IMPERFECT MAINTENANCE

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1. Introduction

An impressive array of mathematical and statistical papers and books have appeared in which a variety of maintenance policies are studied to determine their performance and to achieve optimization. In most of the models treated, it is assumed that the relevant information to be used is available and correct, and that maintenance actions are carried out as specified in the maintenance policy being used or to be used.

Unfortunately, the most important factor in a great many actual maintenance operations is omitted, thus vitiating the solution theoretically determined.

The most important factor, inadvertently overlooked or deliberately ignored for the sake of mathematical tractability, is the fallibility of the maintenance performer. In actual practice (as contrasted with the model formulation), the maintenance performer may:

(1) Repair the wrong part.

(2) Only partially repair the faulty part.

(3) Repair (partially or completely) the faulty part, but damage adjacent parts.

(4) Incorrectly assess the condition of the unit inspected.

(5) Perform the maintenance action not when called for, but at his convenience.

Clearly, the list of imperfect and even destructive repair actions occurring in actual maintenance may be extended much further. Apparently, we need mathematical models for maintenance which take explicit account of imperfect and/or destructive repair and faulty inspection.

In this paper, we formulate a variety of more realistic models which incorporate explicitly imperfect maintenance actions, postulating a probabilistic basis for their occurrence. We do not attempt to carry out solutions to the problems arising within these models, unselfishly leaving this challenging and enjoyable (?) task to the eager doctoral candidate looking for a dissertation topic and the young professor searching for problem areas yielding publications so vital for tenure and promotion. We do, however, summarize the main results of one study we carried out in detail of the type described above.

Throughout we assume a life distribution of F(t) for the unit being maintained.

2. Planned Replacement Based on Time Elapsed

First we succinctly describe three basic maintenance models which have been widely used. (See Barlow and Proschan, 1965, Chapters 3 and 4). In these classical models, no provision has been made for imperfect maintenance. We then describe a variety of imperfect repair or inspection actions on a probabilistic basis with their resultant adverse effects and costs. Some or all of these difficulties may be incorporated into the three models as originally formulated to achieve more realistic descriptions of maintenance situations as they actually occur in practice.

2.1 Age Replacement

A unit is replaced at failure or at age T (constant), whichever comes first. Operation and replacement continue indefinitely in this fashion. Given the cost c_1 of failure during operation and the cost $c_2(< c_1)$ of planned replacement, describe the operating characteristics of this policy and determine the optimal value of T, i.e., the value minimizing the long-run cost per unit of time. Compute the resulting minimal cost per unit of time in the long run.

2.2 Block Replacement

A unit is replaced at fixed times T, 2T,..., . In addition, it is replaced at failure. The cost of failure replacement is c_1, while the cost of a planned replacement is $c_2(< c_1)$. Describe the operating characteristics of the policy and find the value of T minimizing the long run cost per unit of time. Compute the resulting minimum long run cost per unit of time.

In both the age replacement and block replacement policies, replacement of a failed unit is by a new unit. Thus the instant of replacement represents a regeneration point in the stochastic process.

2.3 Maintenance with Minimal Repair

A unit is replaced by a new unit at fixed time T. If failure occurs at time $t < T$, minimal repair occurs, returning the unit to the functioning state, but the condition of the unit is that of a unit of age t Any number of failures resulting in minimal repair may occur during [0,T]. The cost of minimal repair is c_2; the cost of planned replacement by a new unit at time T is c_1, and $c_1 > c_2$. As before, derive the operating characteristics of the policy, find the optimizing value of T, and compute the resulting minimal cost per unit of time in the long run.

We now formulate imperfect maintenance features on a probabilistic basis. These may be incorporated into any of the three basic models above to achieve a more realistic description of maintenance as it occurs in practice.

2.4 Imperfect Maintenance Features

(a) Planned replacement may occur not at the time or age planned but may deviate from this time by an amount D which is governed by a probability distribution F_D.

(b) The replacement is not always installed properly. With probability p it is installed properly, and with probability $q = 1 - p$ it is faultily installed; the resulting failure rate function $r_I(t) = ar(t)$, where $r(t)$ is the failure rate function corresponding to proper installation and $a > 1$.

(c) With probability d, $0 < d < 1$, failure of a unit is detected immediately; with probability density $g(t)$, failure of a unit is detected t units of time following its occurrence, $t > 0$. A cost of $c_3 t$ is incurred if failure remains undetected for t units of time. This cost is in addition to the cost c_1 of failure replacement.

(d) With probability q', minimal repair at time t_0 may be imperfectly performed, leading to a functioning unit of <u>effective</u> age $t_0 + m$, rather than the actual age t_0, $m > 0$. Each time a minimal repair is imperfectly performed, the repaired unit's effective age increases by m units of time.

(e) In addition to the random features corresponding to imperfect maintenance, information as to costs may be uncertain. Thus c_1, c_2, c_3 may be random, governed by distributions F_1, F_2, F_3, respectively.

3. Maintenance Based on Inspection

The models of Section 2 called for maintenance actions based on unit age or chronological time. Another class of maintenance policies calls for maintenance actions based on the physical condition of the unit. Of course, these policies require inspection of the unit according to some plan.

3.1 Periodic Inspection

The simplest plan of inspection is to inspect the unit periodically at interval I, say. For simplicity, we assume that the purpose of the inspection is to determine whether the unit is functioning or failed; a failed unit would remain undetected unless specifically inspected. (An example is a battery serving as a spare; another example is a fire detection device. In the biological realm, the occurrence of cancer in the early stages is an example).

Suppose the cost of each inspection is c_I and the cost of undetected failure is c_U per unit of time between failure and its detection. We wish to determine the optimal value of I (periodic inspection interval) to minimize total expected costs.

3.2 Inspection Interval Dependent on Age

For an item with increasing failure rate, it would be reasonable to schedule inspections more and more frequently as the age of the item increased. Thus, inspections might be scheduled at ages $x_1 < x_2, \ldots$, where $(x_2 - x_1) > (x_3 - x_2) > \cdots$. From a knowledge of c_I, c_U, and the failure rate function r(t) of the unit, we wish to find the values of $x_1, x_2, \ldots$, which minimize total expected costs.

3.3 Identifying Failed Unit(s)

A system fails. We inspect the components in sequence to identify the failed units and then replace them in order to resume system functioning. The reliability of component i is p_i and the cost of inspecting component i is c_i, $i = 1, \ldots, n$. Our aim is not to identify all failures (and, of course, to replace them), but only a set sufficient to permit the resumption of system functioning. For example, consider the following system.

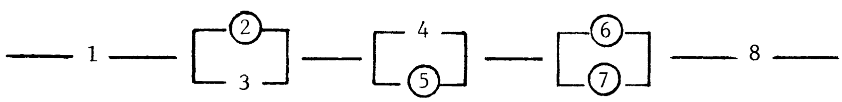

Diagram 1. System diagram showing failed components. Failure is indicated by a circle around component number.

It suffices to identify components 6 and 7 as failed and to replace them to initiate resumption of system functioning.

What is the optimal sequence of component inspection? That is, what sequence of component inspection will result in resumption of system functioning at minimal expected cost?

3.4 Imperfect Inspection Features

(a) Inspection scheduled for time t_0 say, may actually occur at time $t_0 + D$, D random.

(b) Inspection may be imperfect so that with probability q_1 a functioning unit is labeled failed and with probability q_2 a failed unit is labeled functioning. In Model 3.3, these probabilities may differ for different components of the system, so that for component i, these probabilities become q_{1i} and q_{2i}, $i = 1,\ldots,n$.

(c) Knowledge of costs may be imperfect, so that the various cost parameters may be considered as random variables.

4. Summary of Results for an Imperfect Repair Model

Brown and Proschan (1980) formulate and analyze an imperfect repair model. They obtain the operating characteristics of the stochastic process generated and monotonicity properties for various parameters and random variables. They do not impose any cost structure nor attempt any optimization. In this section we describe the model and summarize the results obtained. Here we attempt to interpret the results; for mathematical proofs we refer the interested reader to the original paper.

4.1 Model

An item is repaired at failure. With probability p, it is returned to the "good as new" state (perfect repair); with probability $q = 1 - p$, it is returned to the functioning state, but is only as good as an item of age equal to its age at failure (imperfect repair). Repair takes negligible time. The process of alternating failure and repair continues indefinitely over time; we call it a failure process. Finally, suppose the item has underlying life distribution F with failure rate function r(t).

4.2 Stochastic Results Obtained

For the case $p = 0$ (all repairs are imperfect), the failure process is a nonhomogeneous Poisson process with intensity function r(t).

For the more realistic case of $0 < p < 1$, we note that the time points of perfect repair are regeneration points. The interval between successive regeneration points is the waiting time for a perfect repair starting with a new component.

Let F_p denote the waiting time distribution for a perfect repair starting with a new component. Let r_p denote its failure rate function. Then we show:

LEMMA 1: (i) $r_p(t) = pr(t)$. (ii) $\overline{F}_p(t) = \overline{F}^p(t)$.

Aside from its immediate value in understanding the imperfect repair model, the results of Lemma 1 are interesting in that they represent a physically motivated example of the well-known and widely used model of proportional hazards. The assumption of proportional hazards is often made for mathematical convenience, especially in competing risk theory. Here we see one realistic instance of its occurrence in maintenance theory.

Since the original failure rate function is simply multiplied by p, it follows that many of the important classes of distributions characterized by aging properties are preserved in the following sense: If F has a given aging property, then F_p also has this property for $0 < p < 1$. Formally stated, we have:

THEOREM 1: Let F be in any of the classes: IFR, DFR, IFRA, DFRA, NBU, NWU, DMRL, or IMRL. Then F_p is in the same class.

Theorem 1 cannot be extended to the NBUE and NWUE cases.

Monotonicity Properties

Let $\mu(p)$ denote the mean of F_p. Clearly $\mu(p)$ is a decreasing function. However, the next theorem shows that a reversal in the direction of monotonicity may be achieved by weighting $\mu(p)$ appropriately.

THEOREM 2: (i) Let F_p be NBUE for all p in (0,1]. Then $p\mu(p)$ is increasing for $p \varepsilon (0,1]$. In particular, this monotonicity holds for F NBU or DMRL.

(ii) As an immediate consequence of (i), when F_p is NBUE for all p in (0,1], we have the bound:

$$\mu(p) \leq \frac{1}{p}\mu(1) \quad .$$

(iii) Dual results hold for the NWUE, NWU, and IMRL classes.

The inequality $p_1\mu(p_1) \leq p_2\mu(p_2)$ for $p_1 < p_2$ when F_p is NBUE for all $p \varepsilon (0,1]$ can be interpreted as: F_{p_1} is smaller in expectation than a geometric sum (with parameter p_1/p_2) of i.i.d. random variables having distribution F_{p_2}. When F is NBU, "smaller in expectation" can be strengthened to "stochastically smaller". A dual result holds for F NWU.

Let $N_p(t)$ denote the number of failures in [0,1) for the failure process in which perfect repair has probability p. We can prove that for F NBU, $N_p(t)$ is stochastically decreasing in p. Intuitively, this is reasonable since greater p leads to a quicker return to the "good as new" state.

A stronger conclusion can be obtained under the stronger assumption of IFR:

THEOREM 3: Let F be IFR. Let Z_p denote the waiting time until the next failure, starting in steady state. Let h_p denote the failure rate function of Z_p. Then (i) for each $t \geq 0$, $h_p(t)$ is decreasing in p; (ii) as a consequence, Z_p is stochastically increasing in p. (iii) Dual results hold for F DFR.

A weaker conclusion is obtained under the weak assumption of F DMRL (IMRL):

COROLLARY 1: Let F be DMRL(IMRL). Then EZ_p is increasing (decreasing) in p.

In all the results above, conclusions were obtained for dual families of distributions corresponding to deterioration with age and improvement with age. The following result applies for DFR distributions, but the dual result is known not to necessarily hold for IFR distributions.

THEOREM 4: Let F be DFR. Let $Z_p(t)$ denote the waiting time at t for the next failure to occur; let $Z_p^*(t)$ denote the waiting time at t for the next perfect repair. Let $m_p(t)$ denote the failure intensity at t and let $m_p^*(t)$ denote the renewal density at t for the renewal process with interarrival time distribution F_p. Finally, let $A_p(t)$ denote the effective age at time t, i.e., the time elapsed since the last perfect repair. Then:

(i) $A_p(t)$, $Z_p(t)$ and $Z_p^*(t)$ are stochastically increasing in t for fixed p; $m_p(t)$ and $m_p^*(t)$ are decreasing in t for fixed p.

(ii) $A_p(t)$ and $Z_p(t)$ are stochastically decreasing in p for fixed t, $m_p(t)$ is increasing in p for fixed t.

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REFERENCES

Barlow, R.E. and Proschan, F. (1965). Mathematical Theory of Reliability. Wiley, New York.

Brown, M. and Proschan, F. (1980). Imperfect repair. Florida State University, Department of Statistics Report AFOSR-78-108. Accepted, Journal of Applied Probability.