Chapter 4

Basic Dense Matrix Operations

The following routines are described in the following pages:

Catch errors		51
Error handlers and extensions		53
Error handling style		57
Copy objects		59
Input object from file		62
Output to file	t in the	65
General input/output		67
Deallocate (destroy) objects		68
Create and initialise objects		70
Extract column/row from matrix		72
Initialisation routines		73
Input object from stdin		62
Inner product		75
Operations on integer vectors		76
Resize data structures		77
Machine epsilon		80
Matrix addition and multiplication		81
Memory allocation information		83
Static workspace control functions		88
Matrix transposes, adjoints and multiplication		93
Matrix norms		94
Matrix-vector multiplication		96
Continued		

CHAPTER 4. BASIC DENSE MATRIX OPERATIONS

• • .

2

Output object to stdout	65
Permutation identity, multiplication and inverse	98
Permute columns/rows & permute vectors	99
Set column/row of matrix	101
Scalar-vector multiplication/addition	102
Componentwise operations	104
Linear combinations of arrays and lists	107
Vector norms	109
Operations on complex numbers	111
Core low level routines	113

To use these routines use the include statement

#include "matrix.h"

To use the complex variants use the include statement

#include "zmatrix.h"

catch, catchall, catch_FPE, tracecatch - catch errors SYNOPSIS

DESCRIPTION

The catch() macro provides a way of interposing your own error-handling routines and code in the usual error-handling procedures. The catch() macro works like this: The global variable restart (of type jmp_buf) is saved. Then the code normal_code_to_execute is executed. If an error with error number err_num is raised, then code_to_execute_if_error is executed. If an error with another error number is raised, an error will be raised with the same error number as the original error, but will appear to have come from the catch() macro. If no error is raised then the macro will exit and restart is reset to its old values.

The catchall() macro works just like the catch() macro except that code_to_execute_if_error is executed if *any* error is raised.

The tracecatch() macro is really a specialised version of the catchall() macro that sets the error-handling flag to print out the underlying error when it is raised.

In every case the old error handling status will be restored on exiting the macro.

The routine catch_FPE() sets up a signal handler so that if a SIGFPE signal is raised, it is caught and error() is called as appropriate. The error raised by error() is an E_SIGNAL error.

EXAMPLE

```
main()
{
    MAT *A;
    PERM *pivot;
    VEC *x, *b;
    .....
    tracecatch(
      LUfactor(A,pivot);
      LUsolve(A,pivot,b,x);
      , "main");
```

would result in the error messages

"lufactor.c", line 28: NULL objects passed in function LUfactor() "junk.c", line 20: NULL objects passed in function main() Sorry, exiting program

being printed to stdout if one of A or pivot or b were NULL. These messages would also be printed out to stderr if stdout is not a terminal.

On the other hand,

```
catch(E_NULL,
```

```
LUfactor(A,pi);
LUsolve(A,pi,b,x);
```

, printf("Ooops, found a NULL object\n"));

simply produces the message Ooops, found a NULL object in this case.

However, if another error occurs (say, **b** is the wrong size) then **LUsolve()** raises an **e_SIZES** error, and

```
"junk.c", line 22: sizes of objects don't match in
function catch()
Sorry, exiting program
```

is printed out.

SEE ALSO

```
signal(), error(), set_err_flag(), ERREXIT() etc.
```

BUGS

If a different error to the one caught in catch() is raised, then the file and line numbers of the original error are lost.

In an if-then-else statement, tracecatch() needs to be enclosed by braces $(\{\ldots\})$.

SOURCE FILE: matrix.h

error, set_err_flag, ev_err, err_list_attach, err_is_list_attached, err_list_free, warning - raise errors and warnings

SYNOPSIS

#inclu	ıde "matrix.h"
int	error(int err_num, char *func_name)
int	<pre>ev_err(char *file, int err_num, int line_num,</pre>
	<pre>char *fn_name, int list_num)</pre>
int	<pre>set_err_flag(int new_flag)</pre>
int	<pre>err_list_attach(int list_num, int list_len,</pre>
	<pre>char **err_ptr, int warn)</pre>
int	err_list_free(int list_num)
int	err_is_list_attached(int list_num)
int	warning(int warn_num, char *func_name)

DESCRIPTION

This is where errors are flagged in the system. The call error(err_num, func_name) is in fact a macro which expands to

ev_err(__FILE__,err_num,__LINE__,func_name,0)

This call does not return.

Warnings are raised by warning (warn_num, func_name) which are expands to

ev_err(__FILE__,warn_num,__LINE__,func_name,1)

This call returns zero.

The call to ev_err() prints out a message to stderr indicating that an error has occurred, and where in which function it occurred, and the list of error messages to use (0 is the default). For example, it could look like:

"test1.c", line 79: sizes of objects don't match in function f()

which indicates that an error was flagged in file "testl.c" at line 79, function "f" where the sizes of two objects (vectors in this case) were incompatible.

Once this information is printed out, control is passed to the the address saved in the buffer called **restart** by the last associated call to **setjmp**. The most convenient way of setting up **restart** is to use a ...catch...() macro or by an **ERREXIT**() or **ERRABORT**() macro. If **restart** has not been set then the program exits.

If you wish to do something particular if a certain error occurs, then you could include a code fragment into main() such as the following:

```
if ( (code=setjmp(restart)) != 0 )
{
    if ( code = E_MEM ) /* memory error, say */
        /* something particular */
        { .... }
    else
        exit(0);
}
else
    /* make sure that error handler does jump */
    set_err_flag(EF_JUMP);
```

The list of standard error numbers is given below:

E_UNKNOWN	0	/*	unknown error (unused) */
E_SIZES	1	/*	incompatible sizes */
E_BOUNDS	2	/*	index out of bounds */
E_MEM	3	/*	memory (de)allocation error */
E_SING	4	/*	singular matrix */
E_POSDEF	5	/*	matrix not positive definite */
E_FORMAT	6	/*	incorrect format input */
E_INPUT	7	/*	<pre>bad input file/device */</pre>
E_NULL	8	/*	NULL object passed */
E_SQUARE	9	/*	matrix not square */
E_RANGE	10	/*	object out of range */
E_INSITU2	11	/*	only in-situ for square matrices */
E_INSITU	12	/*	can't do operation in-situ */
E_ITER	13	/*	too many iterations */
E_CONV	14	/*	convergence criterion failed */
E_START	15	/*	bad starting value */
E_SIGNAL	16	/*	floating exception */
E_INTERN	17	/*	some internal error */
E_EOF	18	/*	unexpected end-of-file */
E_SHARED_VECS	19	/*	cannot release shared vectors */
E_NEG	20	/*	negative argument */
E_OVERWRITE	21	/*	cannot overwrite object */

The set_err_flag() routine sets a flag which controls the behaviour of the error handling routine. The old value of this flag is returned, so that it can be restored if necessary.

The list of values of this flag are given below:

EF_EXIT	0	/*	exit on error default */	
EF_ABORT	1	/*	abort on error dump core */	
EF_JUMP	2	/*	do longjmp() see above code */	
EF_SILENT	3	/*	do not report error, but do longjmp() *	*/

If there is a just a warning, then the default behaviour is to print out a message to **stdout**, and possibly **stderr**; the only value of the flag which has any effect is **EF_SILENT**. This suppresses the printing.

The set of error messages, and the set of errors, can be expanded on demand by the user by means of err_list_attach(list_num, list_len, err_ptr, warn). The list number list_num should be greater than one (as numbers zero and one are taken by the standard system). The parameter list_len is the number of errors and error messages. The parameter err_ptr is an array of list_len strings. The parameter warn is TRUE or FALSE depending on whether this class of "errors" should be regarded as being just warnings, or whether they are (potentially) fatal. Then when an "error" should be raised, call

```
ev_err(___FILE__, err_num, __LINE__, func_name, list_num);
```

It may well be worthwhile to write a macro such as:

```
#define my_error(my_err_num,func_name) \
        ev_err(__FILE__,err_num,__LINE__,func_name,list_num)
```

If when originally set, the warn parameter was TRUE, then these calls behave similarly to warning(), and if it was FALSE, then these calls behave similarly to error(). These errors and exceptions are controlled using catch(), catchall() and tracecatch() (if warn was FALSE), just as for error() calls.

The call err_list_free(list_num) unattaches the error list numbered list_num, and allows it to be re-used.

The call err_is_list_attached(list_num) returns TRUE if error list list_num is attached, and FALSE otherwise. This can be used to find the next available free list.

EXAMPLE

Use of error() and warning():

```
if ( ! A ) error(E_NULL, "my_function");
if ( A->m != A->n ) error(E_SQUARE, "my_function");
if ( i < 0 || i >= A->m ) error(E_BOUNDS, "my_function");
/* this should never happen */
if ( panic && something_really_bad )
error(E_INTERN, "my_function");
/* issue a warning -- can still continue */
warning(WARN UNKNOWN, "my function");
```

```
Use of err_list_attach():
char *my_list[] = { "short circuit", "open circuit" };
      my_list_num = 0;
int
main()
{
    for ( my_list_num = 0; ; my_list_num++ )
        if ( ! err is list attached(my list num) )
             break;
    err_list_attach(my_list_num,2,my_list,FALSE);
       . . . . . .
    tracecatch(circuit_simulator(....), "main");
       .......
    err list free(my_list_num);
}
void circuit simulator(....)
ſ
       . . . . . .
    /* open circuit error */
    ev_err(___FILE___,1,___LINE___,
            "circuit_simulator",my_list_num);
       . . . . .
}
```

SEE ALSO

```
ERREXIT(), ERRABORT(), setjmp() and longjmp().
```

BUGS

Not many routines use tracecatch(), so that the trace is far from complete. Debuggers are needed in this case, if only to obtain a backtrace.

SOURCE FILE: err.c

ERREXIT, ERRABORT, ON_ERROR – what to do on error

SYNOPSIS

```
#include "matrix.h"
ERREXIT();
ERRABORT();
ON_ERROR();
```

DESCRIPTION

If ERREXIT() is called, then the program exits once the error occurs, and the error message is printed. This is the default.

If ERRABORT() is called, then the program aborts once the error occurs, and the error message is printed. Aborting in Unix systems means that a core file is dumped and can be analysed, for example, by (symbolic) debuggers. Behaviour on non-Unix systems is undefined.

If ON_ERROR() is called, the current place is set as the default return point if an error is raised, though this can be modified by the catch() macro. The ON_ERROR() call can be put at the beginning of a main program so that control always returns to the start. One way of using it is as follows:

```
main()
{
    .....
    ON_ERROR();
    printf("At start of program; restarts on error\n");
    /* initialisation stuff here */
    .....
    /* real work here */
    .....
}
```

This is a slightly dangerous way of doing things, but may be useful for implementing matrix calculator type programs.

Other, more sophisticated, things can be done with error handlers and error handling, though the topic is too advanced to be treated in detail here.

SEE ALSO

```
error() and ev_err().
```

BUGS

With all of these routines, care must be taken not to use them inside called functions, unless the calling function immediately re-sets the **restart** buffer after the called

function returns. Otherwise the **restart** buffer will reference a point on the stack which will be overwritten by subsequent calculations and function calls. This is a problem inherent in the use of **setjmp()** and **longjmp()**. The only way around this problem is through the implementation of co-routines.

With ON_ERROR(), infinite loops can occur very easily.

SOURCE FILE: matrix.h

bd_copy, iv_copy, px_copy, m_copy, v_copy, zm_copy, zv_copy, m_move, v_move, zm_move, zv_move - copy objects SYNOPSIS

#include "matrix.h" BAND *bd_copy(BAND *in, BAND *out) IVEC *iv_copy(IVEC *in, IVEC *out) MAT *m_copy (MAT *in, MAT *out) MAT *_m_copy(MAT *in, MAT *out, int i0, int j0) *px copy(PERM *in, PERM *out) PERM *v_copy (VEC *in, VEC *out) VEC VEC *_v_copy(VEC *in, VEC *out, int i0) *m_move (MAT *in, int i0, int j0, int m0, int n0, MAT MAT *out, int i1, int j1) *v move (VEC *in, int i0, int dim0, VEC VEC *out, int i1) *mv move(MAT *in, int i0, int j0, int m0, int n0, VEC VEC *out, int i1) *vm move(VEC *in, int i0, MAT MAT *out, int i1, int j1, int m1, int n1) #include "zmatrix.h" ZMAT *zm_copy(ZMAT *in, ZMAT *out) ZMAT * zm copy(ZMAT *in, ZMAT *out, int i0, int j0) ZVEC *zv_copy(ZVEC *in, ZVEC *out) * zv copy(ZVEC *in, ZVEC *out) ZVEC *zm_move (ZMAT *in, int i0, int j0, int m0, int n0, ZMAT ZMAT *out, int i1, int j1) *zv_move (ZVEC *in, int i0, int dim0, ZVEC ZVEC *out, int i1) *zmv_move(ZMAT *in, int i0, int j0, int m0, int n0, ZVEC ZVEC *out, int i1) *zvm_move(ZVEC *in, int i0, ZMAT ZMAT *out, int i1, int j1, int m1, int n1)

DESCRIPTION

All the routines bd_copy(), iv_copy(), m_copy(), px_copy(), v_copy(), zm_copy() and zv_copy() copy all of the data from one data structure to another, creating a new object if necessary (i.e. a NULL object is passed or out is not sufficiently big), by means of a call to bd_get(), iv_get(), m_get(), px_get() or v_get() etc. as appropriate. For m_copy(), v_copy(), bd_copy(), iv_copy(), zm_copy(), and zv_copy() if in is smaller than the object out, then it is copied into a region in out of the same size. If the sizes of the permutations differ in px_copy() then a new permutation is created and returned.

The "raw" copy routines are _m_copy(in,out,i0,j0) and _v_copy(in,out,i0). Here (i0,j0) is the position where the (0,0) element of the in matrix is copied to; in is copied into a block of out. Similarly, for _v_copy(), i0 is the position of out where the zero element of in is copied to; in is copied to a block of components of out.

The .._copy() routines all work *in situ* with in == out, however, the _._copy() routines will only work *in situ* if i0 (and also j0 if this is also passed) is (are) zero.

The complex routines $zm_copy(in,out)$, $zv_copy(in,out)$, and their "raw" versions $_zm_copy(in,out,i0,j0)$ and $_zv_copy(int,out,i0)$ operate entirely analogously to their real counterparts.

The routines .._move() move blocks between matrices and vectors. A source block in a matrix is identified by the matrix structure (in), the co-ordinates ((i0, j0)) of the top left corner of the block and the number of rows (m0) and columns (n0) of the block. The target block of a matrix is identified by out and the co-ordinates of the top left corner of the block ((i1, j1)), except in the case of moving a block from a vector to a matrix (vm_move()). In that case the number of rows and columns of the target need to be specified.

The source block of a vector is identified by the source vector (in), the starting index of the block (i0) and the dimension of the block (dim0). The target block of a vector is identified by the target vector out and the starting index (i1).

The routine m_move() moves blocks between matrices, v_move() moves blocks between vectors, mv_move() moves blocks from matrices to vectors (copying by rows), and vm_move() moves blocks from vectors to matrices (again copying by rows). The routine zm_move() moves blocks between complex matrices, zv_move() moves blocks between complex vectors, zmv_move() moves blocks from complex matrices to complex vectors (copying by rows), and zvm_move() moves blocks from complex vectors to complex matrices (again copying by rows).

EXAMPLE

```
/* copy x to y */
v_copy(x,y);
/* create a new vector z = x */
z = v_copy(x,VNULL);
/* copy A to the block in B with top-left corner (3,5) */
_m_copy(A,B,3,5);
/* an equivalent operation with m_move() */
m_move(A,0,0,A->m,A->n, B,3,5);
```

/* copy a matrix into a block in a vector ... */
mv_move(A,0,0,A->m,A->n, y,3);
/* ... and restore the matrix */
vm_move(y,3,A->m*A->n, A,0,0,A->m,A->n);
/* construct a block diagonal matrix C = diag(A,B) */
C = m_get(A->m+B->m,A->n+B->n);
m_move(A,0,0,A->m,A->n, C,0, 0);
m_move(B,0,0,B->m,B->n, C,A->m,A->n);

SEE ALSO

.._get() routines

SOURCE FILE: copy.h, ivecop.c, zcopy.c, bdfactor.c

이 이 가지 않는 것이 같아.

61

```
NAME
  iv_finput, m_finput, px_finput, v_finput, z_finput,
  zm_finput, zv_finput - input object from a file
SYNOPSIS
#include
          <stdio.h>
           "matrix.h"
#include
        *iv_finput(FILE *fp, IVEC *iv)
IVEC
iv = iv_finput(fp,VNULL);
        *m_finput(FILE *fp, MAT *A)
MAT
A = m finput(fp, MNULL);
        *px_finput(FILE *fp, PERM *pi)
PERM
pi = px_finput(fp,PXNULL);
        *v_finput(FILE *fp, VEC *v)
VEC
v = v_finput(fp,VNULL);
complex
       z_finput(FILE *)
z = z_finput(fp);
ZMAT
        *zm_finput(FILE *fp, ZMAT *A)
A = zm_finput(fp, ZMNULL);
        *zv_finput(FILE *fp, ZVEC *v)
ZVEC
v = zv_finput(fp,ZVNULL);
```

DESCRIPTION

These functions read in objects from the specified file. These functions first determine if fp is a file pointer for a "tty" (i.e. keyboard/terminal). There are also the macros m_input(A), px_input(pi), v_input(x), zm_input(A), zv_input(x), and which are equivalent to m_finput(stdin,A), px_finput(stdin,pi), v_finput(stdin,x), zm_finput(stdin,A), and zv_finput(stdin,x) respectively. If so, then an interactive version of the input functions is called; if not, then a "file" version of the input functions is called.

The interactive input prompts the user for input for the various entries of an object; the file input simply reads input from the file (or pipe, or device etc.) and parses it as necessary. For complex numbers, the format is different between interactive and file input: interactive input has the format "x y" or just "x" for zero real part. File input of complex numbers uses (x, y). For example, -3.2 + 5.1i is entered as -3.2 + 5.1 in interactive mode, and as (-3.2, 5.1) in file mode.

62

Note that the format for file input is essentially the same as the output produced by the .._foutput() and .._output() functions. This means that if the output is sent to a file or to a pipe, then it can be read in again without modification. Note also that for file input, that lines before the start of the data that begin with a "#" are treated as comments and ignored. For example, this might be the contents of a file my.dat:

```
# this is an example
# of a matrix input
Matrix: 3 by 4
row 0: 0
             1
                  -2
                        -1
                         2
row 1:-2
             0
                  1.5
row 2: 5
                  0.5
                         0
            -4
# this is an example
# a vector input
Vector: dim: 4
               -1.372
2
        7
                         3.4
# this is an example
# of a permutation input
Permutation: size: 4
 0 \rightarrow 1 \ 1 \rightarrow 3 \ 2 \rightarrow 0 \ 3 \rightarrow 2
# this is a complex number
(3.765, -1.465324)
# this is a complex matrix
ComplexMatrix: 3 by 4
row 0:
         (1, 0)
                  (-2, 0)
                           (3, 0)
                                    (-1,0)
row 1:
         (5, 3)
                  (-2, -3) (1, -4) (2, 1)
         (1, 0)
row 2:
                  (2.5, 0)
                              (2.5, -3.56)
                                               (2.5,0)
# and this is a complex vector...
ComplexVector: dim: 3
        -1.342235,
 (
                               -1.342)
                                         (2.3, -5)
 (
                  1,
                                     1)
```

Interactive input is read line by line. This means that only one data item can be entered at a time. A user can also go backwards and forwards through a matrix or vector by entering "b" or "f" instead of entering data. Entering invalid data (such as hitting the return key) is not accepted; you must enter valid data before going on to the next entry. When permutations are entered, the value given is checked to see if lies within the acceptable range, and if that value had been given previously.

If the input routines are passed a NULL object, they create a new object of the size determined by the input. Otherwise, for interactive input, the size of the object passed must have the same size as the object being read, and the data is entered into the object

passed to the input routine. For file input, if the object passed to the input routine has a different size to that read in, a new object is created and data entered in it, which is then returned.

EXAMPLE

The above input file can be read in from **stdin** using:

```
complex z;
MAT
      *A:
VEC
      *b;
PERM *pi;
ZMAT *zA;
ZVEC *zv;
     m input(MNULL);
A
   007330
636385
   = v_input(VNULL);
b
pi = px_input(PXNULL);
   = z_input();
\mathbf{z}
zA = zm input(ZMNULL);
zv = zv input(ZVNULL);
```

If you know that a vector must have dimension *m* for interactive input, use:

```
b = v_get(m);
v_input(b); /* use b's allocated memory */
```

SEE ALSO

.._output() entries, .._input() entries

BUGS

Memory can be lost forever; objects should be resize'd.

On end-of-file, an "unexpected end-of-file" error (E_EOF) is raised.

Note that the test for whether the input is an interactive device is made by **isatty(fileno(fp))**. This may not be portable to some systems.

Interactive complex input does not allow (x, y) format; nor does it allow entry of the imaginary part without the real part.

SOURCE FILE: matrixio.c, zmatio.c

NAME iv_foutput, m_foutput, px_foutput, v_foutput, z_foutput, zm_foutput, zv_foutput, iv_dump, m_dump, px_dump, v_dump, **zm_dump**, **zv_dump** – output to a file or stream SYNOPSIS #include "matrix.h" void iv_foutput(FILE *fp, IVEC *v) void m foutput(FILE *fp, MAT *A) voiđ px foutput(FILE *fp, PERM *pi) v_foutput(FILE *fp, VEC void *v) #include "zmatrix.h" z_foutput(FILE *fp, complex z) void zm foutput(FILE *fp, ZMAT *A) void void zv foutput(FILE *fp, ZVEC *v)

DESCRIPTION

These output is a representation of the respective objects to the file (or device, or pipe etc.) designated by the file pointer fp. The format in which data is printed out is meant to be both human and machine readable; that is, there is sufficient information for people to understand what is printed out, and furthermore, the format can be read in by the .._finput() and .._input() routines.

An example of the format for matrices is given in the entry for the .._finput() routines.

There are also the routines m_output (A), px_output (pi) and v_output (x) which are equivalent to m_foutput(stdout, A), px_foutput(stdout, pi) and v_foutput(stdout, x) respectively.

Note that the .._output() routines are in fact just macros which translate into calls of these .._foutput() routines with "fp = stdin".

In addition there are a number of routines for dumping the data structures in their entirety for debugging purposes. These routines are m_dump(fp,A),px_dump(fp,px), v_dump(fp,x), zm_dump(fp, zA) and zv_dump(fp, zv) where fp is a FILE *, A is a MAT *, px is a PERM * and x is a VEC *, zA is a ZMAT *, and zv is a ZVEC *. These print out pointers (as hex numbers), the maximum values of various quantities (such as max_dim for a vector), as well as all the quantities normally printed out. The output from these routines is not machine readable, and can be quite verbose.

EXAMPLE

/* output A to stdout */

m_output(A);
/* ...or to file junk.out */
if ((fp = fopen("junk.out","w")) == NULL)
 error(E_EOF,"my_function");
m_foutput(fp,A);
/* ...but for debugging, you may need... */
m_dump(stdout,A);

SEE ALSO

.._finput(), .._input()

SOURCE FILE: matrixio.c, zmatio.c

finput, input, fprompter, prompter – general input/output routines SYNOPSIS

#include <stdio.h>

```
#include "matrix.h"
```

```
int finput(FILE *fp, char *prompt, char *fmt, void *var)
int input(char *prompt, char *fmt, void *var)
int fprompter(FILE *fp, char *prompt)
int prompter(char *prompt)
```

DESCRIPTION

The macros finput() and input() are for general input, allowing for comments as accepted by the .._finput() routines. That is, if input is from a file, then comments (text following a '#' until the end of the line) are skipped, and if input is from a terminal, then the string prompt is printed to stderr. The input is read for the file/stream fp by finput() and by stdin by input(). The fmt argument is a string containing the scanf() format, and var is the argument expected by scanf() according to the format string fmt.

For example, to read in a file name of no more than 30 characters from stdin, use

```
char fname[31];
.....
input("Input file name: ","%30s",fname);
```

The macros fprompter() and prompter() send the prompt string to stderr if the input file/stream (fp in the case of fprompter(), stdin for prompter()) is a terminal; otherwise any comments are skipped over.

SEE ALSO

scanf(), .._finput()

SOURCE FILE: matrix.h

IV_FREE, M_FREE, PX_FREE, V_FREE, ZM_FREE, ZV_FREE, iv_free_vars, m_free_vars, px_free_vars, v_free_vars, zm_free_vars, zv_free_vars - destroy objects and free up memory SYNOPSIS

```
#include "matrix.h"
void IV_FREE(IVEC *iv)
void M_FREE (MAT *A)
void PX FREE(PERM *pi)
void V FREE (VEC *v)
int iv_free_vars(IVEC **iv1, IVEC **iv2, ..., NULL)
                                   **A2,
                       **A1,
int m_free_vars(MAT
                              MAT
                                          ..., NULL)
int px_free_vars(PERM **pi1, PERM **pi2,
                                          ..., NULL)
     v_free_vars(VEC
                       **v1, VEC
                                   **v2,
int
                                          ..., NULL)
#include "zmatrix.h"
void ZM_FREE(ZMAT *A)
void ZV FREE(ZVEC *v)
int zm_free_vars(ZMAT
                        **A1, ZMAT
                                   **A2, ..., NULL)
int zv_free_vars(ZVEC
                        **v1, ZVEC
                                    **v2, ..., NULL)
```

DESCRIPTION

The .._FREE() routines are in fact all macros which result in calls to thje corresponding .._free() function, so that IV_FREE(iv) calls iv_free(iv). The effect of calling .._free() is to release all the memory associated with the object passed. The effect of the macros .._FREE(object) is to firstly release all the memory associated with the object passed, and to then set object to have the value NULL. The reason for using macros is to avoid the "dangling pointer" problem.

The problems of dangling pointers cannot be entirely overcome within a conventional language, such as 'C', as the following code illustrates:

The .._free_vars() functions free a NULL-terminated list of pointers to variables all of the same type. Calling

.._free_vars(&x1,&x2,...,&xN,NULL)

is equivalent to

.._free(x1); x1 = NULL; .._free(x2); x2 = NULL;_free(xN); xN = NULL;

The returned value of the .._free_vars() routines is the number of objects freed.

SEE ALSO

.._get() routines

BUGS

Dangling pointer problem is neither entirely fixed, nor is it fixable.

SOURCE FILE: memory.c, zmemory.c

NAME bd_get, iv_get, m_get, px_get, v_get, zm_get, zv_get, iv_get_vars, m_get_vars, px_get_vars, v_get_vars, zm_get_vars, zv_get_vars - create and initialise objects SYNOPSIS #include "matrix.h" BAND *bd_get(int lb, int ub, int n)

```
IVEC
       *iv_get(unsigned dim)
MAT
       * m_get(unsigned m, unsigned n)
PERM
       *px get(unsigned size)
VEC
       * v get(unsigned dim)
int
       *iv_get_vars(unsigned dim,
                     IVEC **x1, IVEC **x2, ..., NULL)
int
       * m_get_vars(unsigned m, unsigned n,
                     MAT **A1, MAT **A2, ..., NULL)
int
       *px get vars(unsigned size,
                     PERM **px1, PERM **px2, ..., NULL)
int
       * v_get_vars(unsigned dim,
                     VEC **x1, VEC **x2, ..., NULL)
```

DESCRIPTION

All these routines create and initialise data structures for the associated type of objects. Any extra memory needed is obtained from malloc() and its related routines.

Also note that *zero relative* indexing is used; that is, the vector \mathbf{x} returned by $\mathbf{x} = \mathbf{v}_{get}(10)$ can have indexes $\mathbf{x}_{ve[i]}$ for i equal to 0, 1, 2, ..., 9, not 1, 2, ..., 9, 10. This also applies for both the rows and columns of a matrix.

The $bd_get(lb, ub, n)$ routine creates a band matrix of size $n \times n$ with a lower bandwidth of lb and an upper banwidth of ub. The $iv_get(dim)$ routine creates an integer vector of dimension dim. Its entries are initialised to be zero. The $m_get(m, n)$ routine creates a matrix of size $m \times n$. That is, it has m rows and n columns. The matrix elements are all initialised to being zero. The $px_get(size)$ routine creates and returns a permutation of size size. Its entries are initialised to being those of an identity permutation. Consistent with C's array index conventions, a permutation of the given size is a permutation on the set $\{0, 1, \ldots, size-1\}$. The

v_get (dim) routine creates and returns a vector of dimension dim. Its entries are all initialised to zero.

The .._get_vars() routines allocate and initialise a NULL-terminated list of pointers to variables, all of the same type. All of the variables are initialised to objects of the same size. Calling

.._get_vars([m,]n,&x1,&x2,...,&xN,NULL)

is equivalent to

x1 = .._get([m,]n); x2 = .._get([m,]n);xN = .._get([m,]n);

(Note that "[m,]" indicates that "m," might or might not be present, depending on whether the data structure involved is a matrix or not.) The returned value of the .._get_vars() routines is the number of objects created.

EXAMPLE

MAT *A; /* allocate 10 x 15 matrix */ A = m_get(10,15);

SEE ALSO

.._free(), .._FREE(), and .._resize().

BUGS

As dynamic memory allocation is used, and it is not possible to build garbage collection into C, memory can be lost. It is the programmer's responsibility to free allocated memory when it is no longer needed.

SOURCE FILE: memory.c, zmemory.c, bdfactor.c

NAME
get_col, get_row, zget_col, zget_row - extract columns or rows
from matrices
SYNOPSIS
#include "matrix.h"
VEC *get_col(MAT *A, int col_num, VEC *v)
VEC *get_row(MAT *A, int row_num, VEC *v)
#include "zmatrix.h"

ZVEC*zget_col(ZMAT *A, int col_num, ZVEC *v)ZVEC*zget_row(ZMAT *A, int row_num, ZVEC *v)

DESCRIPTION

These put the designated column or row of the matrix **A** and puts it into the vector **v**. If **v** is NULL or too small, then a new vector object is created and returned by $get_col()$ and $get_row()$. Otherwise, **v** is filled with the necessary data and is then returned. If **v** is larger than necessary, then the additional entries of **v** are unchanged.

The complex routines operate exactly analogously to the real routines.

EXAMPLE

```
MAT *A;
VEC *row, *col;
int row_num, col_num;
.....
row = v_get(A->n);
col = v_get(A->m);
get_row(A, row_num, row);
get_col(A, col_num, col);
```

SEE ALSO

set_col(), set_row(), and zset_col(), zset_row().

SOURCE FILE: matop.c, zmatop.c

72

m_ident, m_ones, v_ones, m_rand, v_rand, m_zero, v_zero, zm_rand, zv_rand, zm_zero, zv_zero, mrand, smrand, mrandlist - initialisation routines SYNOPSIS

#include	e "matrix.h"
MAT	*m_ident(MAT *A)
MAT	*m_ones(MAT *A)
VEC	*v_ones(VEC *x)
MAT	*m_rand(MAT *A)
VEC	*v_rand(VEC *x)
MAT	*m_zero(MAT *A)
VEC	*v_zero(VEC *x)
Real	mrand()
void	smrand(int seed)
void	<pre>mrandlist(Real a[], int len)</pre>

```
#include "zmatrix.h"
ZMAT *zm rand(ZMAT *A)
```

	same an dense de l'annue a se	,
ZVEC	*zv_rand(ZVEC	*x)
ZMAT	*zm_zero(ZMAT	*A)
ZVEC	*zv_zero(ZVEC	*x)

DESCRIPTION

The routine m_ident() sets the matrix A to be the identity matrix. That is, the diagonal entries are set to 1, and the off-diagonal entries to 0.

The routines m_ones(), v_ones() fill A and x with ones.

The routines v_rand(), m_rand() and zv_rand(), zm_rand() fill A and x with random entries. For real vectors or matrices the entries are between zero and one as determined by the mrand() function. For complex vectors or matrices, the entries have both real and imaginary parts between zero and one as determined by the mrand() function.

The routines $m_zero()$, $v_zero()$ and $zm_zero()$, $zv_zero()$ fill A and x with zeros.

These routines will raise an **E_NULL** error if **A** is NULL.

The routine mrand() returns a pseudo-random number in the range [0, 1) using an algorithm based on Knuth's lagged Fibonacci method in *Seminumerical Algorithms:* The Art of Computer Programming, vol. 2 §§3.2–3.3. The implementation is based on that in Numerical Recipes in C, pp. 212–213, §7.1. Note that the seeds for mrand() are initialised using smrand() with a fixed seed. Thus mrand() will produce the

same pseudo-random sequence (unless **smrand()** is called) in different runs, different programs, and but for differences in floating point systems, on different machines.

The routine **smrand()** allows the user to re-set the seed values based on a userspecified **seed**. Thus **mrand()** can produce a wide variety of reproducible pseudorandom numbers.

The routine mrandlist() fills an array with pseudo-random numbers using the same algorithm as mrand(), but is somewhat faster for reasonably long vectors.

EXAMPLE

Let $e = [1, 1, ..., 1]^T$.

```
MAT
     *A;
ZMAT *zA;
VEC
     *x;
ZVEC *zx;
PERM *pi;
  . . . . . .
            /* A == zero matrix */
m_zero(A);
m ident(A);
               /* A == identity matrix */
m ones(A); /* A == e.e^T */
m rand(A);
            /* A[i][j] is random in interval [0,1) */
zm_rand(zA);/* zA[i][j] is random in [0,1) x [0,1) */
            /* x == zero vector */
v_zero(x);
v ones(x);
            /* x == e */
            /* x[i] is random in interval [0,1) */
v rand(x);
zv_rand(zx);/* zx[i] is random in [0,1) x [0,1) */
```

BUGS

The routine m_ident() "works" even if A is not square.

There is also the observation of von Neumann, Various techniques used in connection with random digits, National Bureau of Standards (1951), p. 36:

"Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin."

SOURCE FILE: init.c, matop.c, zmatop.c, zmemory.c, zvecop.c

in_prod, zin_prod – inner product

SYNOPSIS

#include "matrix.h"
double in_prod(VEC *x, VEC *y)

#include "zmatrix.h"
complex zin_prod(ZVEC *x, ZVEC *y)

DESCRIPTION

The inner product $x^T y = \sum_i x_i y_i$ of **x** and **y** is returned by **in_prod()**. The complex inner product $\bar{x}^T y = \sum_i \bar{x}_i y_i$ of **x** and **y** is returned by **zin_prod()**. This will fail if **x** or **y** is NULL.

These are built on the "raw" inner product routines:

double __in_prod (VEC *x, VEC *y, int i0)
complex __zin_prod(ZVEC *x, ZVEC *y, int i0, int conj)

which compute the inner products ignoring the first i0 entries. For the routine $_zin_prod()$ if the flag conj is Z_CONJ (or TRUE) then the entries in the x vector are conjugated and $\sum_{i\geq i0} \bar{x}_i y_i$ is returned; otherwise if conj is Z_NOCONJ (or FALSE) then $\sum_{i\geq i0} x_i y_i$ is returned.

EXAMPLE

```
VEC *x, *y;
ZVEC *zx, *zy;
Real x_dot_y;
complex zx_do_zy;
.....
x_dot_y = in_prod(x,y);
zx_dot_zy = zin_prod(zx,zy);
```

SEE ALSO

___ip__(), ___zip__() and the core routines.

BUGS

The accumulation is not guaranteed to be done in a higher precision than **Real**, although the return type is **double**. To guarantee more than this, we would either need an explicit extended precision **long double** type or force the accumulation to be done in a single register. While this is in principle possible on IEEE standard hardware, the routines to ensure this are not standard, even for IEEE arithmetic.

SOURCE FILE: vecop.c, zvecop.c

iv_add, iv_sub – Integer vector operations

SYNOPSIS

```
#include "matrix.h"
IVEC *iv_add(IVEC *iv1,IVEC *iv2, IVEC *out)
IVEC *iv_sub(IVEC *iv1, IVEC *iv2, IVEC *out)
```

DESCRIPTION

The two arithmetic operations implemented for integer vectors are addition (iv_add()) and subtraction (iv_sub()). In each of these routines, out is resized to be of the correct size if it does not have the same dimension as iv1 and iv2.

This dearth of operations is because it is envisaged that the main purpose for using integer vectors is to hold indexes or to represent combinatorial objects.

EXAMPLE

```
IVEC *x, *y, *z;
.....x = ...;
y = ...;
/* z = x+y, allocate z */
z = iv_add(x,y,IVNULL);
/* z = x-y, z already allocated */
iv_sub(x,y,z);
```

SEE ALSO

Vector operations v_...() and iv_resize().

SOURCE FILE: ivecop.c

NAME bd_resize, iv_resize, m_resize, px_resize, v_resize, zm_resize, zv_resize, iv_resize_vars, m_resize_vars, px_resize_vars, v_resize_vars, zm_resize_vars, **zv_resize_vars** – Resizing data structures **SYNOPSIS** #include "matrix.h" *bd resize(BAND *A, BAND int new_lb, int new_ub, int new_n) *iv_resize(IVEC *iv, int new_dim) IVEC MAT *m_resize (MAT *A, int new_m, int new_n) PERM *px_resize(PERM *px, int new_size) VEC *v_resize (VEC *x, int new_dim) *iv resize vars(unsigned new dim, int IVEC **x1, IVEC **x2, ..., NULL) *m_resize_vars (unsigned new_m, unsigned new_n, int MAT **A1, MAT **A2, ..., NULL) int *px_resize_vars(unsigned new_size, PERM **px1, PERM **px2, ..., NULL) int *v_resize_vars (unsigned new_dim, VEC **x1, VEC **x2, ..., NULL) #include "zmatrix.h" ZMAT *zm_resize(ZMAT *A, int new_m, int new_n) ZVEC *zv_resize(ZVEC *x, int new_dim) *zm resize vars(unsigned new m, unsigned new n, int ZMAT **A1, ZMAT **A2, ..., NULL) *zv_resize_vars(unsigned new_dim, int ZVEC **x1, ZVEC **x2, ..., NULL)

77

DESCRIPTION

Each of these routines sets the (apparent) size of data structure to be identical to that obtained by using .._get (new_...). Thus the VEC * returned by v_resize(x,new_dim) has x->dim equal to new_dim. The MAT * returned by m_resize(A,new_m,new_n) is a new_m × new_n matrix.

The following rules hold for all of the above functions except for $px_resize()$. Whenever there is overlap between the object passed and the re-sized data structure, the entries of the new data structure are identical, and elsewhere the entries are zero. So if **A** is a 5 × 2 matrix and new_A = m_resize(A, 2, 5), then new_A->me[1][0] is identical to the old A->me[1][0]. However, new_A->me[1][3] is zero.

For px_resize() the rules are somewhat different because permutations do not remain permutations under such arbitrary operations. Instead, if the size is *reduced*,

then the returned permutation is an identity permutation. If **size** is *increased*, then **new_px->pe[i]** == i for i greater than or equal to the old **size**.

Allocation or reallocation and copying of data structure entries is avoided if possible (except, to some extent, in m_resize()). There is a "high-water mark" field contained within each data structure; for the VEC and IVEC data structures it is max_dim, which contains the actual amount of memory that has been allocated (at some time) for this data structure. Thus resizing does not deallocate memory! To actually free up memory, use one of the .._free() routines or the .._FREE() macros.

You should not rely on the values of entries outside the apparent size of the data structures but inside the maximum allocated area. These areas may be zeroed or overwritten, especially by the m_resize() routine.

The .._resize_vars() routines resize a NULL-terminated list of pointers to variables, all of the same type. The new sizes of the all variables in the list are the same. Calling

.._resize_vars([m,]n,&x1,&x2,...,&xN,NULL)

is equivalent to

78

x1 = .._resize(x1, [m,]n); x2 = .._resize(x2, [m,]n);xN = .._resize(xN, [m,]n);

(Note that "[m,]" indicates that "m," might or might not be present, depending on whether the data structure involved is a matrix or not.) The returned value of the .._resize_vars() routines is the number of objects resized.

EXAMPLE

```
/* an alternative to workspace arrays */
... my_function(...)
{
    static VEC *x = VNULL;
    .....
    x = v_resize(x,new_size);
    MEM_STAT_REG(x,TYPE_VEC);
    .....
    v_copy(..., x);
    .....
}
```

BUGS

Note the above comment: **resizing does not deallocate memory**! To free up the actual memory allocated you will need to use the .._FREE() macros or the .._free() function calls.

SEE ALSO

.._get() routines; MEM_STAT_REG().

SOURCE FILE: memory.c, zmemory.c, bdfactor.c and ivecop.c

MACHEPS – machine epsilon

SYNOPSIS

#include "matrix.h"
Real macheps = MACHEPS;

DESCRIPTION

The quantity **MACHEPS** is a **#define**'d quantity which is the "machine epsilon" or "unit roundoff" for a given machine. For more information on this concept, see, e.g., Introduction to Numerical Analysis by K. Atkinson, or Matrix Computations by G. Golub and C. Van Loan. The value given is for the standard floating point type **Real** only. Normally the standard floating point type is **double**, but in the installation this can be changed to be **float** or **long double**. (See the chapter on installation.)

For ANSI C implementations, this is set to the value of the DBL_EPSILON or FLT_EPSILON macro defined in <float.h>.

EXAMPLE

```
while ( residual > 100*MACHEPS )
{ /* iterate */ }
```

BUGS

The value of **MACHEPS** has to be modified in the source whenever moving to another machine if the floating point processing is different.

SOURCE FILE: machine.h

m_add, m_mlt, m_sub, sm_mlt, zm_add, zm_mlt, zm_sub, zsm_mlt - matrix addition and multiplication

SYNOPSIS

```
#include "matrix.h"
MAT *m_add(MAT *A, MAT *B, MAT *C)
MAT *m_mlt(MAT *A, MAT *B, MAT *C)
MAT *m_sub(MAT *A, MAT *B, MAT *C)
MAT *sm_mlt(double s, MAT *A, MAT *OUT)
#include "zmatrix.h"
ZMAT *zm_add(ZMAT *A, ZMAT *B, ZMAT *C)
```

ZMAT*zm_mlt(ZMAT *A, ZMAT *B, ZMAT *C)ZMAT*zm_sub(ZMAT *A, ZMAT *B, ZMAT *C)ZMAT*zsm_mlt(complex s, ZMAT *A, ZMAT *OUT)

DESCRIPTION

The functions m_add(), zm_add() adds the matrices A and B and puts the result in C. If C is NULL, or is too small to contain the sum of A and B, then the matrix is resized to the correct size, which is then returned. Otherwise the matrix C is returned.

The functions, m_sub(), zm_sub() subtracts the matrix B from A and puts the result in C. If C is NULL, or is too small to contain the sum of A and B, then the matrix is resized to the correct size, which is then returned. Otherwise the matrix C is returned. Similarly, m_mlt() multiplies the matrices A and B and puts the result in C. Again, if C is NULL or too small, then a matrix of the correct size is created which is returned.

The routines $sm_mlt()$, $zsm_mlt()$ above puts the results of multiplying the matrix A by the scalar s in the matrix OUT. If, on entry, OUT is NULL, or is too small to contain the results of this operation, then OUT is resized to have the correct size. The result of the operation is returned. This operation may be performed *in situ*. That is, you may use A == OUT.

The routines m_add(), m_sub() and sm_mlt() routines and their complex counterparts can work *in situ*; that is, C need not be different to either A or B. However, m_mlt() and zm_mlt() will raise an E_INSITU error if A == C or B == C.

These routines avoid thrashing on virtual memory machines.

EXAMPLE

```
MAT *A, *B, *C;
Real alpha;
.....
C = m_add(A,B,MNULL); /* C = A+B */
```

 $\mathbf{A}_{i} = \{i_{i}, i_{j}\}$

$m_{sub}(A, B, C);$	/*	С	0.000 0.000	A-B */	
<pre>sm_mlt(alpha,A,C);</pre>		C	0000 4000	alpha.A	*/
m_mlt(A,B,C);	/*	С	-	A.B */	

SEE ALSO

v_add(), mv_mlt(), sv_mlt(), zv_add(), zmv_mlt(), zv_mlt().

SOURCE FILE: matop.c, zmatop.c

```
NAME
  mem_info, mem_info_on, mem_info_is_on, mem_info_bytes,
  mem_info_numvar, mem_info_file, mem_attach_list,
  mem_free_list, mem_bytes_list, mem_numvar_list,
  mem_dump_list, mem_is_list_attached – Meschach dynamic memory
  information
SYNOPSIS
#include "matrix.h"
void mem info()
int
     mem_info_on(int true_or_false)
     mem info is on(void)
int
void mem info file(FILE *fp, int list_num)
void mem dump list(FILE *fp, int list_num)
long mem_info_bytes (int type_num, int list_num)
     mem_info_numvar(int type_num, int list_num)
int
int
     mem attach_list(int list_num, int ntypes, char *names[],
                      int (*frees[])(), MEM_ARRAY info_sum[])
int
     mem free list(int list num)
int
     mem is list_attached(int list_num)
void mem_bytes(int type_num, int old_size, int new_size)
void mem_bytes_list(int type_num, int old_size, int new_size,
                     int list num)
void mem numvar(int type_num, int diff_numvar)
void mem numvar_list(int type_num, int diff_numvar,
                      int list num)
```

DESCRIPTION

These routines allow the user to obtain information about the amount of memory allocated for the Meschach data structures (VEC, BAND, MAT, PERM, IVEC, ITER, SPMAT, SPROW, ZVEC and ZMAT). The call mem_info_on (TRUE); sets a flag which directs the allocation and deallocation and resizing routines to store information about the memory that is (de)allocated and resized. The call mem_info_on(FALSE); turns the flag off.

The routine mem_info_is_on() returns the status of the memory information flag.

To get a general picture of the state of the memory allocated by Meschach data structures call mem_info_file(fp,list_num) which prints a summary of the amount of memory used for the different types of data structures to the file or stream fp. The list_num parameter indicates which list of types to use; use zero for the list of standard Meschach data types. The printout for mem_info_file(stdout,0), or the equivalent macro mem_info() looks like this for one real and one complex vector of dimension 10 allocated (with the full system installed on an RS/6000):

MEMORY INFORMA	ATION (s	standard	l types):			
type MAT	0	alloc.	bytes	0	alloc.	variables
type BAND	0,0	alloc.	bytes	0	alloc.	variables
type PERM	0	alloc.	bytes	0	alloc.	variables
type VEC	92	alloc.	bytes	1	alloc.	variable
type IVEC	0	alloc.	bytes	0	alloc.	variables
type ITER	0	alloc.	bytes	0	alloc.	variables
type SPROW	0	alloc.	bytes	0	alloc.	variables
type SPMAT	0	alloc.	bytes	0	alloc.	variables
type ZVEC	204	alloc.	bytes	1	alloc.	variable
type ZMAT	0	alloc.	bytes	0	alloc.	variables
total:	296	alloc.	bytes	2	alloc.	variables

(Note that this is for the system built with all of Meschach, including the sparse part: ITER, SPMAT; and the complex part: ZVEC, ZMAT. The mem_info_...() routines also work for partial installations of Meschach.) There is also the routine mem_dump_list() which provides a more complete printout, which is suitable for debugging purposes.

To obtain information about the amount of memory allocated for objects of a particular type, use mem_info_bytes() (with list_num equal to zero for a standard Meschach structures). To find out the amount of memory allocated for ordinary vectors, use

printf("Bytes in VEC'S = %ld = %ld\n", mem_info_bytes(TYPE_VEC,0));

The routine mem_info_numvar() returns the number of data structures that are allocated for each type. Use list_num equal to zero for standard Meschach structures.

Each Meschach type has an associated type macro **TYPE_...** which is a small integer. The "..." is the ordinary name of the type, such as **VEC**, **MAT** etc. This is the complete list of **TYPE_...** macros:

TYPE_MAT	0	/*	real dense matrix */
TYPE_BAND	1	/*	real band matrix */
TYPE_PERM	2	/*	permutation */
TYPE_VEC	3	/*	real vector */
TYPE_IVEC	4	/*	integer vector */
TYPE_ITER	5	**/*	iteration structure */
TYPE_SPROW	6	/*	real sparse matrix row */
TYPE_SPMAT	7	/*	real sparse matrix */
TYPE_ZVEC	8	/*	complex vector */
TYPE_ZMAT	9	/*	complex dense matrix */

This is how different types are distinguished within the mem_info_... system.

Note that **SPROW** is an auxiliary type; when an **SPROW** (sparse row) is allocated as part of a **SPMAT** (sparse matrix), then the memory allocation is entered under **SPMAT**; only "stand-alone" **SPROW**'s have their memory allocation entered under the typer **SPROW**.

The routine mem_attach_list() can be used to add new lists of types to the Meschach system for both tracking memory usage, and also for registering static workspace arrays with MEM_STAT_REG(). The routine is passed a collection of arrays: names is an array of strings being the names of the different types, frees is an array of the .._free() routines which deallocate and destroy objects of the corresponding types, info_sum is an array in which the memory allocation information is stored. This array has the component type MEM_ARRAY which is defined as

```
typedef struct {
   long bytes; /* # allocated bytes for each type */
   int numvar; /* # allocated variables for each type */
} MEM_ARRAY;
```

This is defined in matrix.h.

The parameter ntypes is the number of types, which should also be the common length of the arrays. The parameter $list_num$ is the list number used to identify which list of types should be used. The routine mem_attach_list() returns the zero on successful completion, and (-1) if there is an invalid parameter. An E_OVERWRITE error will be raised if the specified list_num has already been used.

To track memory usage for any new types, the allocation, deallocation and resizing routines for these types you should use mem_bytes_list() and

mem_numvar_list() to inform the mem_info_...() system of the change in the number of bytes allocated, and number of structures allocated, respectively, of an object of a particular type (as specified by the type_num and list_num parameters). In mem_bytes_list(), the parameter old_size should contain the old size in bytes, and new_size should contain the new size in bytes. In mem_numvar_list(), the parameter diff_numvar is the change in the number of allocated structures: +1 for allocating a new structure, and -1 for destroying a structure.

The routines mem_bytes() and mem_numvar() are just macros that call mem_bytes_list() and mem_numvar() respectively, with list_num zero for the standard Meschach structures.

The routine mem_attach_list() should be used once at the beginning of a program using these additional types.

Here is an example of how this might be used to extend Meschach with three types for nodes, edges and graphs:

```
/* Example with three new types: NODE, EDGE and GRAPH */
#define MY_LIST 1
#define TYPE_NODE 0
```

```
#define TYPE EDGE
                   1
#define TYPE GRAPH 2
static char *my_names[] = { "NODE", "EDGE", "GRAPH"
                                                        };
static int (*my_frees[]) = { n_free, e_free, gr_free };
static MEM_ARRAY my_tnums[3]; /* initialised to zeros */
main(...)
{
      ..... /* declarations */
    mem_attach_list(MY_LIST,3,my_names,my_frees,my_tnums);
      ..... /* actual work */
    mem_info_file(stdout,MY_LIST); /* list memory used */
}
/* n_get -- get a node data structure;
            NODE has type number 0 */
NODE *n_get(...)
{
    NODE *n;
    n = NEW(NODE);
    if (n == NULL)
      error(E_MEM, "n_get"); /* can't allocate memory */
    mem_bytes_list(TYPE_NODE, 0, sizeof(NODE), MY_LIST);
    mem numvar list(TYPE NODE, 1, MY_LIST);
      . . . . . .
}
/* n_free -- deallocate node data structure */
int
     n_free(NODE *n)
{
    if (n != NULL)
    {
        free(n);
        mem_res_elem_list(TYPE_NODE, sizeof(NODE), 0, MY_LIST);
        mem_numvar_list(TYPE_NODE, -1, MY_LIST);
    }
    return 0;
}
```

For more information see chapter 8.

BUGS

Memory used by the underlying memory (de)allocation system (malloc(),

calloc(), realloc(), sbrk() etc.) for headers are not included in the amounts of allocated memory.

The numbers of vectors, matrices etc. currently allocated cannot be found by this system.

SEE ALSO

.._get(), .._free(), .._resize() routines; MEM_STAT_REG() and the mem_stat_...() routines.

SOURCE FILE: meminfo.c, meminfo.h

MEM_STAT_REG, mem_stat_reg_list, mem_stat_reg_vars, mem_stat_mark, mem_stat_free, mem_stat_dump, mem_stat_show_mark - Static workspace control routines

SYNOPSIS

DESCRIPTION

Older versions of Meschach (v.1.1b and previous) had a limitation in that it was essentially impossible to control the use of static workspace arrays used within Meschach functions. This can lead to problems where too much memory is taken up by these workspace arrays for memory intensive problems. The obvious alternative approach is to deallocate workspace at the end of every function, which can be quite expensive because of the time taken to deallocate and the reallocate the memory on every usage.

These functions provide a way of avoiding these problems, by giving users control over the (selective) destruction of workspace vectors, matrices, etc.

The simplest way to use this to deallocate workspace arrays in a routine hairy1(...) is as follows:

```
.....
mem_stat_mark(1); /* ''group 1'' of workspace arrays */
for ( i = 0; i < n; i++ )
        hairy1(...); /* workspace registered as ''group 1'' */
mem_stat_free(1); /* deallocate ''group 1'' workspace */</pre>
```

The call mem_stat_mark(num) sets the current workspace group number. This number must be a positive integer. Provided the appropriate workspace registration routines are used in hairy1(...) (see later), then the workspace arrays are registered as being in the current workspace group as determined by mem_stat_mark(). If mem_stat_mark() has not been called, then there is no current group number and the variables are not registered. The call mem_stat_free(num) deallocates all static workspace arrays allocated in workspace group num, and also unsets the current workspace group. So, to continue registering static workspace variables, mem_stat_mark(num), or

mem_stat_mark(new_num) should follow.

Keeping two groups of registered static workspace variables (one for hairy1() and another for hairy2()) can be done as follows:

```
for ( i = 0; i < n; i++ )
{
    mem_stat_mark(1);
    hairy1(...);
    mem_stat_mark(2);
    hairy2(...);
}
mem_stat_free(2); /* don't want hairy2()'s workspace */
hairy1(...); /* keep hairy1()'s workspace */</pre>
```

For the person writing routines to use workspace arrays, there are a number of rules that must be followed if these routines are to be used.

- the workspace variables must be static pointers to Meschach data structures.
- they must be initialised to be NULL vectors in the type declaration.
- they are allocated using a .._resize() routine.
- they are allocated before registering.
- the pointer variable is passed to MEM_STAT_REG(), but mem_stat_reg_vars() and mem_stat_reg_vars() require the *address* of the pointer to be passed.

The type parameter of MEM_STAT_REG() should be a macro of the form TYPE_... where the "..." is the name of the type used. An example of its use follows:

```
VEC *hairy1(x, y, out)
VEC *x, *y, *out;
{
   static VEC *wkspace = VNULL;
   int new_dim;
    .....
   wkspace = v_resize(wkspace,new_dim);
   MEM_STAT_REG(wkspace,TYPE_VEC);
    .....
   mv_mlt(....,wkspace); /* use of wkspace */
    .....
   /* no need to deallocate wkspace */
   return out;
}
```

MEM_STAT_REG() is a macro which calls mem_stat_reg_list() with list_num set to zero.

The call mem_stat_dump(fp) prints out a representation of the registered workspace variables onto the file or stream fp suitable for debugging purposes. It is not expected that this would be needed by most users of Meschach.

The routine mem_stat_show_mark() returns the current workspace group, and zero if no group is active.

A NULL terminated list of variables can be registered at once using mem_stat_reg_vars(). The call

mem_stat_reg_vars(list_num,type_num,&x1,&x2,...,&xN,NULL);

is equivalent to

```
mem_stat_reg_list(&x1,type_num,list_num);
mem_stat_reg_list(&x2,type_num,list_num);
.....
mem_stat_reg_list(&xN,type_num,list_num);
```

Note that $x1, x2, \ldots, xN$ must be of the same type.

For non-Meschach data structures, you can use mem_stat_reg_list() in conjunction with mem_attach_list(). For more information on the use of this function see chapter 8.

SEE ALSO

mem_info_...() routines.

BUGS

There is a static registration area for workspace variables, so there is a limit on the number of variables that can be registered. The default limit is 509. If it is too small, an appropriate message will appear and information on how to change the limit will follow.

Attempts to register a workspace array that is neither **static** or global will most likely result in a crash when **mem_stat_free()** is called for the workspace group containing that variable.

SOURCE FILE: memstat.c

NAME m_load, m_save, v_save, d_save, zm_load, z_save, zm_save, zv_save - MATLAB save/load to file SYNOPSIS

```
#include "matlab.h"
MAT
        *m load(FILE *fp, char **name)
MAT
        *m save(FILE *fp, MAT *A, char **name)
VEC
        *v_save(FILE *fp, VEC *x, char **name)
         d_save(FILE *fp, double d, char **name)
double
#include "matlab.h"
        *zm_load(FILE *fp, char **name)
ZMAT
        *zm_save(FILE *fp, ZMAT *A, char **name)
ZMAT
        *zv save(FILE *fp, ZVEC *x, char **name)
ZVEC
         z_save (FILE *fp, complex z, char **name)
complex
```

DESCRIPTION

These routines read and write MATLABTM load/save files. This enables results to be transported between MATLAB and Meschach. The routine m_load() loads in a matrix from file fp in MATLAB save format. The matrix read from the file is returned, and name is set to point to the saved MATLAB variable name of the matrix. Both the matrix returned and name have allocated memory as needed. An example of the use of the routine to load a matrix **A** and a vector **x** is

```
MAT *A, *Xmat;
VEC *x;
FILE *fp;
char *name1, *name2;
if ( (fp=fopen("fred.mat", "r")) != NULL )
{
         = m_load(fp,&name1);
    Α
    Xmat = m load(fp,&name2);
    if (Xmat->n != 1)
    {
       printf("Incorrect size matrix read in\n");
       exit(0);
                  }
    x = v get(Xmat -> m);
    x = mv move(Xmat, 0, 0, Xmat -> m, 1, x, 0);
}
```

The m_save() routine saves the matrix A to the file/stream fp in MATLAB save format. The MATLAB variable name is name.

The $v_save()$ routine saves the vector x to the file/stream fp as an $x->\dim \times 1$ matrix (i.e. as a column vector) in MATLAB save format. The MATLAB variable name is name.

The d_save() routine saves the double precision number d to the file/stream fp in MATLAB save format. The MATLAB variable name is name.

The MATLAB save format can depend in subtle ways on the type of machine used, so you may need to set the machine type in machine.h. This should usually just mean adding a line to machine.h to be one of

#define MACH_ID	INTEL	/*	80x87	format	*/
#define MACH_ID	MOTOROLA	/*	6888x	format	*/
#define MACH_ID	VAX_D	/*	VAX D	format	*/
#define MACH_ID	VAX_G	/*	VAX G	format	*/

to be the appropriate machine. The machine dependence involves both whether IEEE or non IEEE format floating point numbers are used, but also whether or not the machine is a "little-endian" or a "big-endian" machine.

BUGS

The m_load() routine will only read in the real part of a complex matrix.

The routines are machine-dependent as described above.

SOURCE FILE: matlab.c, zmatlab.c

bd_transp, m_transp, mmtr_mlt, mtrm_mlt, zm_adjoint, zmma_mlt, zmam_mlt - matrix transposes, adjoints and multiplication SYNOPSIS

```
#include "matrix.h"
BAND *bd_transp(BAND *A, BAND *OUT)
MAT *m_transp(MAT *A, MAT *OUT)
MAT *mmtr_mlt(MAT *A, MAT *B, MAT *OUT)
MAT *mtrm_mlt(MAT *A, MAT *B, MAT *OUT)
#include "zmatrix.h"
ZMAT *zm_adjoint(ZMAT *A, ZMAT *OUT)
```

```
ZMAT *zmma_mlt(ZMAT *A, ZMAT *B, ZMAT *OUT)
ZMAT *zmam_mlt(ZMAT *A, ZMAT *B, ZMAT *OUT)
```

DESCRIPTION

The routine bd_transp() computes the transpose of the banded matrix A and puts the result in OUT. Both are BAND structures.

The routine $m_transp()$ transposes the matrix A and stores the result in OUT. The routine $m_adjoint()$ takes the complex conjugate transpose (or complex adjoint) of A and stores the result in OUT. These routines may be *in situ* (i.e. A == OUT) only if A is square. (Note that BAND matrices are always square.) The complex adjoint of A is denoted A^* .

The routine $mmtr_mlt()$ forms the product AB^T , which is stored in OUT. The routine $mma_mlt()$ forms the product AB^* , which is stored in OUT. The routine $mtrm_mlt()$ forms the product A^TB , which is stored in OUT. The routine $mam_mlt()$ forms the product A^*B , which is stored in OUT. Neither of these routines can form the product *in situ*. This means that they must be used with A != OUT and B != OUT. However, you can still use A == B.

For all the above routines, if **OUT** is NULL or too small to contain the result, then it is resized to the correct size, and is then returned.

EXAMPLE

```
MAT *A, *B, *C;
.....
C = m_transp(A,MNULL); /* C = A^T */
mmtr_mlt(A,B,C); /* C = A.B^T */
mtrm_mlt(A,B,C); /* C = A^T.B */
```

SOURCE FILE: matop.c, zmatop.c

m_norm1, m_norm_inf, m_norm_frob, zm_norm1, zm_norm_inf, zm_norm_frob - matrix norms

SYNOPSIS

```
#include "matrix.h"
Real m_norm1(MAT *A)
Real m_norm_inf(MAT *A)
Real m_norm_frob(MAT *A)
```

```
#include "zmatrix.h"
Real zm_norm1(ZMAT *A)
Real zm_norm_inf(ZMAT *A)
Real zm_norm_frob(ZMAT *A)
```

DESCRIPTION

These routines compute matrix norms. The routines m_norm1() and zm_norm1() compute the matrix norm of A in the matrix 1-norm; m_norm_inf() and zm_norm_inf() compute the matrix norm of A in the matrix ∞ -norm; m_norm_frob() and zm_norm_frob() compute the Frobenius norm of A. All of these routines are unscaled; that is, there is no scaling vector for weighting the elements of A.

These norms are defined through the following formulae:

(4.1)
$$||A||_1 = \max_j \sum_i |a_{ij}|, \qquad ||A||_{\infty} = \max_i \sum_j |a_{ij}|,$$

(4.2) $||A||_F = \sqrt{\sum_{ij} |a_{ij}|^2}.$

The matrix 2-norm is not included as it requires the calculation of eigenvalues or singular values.

EXAMPLE

MAT *A;

```
printf("||A||_1 = %g\n", m_norm1(A));
printf("||A||_inf = %g\n", m_norm_inf(A));
printf("||A||_F = %g\n", m_norm_frob(A));
```

SEE ALSO

v_norm1(), v_norm_inf(), zv_norm1(), zv_norm_inf().

BUGS

The Frobenius norm calculations may overflow if the elements of **A** are of order $\sqrt{\text{HUGE}}$.

SOURCE FILE: norm.c, znorm.c

NAME mv_ml zmv_m	t, vm_mlt, my_mltadd, vm_mltadd, zmv_mlt, zvm_mlt, ltadd, zvm_mltadd - matrix-vector multiplication	
SYNOPSI		
011101 0.		
<pre>#include "matrix.h"</pre>		
VEC	<pre>*mv_mlt(MAT *A, VEC *x, VEC *out)</pre>	
VEC	<pre>*vm_mlt(MAT *A, VEC *x, VEC *out)</pre>	
VEC	*mv_mltadd(VEC *v1, VEC *v2, MAT *A,	
	double s, VEC *out)	
VEC	<pre>*vm_mltadd(VEC *v1, VEC *v2, MAT *A,</pre>	
	double s, VEC *out)	
#include	"zmatrix.h"	
ZVEC	<pre>*zmv_mlt(ZMAT *A, ZVEC *x, ZVEC *out)</pre>	
ZVEC	*zvm_mlt(ZMAT *A, ZVEC *x, ZVEC *out)	
ZVEC	*zmv_mltadd(ZVEC *v1, ZVEC *v2, ZMAT *A,	
	complex s, ZVEC *out)	
ZVEC	*zvm_mltadd(ZVEC *v1, ZVEC *v2, ZMAT *A,	
	complex s, ZVEC *out)	

DESCRIPTION

The routines $mv_mlt()$ and $vm_mlt()$ form Ax and $A^Tx = (x^TA)^T$ respectively and store the result in out. The routines $zmv_mlt()$ and $zvm_mlt()$ form Ax and $A^*x = (x^*A)^*$ respectively and store the result in out. The routines $mv_mltadd()$ and $vm_mltadd()$ form $v_1 + sAv_2$ and $v_1 + sA^Tv_2$ respectively, and stores the result in out. The routines $zmv_mltadd()$ and $zvm_mltadd()$ form $v_1 + sAv_2$ and $v_1 + sA^Tv_2$ respectively, and stores the result in out. If out is NULL or too small to contain the product, then it is resized to the correct size.

These routines do not work *in situ*; that is, out must be different to x for $mv_mlt()$ and $vm_mlt()$, and in the case of $mv_mltadd()$ and $vm_mltadd()$, out must be different to v2.

These routines avoid thrashing virtual memory machines.

EXAMPLE

```
MAT *A;
VEC *x, *y, *out;
Real alpha;
.....
out = mv_mlt(A,x,VNULL); /* out = A.x */
vm_mlt(A,x,out); /* out = A^T.x */
mv_mltadd(x,y,A,out); /* out = x + A.y */
vm_mltadd(x,y,A,out); /* out = x + A^T.y */
```

SOURCE FILE: matop.c, zmatop.c

px_ident, px_inv, px_mlt, px_transp, px_sign - permutation identity, inverse and multiplication

SYNOPSIS

```
#include "matrix.h"
PERM *px_ident(PERM *pi)
PERM *px_mlt(PERM *pi1, PERM *pi2, PERM *out)
PERM *px_inv(PERM *pi, PERM *out)
PERM *px_transp(PERM *pi, int i, int j)
int px_sign(PERM *pi)
```

DESCRIPTION

The routine px_ident() initialises pi to be the identity permutation of the size of pi->size on entry. The permutation pi is returned. If pi is NULL then an error is generated.

The routine px_mlt() multiplies pil by pil to give out. If out is NULL or too small, then out is resized to be a permutation of the correct size. This cannot be done *in situ*.

The routine $px_{inv}()$ computes the inverse of the permutation pi. The result is stored in out. If out is NULL or is too small, a permutation of the correct size is created, which is returned. This can be done *in situ* if pi = out.

The routine $px_transp()$ swaps pi->pe[i] and pi->pe[j]; it is a multiplication by the transposition $i \leftrightarrow j$.

The routine $px_sign(pi)$ computes the sign of the permutation pi. This sign is $(-1)^p$ where pi can be written as the product of p permutations. This is done by sorting the entries of pi using quicksort, and counting the number of transpositions used. This is also the determinant of the permutation matrix represented by pi.

EXAMPLE

SOURCE FILE: pxop.c

px_cols, px_rows, px_vec, pxinv_vec, px_zvec, pxinv_zvec – permute rows or columns of a matrix, or permute a vector

SYNOPSIS

```
#include "matrix.h"
MAT *px_rows(PERM *pi, MAT *A, MAT *OUT)
MAT *px_cols(PERM *pi, MAT *A, MAT *OUT)
VEC *px_vec (PERM *pi, VEC *x, VEC *out)
VEC *pxinv_vec(PERM *pi, VEC *x, VEC *out)
#include "zmatrix.h"
ZVEC *px_zvec (PERM *pi, ZVEC *x, ZVEC *out)
ZVEC *pxinv_zvec(PERM *pi, ZVEC *x, ZVEC *out)
```

DESCRIPTION

The routines $px_rows()$ and $px_cols()$ are for permuting matrices, permuting respectively the rows and columns of the matrix **A**. In particular, for $px_rows()$ the i-th row of OUT is the pi->pe[i]-th row of **A**. Thus OUT = PA where P is the permutation matrix described by pi. The routine $px_cols()$ computes OUT = AP.

The result is stored in **OUT** provide it has sufficient space for the result. If **OUT** is NULL or too small to contain the result then it is replaced by a matrix of the appropriate size. In either case the result is returned.

Similarly, px_vec() and px_zvec() permute the entries of the vector x into the vector out by the rule that the i-th entry of out is the pi->pe[i]-th entry of x. Conversely, pxinv_vec() and pxinv_zvec() permute x into out by the rule that the pi->pe[i]-th entry of out is the i-th entry of x. This is equivalent to inverting the permutation pi and then applying px_vec(), respectively, px_zvec() for real, resp., complex vectors.

If out is NULL or too small to contain the result, then a new vector is created and the result stored in it. In either case the result is returned.

EXAMPLE

```
PERM *pi;
VEC *x, *tmp;
ZVEC *z, *ztmp;
MAT *A, *B;
.....
/* permute x to give tmp */
tmp = px_vec(pi,x,tmp);
ztmp = px_zvec(pi,z,ZVNULL);
/* restore x & z */
```

x = pxinv_vec(pi,tmp,x); pxinv_zvec(pi,ztmp,z); /* symmetric permutation */ B = px_rows(pi,A,MNULL); A = px_cols(pi,B,A);

SEE ALSO

The **px_...**() operations; in particular **px_inv(**)

SOURCE FILE: pxop.c, zvecop.c

set_col, set_row, zset_col, zset_row - set rows and columns of
matrices

SYNOPSIS

#include "matrix.h"
MAT *set_col(MAT *A, int k, VEC *out)
MAT *set_row(MAT *A, int k, VEC *out)

#include "zmatrix.h"
ZMAT *zset_col(ZMAT *A, int k, ZVEC *out)
ZMAT *zset_row(ZMAT *A, int k, ZVEC *out)

DESCRIPTION

The routines $set_col()$ and $zset_col()$ above sets the value of the kth column of A to be the values of out. The A matrix so modified is returned.

The routine **set_row()** above sets the value of the kth row of A to be the values of out. The A matrix so modified is returned.

If out is NULL, then an **E_NULL** error is raised. If **k** is negative or greater than or equal to the number of columns or rows respectively, an **E_BOUNDS** error is raised.

As the MAT and ZMAT data structures are row-oriented data structures, the set_row() routine is faster than the set_col() routine.

EXAMPLE

MAT *A; VEC *tmp; /* scale row 3 of A by 2.0 */ tmp = get_row(A,3,VNULL); sv_mlt(2.0,tmp,tmp); set_row(A,3,tmp);

SEE ALSO

```
get_col() and get_row()
```

SOURCE FILE: matop.c, zmatop.c

sv_mlt, v_add, v_mltadd, v_sub, zv_mlt, zv_add, zv_mltadd, zv_sub - scalar-vector multiplication and addition

SYNOPSIS

```
#include "matrix.h"
```

VEC	<pre>*sv_mlt(double s, VEC *x, VEC *out)</pre>
VEC	<pre>*v_add(VEC *v1, VEC *v2, VEC *out)</pre>
VEC	<pre>*v_mltadd(VEC *v1, VEC *v2, double s, VEC *out)</pre>
VEC	*v_sub(VEC *v1, VEC *v2, VEC *out)

#include "zmatrix.h"

ZVEC	<pre>*zv_mlt(complex s, ZVEC *x, ZVEC *out)</pre>
ZVEC	*zv_add(ZVEC *v1, ZVEC *v2, ZVEC *out)
ZVEC	<pre>*zv_mltadd(ZVEC *v1, ZVEC *v2, complex s, ZVEC *out)</pre>
ZVEC	<pre>*zv_sub(ZVEC *v1, ZVEC *v2, ZVEC *out)</pre>

DESCRIPTION

The routines $sv_mlt()$ and $zv_mlt()$ perform the scalar multiplication of the scalar s and the vector x and the results are placed in out.

The routines $v_add()$ and $zv_add()$ adds the vectors v1 and v2, and the result is returned in out.

The routines **v_mltadd()** and **zv_mltadd()** set out to be the linear combination **v1+s.v2**.

The routines $v_{sub}()$ and $zv_{sub}()$ subtract v2 from v1, and the result is returned in out.

For all of the above routines, if out is NULL, then a new vector of the appropriate size is created. For all routines the result (whether newly allocated or not) is returned. All these operations may be performed *in situ*. Errors are raised if **v1** or **v2** are NULL, or if **v1** and **v2** have different dimensions.

EXAMPLE

/* z = x + alpha.y */
v_mltadd(x,y,alpha,z);
/* ...or equivalently */
sv_mlt(alpha,y,z);
v_add(x,z,z);
zv_mltadd(v,w,beta,v);

SOURCE FILE:

vecop.c, zvecop.c

v_conv, v_map, v_max, v_min, v_pconv, v_star, v_slash, v_sort, v_sum, zv_map, zv_star, zv_slash, zv_sum -Componentwise operations

SYNOPSIS

```
#include
          "matrix.h"
VEC
        *v_conv (VEC *x, VEC *y, VEC *out)
VEC
        *v_pconv(VEC *x, VEC *y, VEC *out)
        *v_map (double (*fn)(double), VEC *x, VEC *out)
VEC
        v_max (VEC *x, int *index)
double
         v min (VEC *x, int *index)
double
VEC
        *v_star (VEC *x, VEC *y, VEC *out)
VEC
        *v_slash(VEC *x, VEC *y, VEC *out)
VEC
        *v_sort (VEC *x, PERM *order)
double
                (VEC *x)
        v sum
#include "mzatrix.h"
        *zv_map(complex (*fn)(complex), ZVEC *x, ZVEC *out)
ZVEC
        *zv_star(ZVEC *x, ZVEC *y, ZVEC *out)
ZVEC
        *zv slash(ZVEC *x, ZVEC *y, ZVEC *out)
ZVEC
complex zv sum(ZVEC *x)
```

DESCRIPTION

The routines $\mathbf{v}_conv()$ and $\mathbf{v}_pconv()$ compute convolution-type products of vectors. The routine $\mathbf{v}_conv()$ computes the vector z where $z_i = \sum_{0 \le j \le i} x_j y_{i-j}$. The routine $\mathbf{v}_pconv()$ computes a periodic convolution with period $\mathbf{y}->dim$. The routine $\mathbf{v}_conv()$ can be used to compute the product of two polynomials, with the polynomial $x(t) = \sum_{i=0}^{\deg x} x_i t^i$ and $y(t) = \sum_{i=0}^{\deg y} y_i t^i$.

The routines $v_map()$ and $zv_map()$ apply the function (*fn)() to the components of x to give the vector out. That is, out ->ve[i] = (*fn)(x->ve[i]). There are also versions

VEC	<pre>*_v_map(double (*fn)(void *,double),</pre>
	<pre>void *fn_params, VEC *x, VEC *out)</pre>
ZVEC	<pre>*_zv_map(complex (*fn)(void *,complex),</pre>
	<pre>void *fn_params, ZVEC *x, ZVEC *out)</pre>

where $out \rightarrow ve[i] = (*fn)(fn_params, x \rightarrow ve[i])$. This enables more flexible use of this function. Both of these functions may be used *in situ* with x = out.

The routine v_{max} () returns the maximum entry of the vector x, and sets index to be the index of this maximum value in x. Note that index is the in-

dex for the *first* entry with this value. Thus $\max_x = v_{\max}(x, \&i)$ means that $x - ve[i] = \max_x$.

The routine $v_min()$ returns the minimum entry of the vector x, and sets index to be the index of this minimum value similarly to $v_max()$. Both $v_min()$ and $v_max()$ raise an E_SIZES error if they are passed zero dimensional vectors.

The routines $v_star()$ and $zv_star()$ compute the componentwise, or Hadamard, product of x and y. That is, out ->ve[i] = x->ve[i]*y->ve[i] for all i. Note that $v_star()$ is equivalent to multiplying y by a diagonal matrix whose diagonal entries are given by the entries of x. This routine may be used *in situ* with x == out.

The routines $v_slash()$ and $zv_slash()$ compute the componentwise ratio of entries of y and x. (Note the order!) That is, out ->ve[i] = y->ve[i]/x->ve[i]for all i. Note that this is equivalent to multiplying y by the inverse of the diagonal matrix described in the previous paragraph. This could be useful for preconditioning, for example. This routine may be used *in situ* with x == out and/or y == out. The routine $v_slash()$ raises an E_sING error if x has a zero entry (the rationale being that it is really solving the system of equations Xz = y where z is out).

The routine $v_sort()$ sorts the entries of the vector x in situ, and sets order to be the permutation that achieves this. Note that the old ordering of x can be obtained by using $pxinv_vec()$ as illustrated in the example below. The algorithm used is a version of quicksort based on that given in *Algorithms in C*, by R. Sedgewick, pp. 116–124 (1990).

The routines $v_sum()$ and $zv_sum()$ return the sum of the entries of x.

Note that there are no complex "min", "max" or "sorting" routines, as there is no suitable ordering on the complex numbers.

EXAMPLE

An alternative way of computing $||x||_{\infty}$ (but slower):

```
VEC *x, *y, *z;
PERM *order;
Real norm;
int i;
.....
y = v_map(fabs,x,VNULL);
norm = v_max(y,&i);
```

```
v_sort(x,order);
/* x now sorted */
y = pxinv_vec(order,x,VNULL);
/* y is now the original x */
```

Using the Hadamard product for setting $y_i = w_i x_i$:

VEC *weights; for (i = 0; i < weights->dim; i++) weights->ve[i] = ...; v_star(weights,x,y);

SEE ALSO

Other componentwise operations: v_add(), v_sub(), sv_mlt().

Iterative routines benefiting from diagonal preconditioning: iter_cg(), iter_cgs(), and iter_lsqr().

SOURCE FILE: vecop.c, zvecop.c

NAME vlincomb, vlinlist, zvlincomb, zvlinlist-linear combinations **SYNOPSIS** #include "matrix.h" VEC *v_lincomb(int n, VEC *v_list[], double a_list[], VEC *out) *v_linlist(VEC *out, VEC *v1, double a1, VEC VEC *v2, double a2, ..., VNULL) #include "zmatrix.h" ZVEC *zv_lincomb(int n, ZVEC *v_list[], complex a_list[], ZVEC *out) ZVEC *zv_linlist(ZVEC *out, ZVEC *v1, complex a1, ZVEC *v2, complex a2, ..., ZVNULL)

DESCRIPTION

The routines $\mathbf{v}_{lincomb}()$ and $\mathbf{zv}_{lincomb}()$ compute the linear combination $\sum_{i=0}^{n-1} a_i v_i$ where v_i is identified with $\mathbf{v}_{list[i]}$ and a_i is identified with $\mathbf{a}_{list[i]}$. The result is stored in out, which is created or resized as necessary. Note that \mathbf{n} is the *length* of the lists.

An E_INSITU error will be raised if out == v_list[i] for any i other than i == 0.

The routines $v_linlist()$ and $zv_linlist()$ are variants of the above which do not require setting up an array before hand. This returns $\sum_i a_i v_i$ where the sum is over i = 1, 2, ... until a VNULL is reached, which should take the place of one of the vk's.

An E_INSITU error will be raised if out == v2, v3, v4,....

EXAMPLE

```
VEC *x[10], *v1, *v2, *v3, *v4, *out;
Real a[10], h;
.....
for ( i = 0; i < 10; i++ )
{ x[i] = ...; a[i] = ...; }
out = v_lincomb(10,x,a,VNULL)
/* for Runge--Kutta code:
        out = h/6*(v1+2*v2+2*v3+v4) */
v_zero(out);
out = v_linlist(out, v1, h/6.0, v2, h/3.0,
        v3, h/3.0, v4, h/6.0,
        VNULL);
```

SEE ALSO

sv_mlt(), v_mltadd(), zv_mlt(), zv_mltadd()

BUGS

SOURCE FILE: vecop.c, zvecop.c

v_norm1, v_norm2, v_norm_inf, zv_norm1, zv_norm2, zv_norm_inf - vector norms SYNOPSIS #include "matrix.h"

```
double v_norm1(VEC *x)
double v_norm2(VEC *x)
double v_norm_inf(VEC *x)
double _v_norm1(VEC *x, VEC *scale)
double _v_norm2(VEC *x, VEC *scale)
double _v_norm_inf(VEC *x, VEC *scale)
#include "zmatrix.h"
double _zv_norm1(ZVEC *x)
double _zv_norm2(ZVEC *x)
double _zv_norm_inf(ZVEC *x)
double _zv_norm1(ZVEC *x, VEC *scale)
double _zv_norm2(ZVEC *x, VEC *scale)
double _zv_norm1(ZVEC *x, VEC *scale)
```

DESCRIPTION

These functions compute vector norms. In particular, $v_norm1()$ and $zv_norm1()$ give the 1-norm, $v_norm2()$ and $zv_norm2()$ give the 2-norm or Euclidean norm, and $v_norm_inf()$ and $zv_norm_inf()$ compute the ∞ -norm. These are defined by the following formulae:

(4.3)
$$||x||_1 = \sum_i |x_i|$$

$$\|x\|_{\infty} = \max |x_i|$$

(4.5)
$$\|x\|_2 = \sqrt{\sum_i |x_i|^2}.$$

There are also *scaled* versions of these vector norms: _v_norm1(), _v_norm2() and _v_norm_inf(), and _zv_norm1(), _zv_norm2() and _zv_norm_inf(). These take a vector **x** whose norm is to be computed, and a scaling vector. Each component of the **x** vector is divided by the corresponding component of the **scale** vector, and the norm is computed for the "scaled" version of **x**. Note that the **scale** vector is a (real) **VEC** since only the magnitudes are important. If the corresponding component of **scale** is zero for that component of **x**, or if **scale** is NULL, then no scaling is done. (In fact, **v_norm1(x)** is a macro that expands to _**v_norm1(x, VNULL)**.)

For example, _v_norm1(x, scale) returns

$$\sum_i |x_i|$$
scale $_i|$

provided scale is not NULL, and no element of scale is zero. The behaviour of _v_norm2() and _v_norm_inf() is similar.

EXAMPLE.

```
VEC *x, *scale;
.....
printf("# 2-Norm of x = %g\n", v_norm2(x));
printf("# Scaled 2-norm of x = %g\n",
_v_norm2(x,scale));
```

SEE ALSO

m_norm1(), m_norm_inf(), zm_norm1(), zm_norm_inf().

BUGS

There is the possibility that v_norm2() may overflow if x has components with size of order $\sqrt{\text{HUGE}}$.

SOURCE FILE: norm.c

zmake, zconj, zneg, zabs, zadd, zsub, zmlt, zinv, zdiv, zsqrt,

zexp, **zlog** – Operations on complex numbers

SYNOPSIS

```
#include "zmatrix.h"
complex zmake(double real, double imag)
complex zconj(complex z)
complex zneg(complex z)
double zabs(complex z)
complex zadd(complex z1, complex z2)
complex zsub(complex z1, complex z2)
complex zmlt(complex z1, complex z2)
complex zinv(complex z1, complex z2)
complex zdiv(complex z1, complex z2)
complex zdiv(complex z1, complex z2)
complex zsqrt(complex z)
complex zexp(complex z)
complex zlog(complex z)
```

DESCRIPTION

These routines provide the basic operations on complex numbers.

Complex numbers are represented by the complex data structure which is defined as

typedef struct { Real re, im; } complex;

and the real part of complex z; is z.re and its imaginary part is z.im. Let z = x + iy.

The routine **zmake(real,imag)** returns the complex number with real part **real** and imaginary part **imag**.

The routine **zconj**(**z**) returns $\bar{z} = x - iy$

The routine zneg(z) returns -z.

The routine **zabs** (z) returns $|z| = \sqrt{x^2 + y^2}$. Note that it is done safely to avoid overflow if |x| or |y| is close to floating point limits.

The routine zadd(z1, z2) returns $z_1 + z_2$.

The routine zsub(z1, z2) returns $z_1 - z_2$.

The routine zmlt(z1,z2) returns z_1z_2 .

The routine zinv(z) returns 1/z. An **E_SING** error is raised if z = 0.

The routine zdiv(z1, z2) returns z_1/z_2 . An **E_SING** error is raised if $z_2 = 0$.

The routine zsqrt(z) returns \sqrt{z} . The principle branch is used for a branch cut along the negative real axis, so the real part of \sqrt{z} as computed is not negative.

The routine $\mathbf{zexp}(\mathbf{z})$ returns $\exp(z) = e^z = e^x(\cos y + i \sin y)$.

The routine zlog(z) returns log(z). The principle branch is used for a branch cut along the negative real axis, so the imaginary part of log(z) lies between or on $\pm \pi$.

EXAMPLE

To compute $\log(z + e^w)/\sqrt{1 + z^2}$:

where ONE is 1 + 0i; ONE = zmake(1.0,0.0);.

SOURCE FILE: zfunc.c

__add__,__ip__ , __mltadd__, __smlt__, __sub__, __zero__, __zadd__, __zconj__, __zip__ , __zmltadd__, __zmlt__, __zsub__, __zzero__ - core routines

SYNOPSIS

#include "machine.h"

/* or #include "matrix.h" */
void __add___ (Real dp1[], Real dp2[], Real out[], int len)
double __ip___ (Real dp1[], Real dp2[], int len)
void __mltadd__(Real dp1[], Real dp2[], double s, int len)
void __smlt__(Real dp[], double s, Real out[], int len)
void __sub___ (Real dp1[], Real dp2[], Real out[], int len)
void __zero__(Real dp[], int len)

```
#include "zmatrix.h"
        ___zadd___ (complex z1[], complex z2[],
void
                 complex out[], int len);
        ___zconj__(complex z[],
void
                                  int len);
complex __zip__ (complex z1[], complex z2[],
                                  int len, int conj);
void
        zmlt (complex z1[], complex s, complex z2[],
                                  int len);
        _____zmltadd___(complex z1[], complex z2[], complex s,
void
                                  int len, int conj);
        ____zsub___ (complex z1[], complex z2[], complex out[],
void
                                  int len);
                                 int len);
        zzero_(complex z[],
void
```

DESCRIPTION

These routines are the underlying routines for almost all dense matrix routines. Unlike the other routines in this library they do not take pointers to structures as arguments. Instead they work directly with arrays of Real's. It is intended that these routines should be *fast*. If you wish to take full advantage of a particular architecture, it is suggested that you modify these routines.

The current implementation does not use any special techniques for boosting speed, such as loop unrolling or assembly code, in the interests of simplicity and portability.

Note that zconj (z), referred to below, returns the complex conjugate of z.

The routine __add__() sets out[i] = dp1[i]+dp2[i] for i ranging from zero to len-1. The routine __zadd__() sets out[i] = z1[i]+z2[i] for i ranging from zero to len-1.

The routine ___ip__() returns the sum of dp1[i]*dp2[i] for i ranging from zero to len-1. The routine ___zip__() returns the sum of z1[i]*z2[i] for

i ranging from zero to len-1 if conj is Z_NOCONJ, and returns the sum of zconj(z1[i])*z2[i] for i ranging from zero to len-1 if conj is Z_CONJ.

The routine __mltadd__() sets dp1[i] = dp1[i]+s*dp2[i] for i ranging from zero to len-1. The routine __zmltadd__() sets z1[i] = z1[i]+s*z2[i] for i ranging from zero to len-1 if conj is Z_NOCONJ, and sets dp1[i] = z1[i]+s*zconj(z2[i]) for i ranging from zero to len-1 if conj is Z_CONJ.

The routine __smlt__() sets out[i] = s*dp[i] for i ranging from zero to len-1. The routine __zmlt__() sets out[i] = s*z[i] for i ranging from zero to len-1.

The routine __sub__() sets out[i] = dp1[i]-dp2[i] for i ranging from zero to len-1. The routine __zsub__() sets out[i] = z1[i]-z2[i] for i ranging from zero to len-1.

The routines __zero__() and __zzero__() set out[i] = 0.0 for i ranging from zero to len-1. These routines should be used instead of the macro MEM_ZERO() or the ANSI C routine memset() for portability, in case the floating point zero is not represented by a bit string of zeros.

EXAMPLE

MAT *A, *B; ZVEC *x, *y; Real alpha; /* set A = A + alpha.B */ for (i = 0; i < m; i++) __mltadd__(A->me[i],B->me[i],alpha,A->n); /* zero row 3 of A */ __zero__(A->me[3],A->n); /* quick complex inner product */ z_output(__zip__(x->ve,y->ve,x->dim,Z_CONJ));

SOURCE FILE: machine.c, zmachine.c