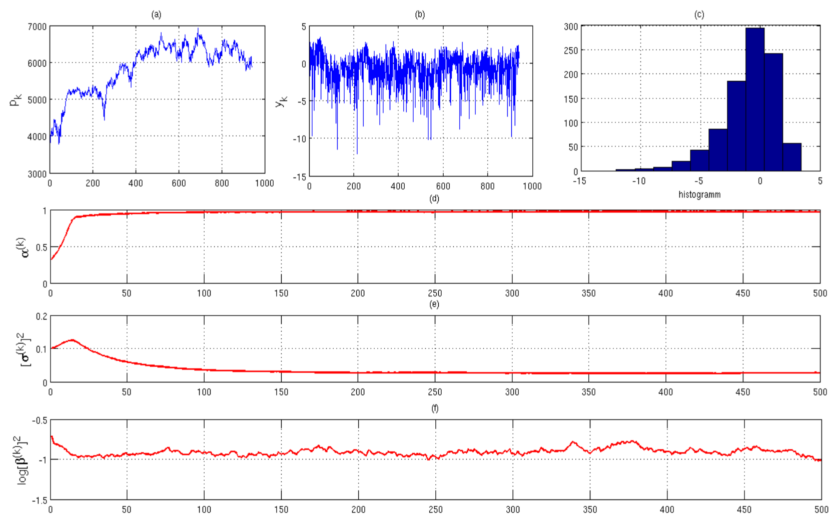

FIGURE 7. Calibrating model (2.5) to FTSE 100.

(4) - Nikkei 225 index. A final application is made to the Nikkei 225 index. The same period used for the FTSE 100 index is also considered. A similar analysis is also conducted.

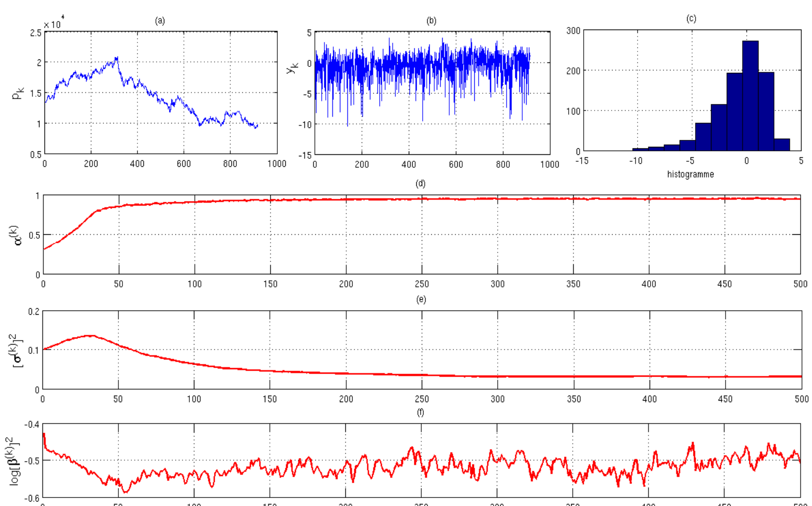

FIGURE 8. Analysis de l'indice Nikkei 225.

Index Estimator Estimators Indexes	Nikkei 225	Dow Jones	FTSE 100
$\hat{\alpha}$	0.944	0.961	0.974
$\hat{\sigma}$	0.178	0.165	0.157
$\exp(\hat{\varpi})$	1.387	1.154	1.182

TABLE 2. MCMC Estimations

Indexes Methods Estimations Indexes	Nikkei 225	Dow Jones	FTSE 100
$\hat{\alpha}$	0.9447	0.9609	0.9692
$\hat{\sigma}$	0.1744	0.1628	0.1631
$\exp(\hat{\varpi})$	1.4770	1.2196	1.1884

TABLE 3. Mean averages over last 40 iterations of MCEM procedure

4.2.1. *Comparison of stock indexes.* We restate the results by Krichene (2003) using MCMC methods to estimate the three parameters in concern on the Dow Jones, FTSE and Nikkei 225 indexes. The posterior averages of these parameters are summarized in Table 2.

Remark. The parameter $\exp(\varpi)$ in Krichene (2003) is linked to the β parameter through the relation $\beta = \exp(-\varpi/2)$.

We could directly compare the last iteration of the MCEM with the averages estimated by MCMC. However, we have averaged the last 40 iterations of the MCEM in order to compare them with the posterior averages of these parameters estimated by MCMC in Krichene (2003). The summary is recorded in Table 3.

It can be seen that the results of the two methods are substantially similar.

REMARK 9. *In these different applications to exchange rate and stock index data, it appears that the parameter α is very close to 1. This tends to* *confirm a hypothesis of persistence of volatility as well as its return to the long-term average. Estimated values of σ are reasonably small (less than 20%). This ensures a certain stability of the calibration of the volatility. Finally, the $\exp(\varpi) = \beta^{-2}$ parameter, whose estimated values are relatively large, reflects the quantifiability of the input of new information on volatility.*

5. Departure from canonical sv model

In order to test the basic volatility model as well as the linearization performed in the observations equation (2.5), we have taken over the non-linearised model (2.4) to which we have added the additive term $X^2+\cos(X^2)$ which is highly nonlinear, thereby increasing the complexity of the model. So that the evolution of observations is governed by:

$$Y_k = \beta \exp\left(\frac{X_k^2 + X_k + \cos(X_k^2)}{2}\right) V_k \tag{5.1}$$

with a complete dynamic given by :

$$\begin{cases} X_k = \alpha X_{k-1} + \sigma W_k \\ Y_k = \beta \exp\left(\frac{X_k^2+X_k+\cos(X_k^2)}{2}\right) V_k. \end{cases} \tag{5.2}$$

In this new configuration, we have replicated 150 Monte Carlo experiments which consist in estimating (5.2) by means of MCEM with random initialization. For each experiment, we run the MCEM procedure 500 times with $N = 200$ particles, $\theta^{(0)} = (\alpha^{(0)}, \sigma^{(0)}, \beta^{(0)})$ with $\alpha^{(0)}, \beta^{(0)} \sim \mathcal{U}([0,1])$ and $\sigma^{(0)} \sim \mathcal{U}([0.01, 1.01])$. Recall that the MCEM procedure follows the usual recurrence pattern. Given the $(k+1)^{\text{th}}$ iteration, the mechanism for sequential updating of the parameters then obeys the following recursion:

$$\begin{cases} \alpha^{(k+1)} = \frac{\sum_{r=2}^n \mathbb{E}_{\theta^{(k)}}[X_{r-1}X_r|Y_{1:n}]}{\sum_{r=2}^n \mathbb{E}_{\theta^{(k)}}\left[X_{r-1}^2|Y_{1:n}\right]} \\ \\ \sigma^{(k+1)} = \sqrt{\frac{1}{n}\sum_{r=2}^n \mathbb{E}_{\theta^{(k)}}\left[(X_r - \alpha^{(k+1)}X_{r-1})^2|Y_{1:n}\right]} \\ \\ \beta^{(k+1)} = \sqrt{\sum_{r=1}^n y_i^2 \mathbb{E}_{\theta^{(k)}}\left[e^{-(X_r^2+\cos(X_r^2)+X_r)}|Y_{1:n}\right]}. \end{cases} \tag{5.3}$$

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.7344	0.2351	0.5215
Bias	0.0344	0.0351	0.0215
Standard deviation	0.0754	0.0674	0.1167
MSER	0.0830	0.0761	0.1187

TABLE 4. Summary of 150 Monte Carlo replications

The parameters of the synthetic model used are $\theta^* = (0.7, 0.2, 0.5)$ with $\alpha^* = 0.7$, $\sigma^* = 0.2$ and $\beta^* = 0.5$ whose path in Figure 9.

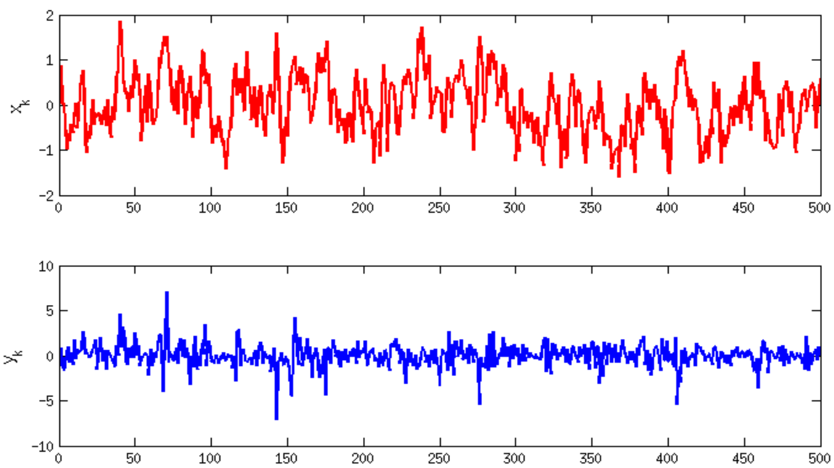

FIGURE 9. Path of model (5.2) over a time horizon of $T = 500$

At the end of each Monte Carlo experiment (500^{th} iteration of the MCEM), we averages the last 50 iterations of the MCEM for each of the 3 estimated parameters. A first summary of the Monte Carlo experiments is given in Table 4.
In Table 4, we put the mean, bias, standard deviation and the mean squared error root obtained from the Monte Carlo experiments for each of the 3 parameters.

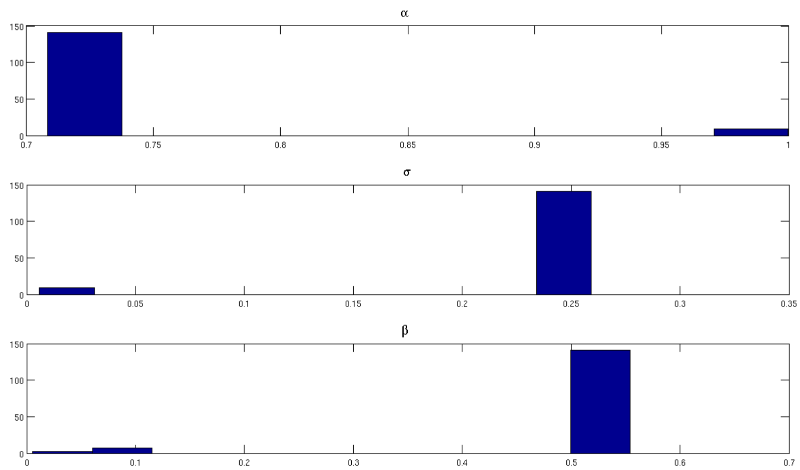

FIGURE 10. Histograms of parameters estimates over 150 Monte Carlo replications of MCEM in model (5.2).

We found a divergence rate to the 3 parameters of 6% which can result, among other things, from initialization problems, likelihood modalities, common to GEM algorithm . This results in the existence of extra classes with very small numbers in histograms. The dominant class of each histogram is that which translates the actual convergence of the parameter in concern. The output parameters are fairly faithful compare to the injected parameters as input of the model in a satisfactorily manner.

5.0.1. *Further Monte Carlo Simulations.* In order to test even more model (5.2), we realized two other series of Monte Carlo experiments. In the first set of 150 replications, we vary the number of particles used from 250 to 2000. The data length remains fixed at $T = 500$. The true parameter vector and the initialization vector are respectively $\theta^* = (0.9, 0.2, 0.6)$ and $\theta^{(0)} = (0, 0.1, 0)$. A first summary of simulation results is shown in Figure 11.

REMARK 10. *One finding that can be made is that as the number of particles increases, there is greater stability in the estimated parameters for fixed T. This reinforces the idea of effective convergence of exhaustive statistics, a function of conditional distributions approximated by particle systems.*

Alongside the graphic illustrations, we have calculated some numerical quantities for a better understanding of the simulations. Tables 5-02-05-tab10 tables illustrate an aggregated summary of the 150 Monte Carlo

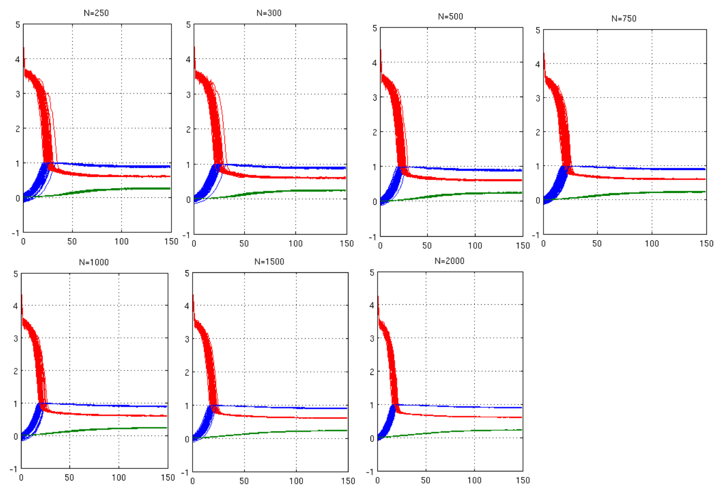

FIGURE 11. 150 Monte Carlo replications : Evolution of ÂĽMCEM algorithm with respect to the number of particles for $T = 500$ fixed

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.8835	0.2452	0.6008
Bias	-0.0164	0.0452	0.0088
Standard deviation	0.0754	0.0674	0.1167
RMSE	0.0198	0.0457	0.0082

TABLE 5. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 500$.

replications across means, bias, root of the mean squared errors, and standard deviations from the last 50 iterations of the MCEM.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.8876	0.2421	0.6028
Bias	-0.0123	0.0421	0.0028
Standard deviation	0.0095	0.0057	0.0067
RMSE	0.0159	0.0426	0.0082

TABLE 6. Aggregation of 150 Monte Carlo replications with $N = 300$ particles and $T = 500$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9008	0.2299	0.6103
Bias	0.0008	0.0299	0.0103
Standard deviation	0.0071	0.0060	0.0052
RMSE	0.0082	0.0306	0.0117

TABLE 7. Aggregation of 150 Monte Carlo replications with $N = 750$ particles and $T = 500$.

The second set of 150 Monte Carlo replications consist in observing the behavior of the estimated parameters over an increasing trajectory length while keeping the number of particles constant. The summary of the different simulations is given in Figure 12.
Similar to previous replications, a numerical view of the last 50 iterations of the MCEM can be considered. Tables 11-16, given after the bibliography, are an aggregate summary.

REMARK 11. *A finding that can be made is that we can correctly estimate the parameters as we increase the length of the trajectory despite the fact that we have frozen the number of particles at $N = 250$. The only apparent disadvantage is the extra effort provided to handle the additional observations.*

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9046	0.2265	0.6131
Bias	0.0046	0.0265	0.0131
Standard deviation	0.0063	0.0057	0.0046
RMSE	0.0087	0.0272	0.0140

TABLE 8. Aggregation of 150 Monte Carlo replications with $N = 1000$ particles and $T = 500$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9086	0.2228	0.6159
Bias	0.0086	0.0228	0.0159
Standard deviation	0.0052	0.0054	0.0040
REQM	0.0103	0.0235	0.0164

TABLE 9. Aggregation of 150 Monte Carlo replications with $N = 1500$ particles and $T = 500$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9103	0.2211	0.6172
Bias	0.0103	0.0211	0.0172
Standard deviation	0.0046	0.0054	0.0035
RMSE	0.0113	0.0218	0.0176

TABLE 10. Aggregation of 150 Monte Carlo replications with $N = 2000$ particles and $T = 500$.

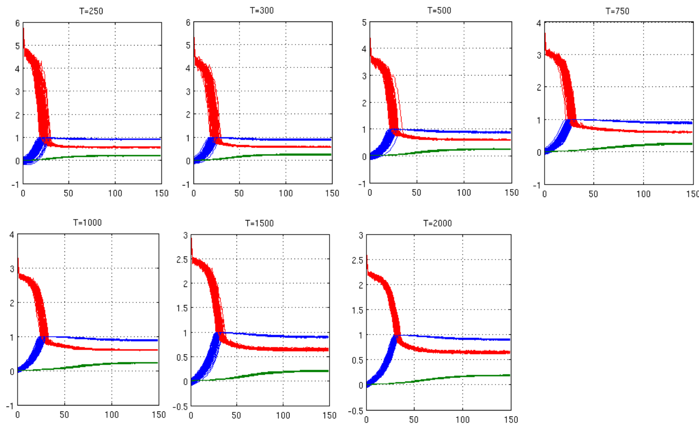

FIGURE 12. 150 Monte Carlo experiments: Evolution of MCEM algorithm with respect to length path for $N = 250$ fixed.

6. Conclusion

In this paper, through the canonical stochastic volatility model we have highlighted the common use of the MCEM algorithm for inference purpose. We investigate the calibration to financial data including stock market indexes and exchange rates. We also compare to MCMC approach by Krichene (2003) dealing with the same dataset within the same model. Finally, we also tested the basic model by adding a strongly non-linear component. On the one hand, to get rid of simplicity of the basic model and on the other hand to gauge the robustness of the MCEM procedure to high non-linearity of the resulting model in the estimated parameters. There are still several points that need more attention. One of them is the reduction of the cost of calculation. For example, an idea on reducing the cost of computing the MCEM estimates would be to make the number of particles used to be adaptive combined along with a forcing function in order to force the stop of unnecessary iterations whenever convergence is taking place. Given that some components of the parameter vector converge faster than others. This can result in a substantial gain in computation time.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9205	0.2017	0.5705
Bias	0.0205	0.0017	−0.0294
Standard deviation	0.0111	0.0048	0.0088
RMSE	0.0235	0.0067	0.0308

TABLE 11. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 250$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.8835	0.2452	0.6008
Bias	−0.0164	0.0452	0.0088
Standard deviation	0.0754	0.0674	0.1167
RMSE	0.0198	0.0457	0.0082

TABLE 12. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 500$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9013	0.2316	0.6115
Bias	0.0013	0.0316	0.0115
Standard deviation	0.0104	0.01092	0.0081
RMSE	0.0122	0.03365	0.0146

TABLE 13. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 750$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.8896	0.2212	0.6038
Bias	-0.0103	0.0212	0.0038
Standard deviation	0.0091	0.0081	0.0058
RMSE	0.0142	0.0229	0.0074

TABLE 14. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 1000$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9042	0.2012	0.6385
Bias	0.0042	0.0012	0.0385
Standard deviation	0.0069	0.0066	0.0115
RMSE	0.0085	0.0075	0.0402

TABLE 15. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 1500$.

Estimated parameters	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$
Mean	0.9080	0.1826	0.6408
Bias	0.0080	-0.0173	0.0408
Standard deviation	0.0067	0.0066	0.0094
RMSE	0.0104	0.0187	0.0419

TABLE 16. Aggregation of 150 Monte Carlo replications with $N = 250$ particles and $T = 2000$.

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