

Contents

| | |
|--|----|
| Preface | ix |
| Chapter 1. Fractional operations | 1 |
| 1.1. Weyl algebra | 1 |
| 1.2. Laplace and gauge transformations and reduced representatives | 2 |
| 1.3. Examples of ordinary differential operators | 4 |
| 1.4. Ordinary differential equations | 9 |
| 1.5. Okubo normal form and Schlesinger canonical form | 13 |
| Chapter 2. Confluences | 17 |
| 2.1. Regular singularities | 17 |
| 2.2. A confluence | 21 |
| 2.3. Versal additions | 22 |
| 2.4. Versal operators | 23 |
| Chapter 3. Series expansion and Contiguity relation | 27 |
| 3.1. Series expansion | 27 |
| 3.2. Contiguity relation | 29 |
| Chapter 4. Fuchsian differential equation and generalized Riemann scheme | 31 |
| 4.1. Generalized characteristic exponents | 31 |
| 4.2. Tuples of partitions | 35 |
| 4.3. Conjugacy classes of matrices | 37 |
| 4.4. Realizable tuples of partitions | 38 |
| Chapter 5. Reduction of Fuchsian differential equations | 43 |
| Chapter 6. Deligne-Simpson problem | 55 |
| 6.1. Fundamental lemmas | 55 |
| 6.2. Existence theorem | 57 |
| 6.3. Divisible spectral types | 62 |
| 6.4. Universal model | 63 |
| 6.5. Simply reducible spectral type | 66 |
| Chapter 7. A Kac-Moody root system | 69 |
| 7.1. Correspondence with a Kac-Moody root system | 69 |
| 7.2. Fundamental tuples | 78 |
| Chapter 8. Expression of local solutions | 81 |
| Chapter 9. Monodromy | 85 |
| 9.1. Middle convolution of monodromies | 85 |
| 9.2. Scott's lemma and Katz's rigidity | 91 |
| Chapter 10. Reducibility | 95 |
| 10.1. Direct decompositions | 95 |

| | |
|--|-----|
| 10.2. Reduction of reducibility | 100 |
| Chapter 11. Shift operators | 109 |
| 11.1. Construction of shift operators and contiguity relations | 109 |
| 11.2. Relation to reducibility | 114 |
| 11.3. Polynomial solutions | 118 |
| Chapter 12. Connection problem | 119 |
| 12.1. Connection formula | 119 |
| 12.2. An estimate for large exponents | 125 |
| 12.3. Zeros and poles of connection coefficients | 129 |
| Chapter 13. Examples | 139 |
| 13.1. Basic tuples | 139 |
| 13.2. Rigid tuples | 143 |
| 13.3. Jordan-Pochhammer family | 147 |
| 13.4. Hypergeometric family | 150 |
| 13.5. Even/Odd family | 156 |
| 13.6. Trigonometric identities | 162 |
| 13.7. Rigid examples of order at most 4 | 163 |
| 13.8. Other rigid examples with a small order | 168 |
| 13.9. Submaximal series and minimal series | 174 |
| 13.10. Appell's hypergeometric functions | 186 |
| 13.11. Okubo and Risa/Asir | 192 |
| Chapter 14. Further problems | 193 |
| 14.1. Multiplicities of spectral parameters | 193 |
| 14.2. Schlesinger canonical form | 193 |
| 14.3. Apparent singularities | 193 |
| 14.4. Irregular singularities | 193 |
| 14.5. Special parameters | 194 |
| 14.6. Shift operators | 195 |
| 14.7. Isomonodromic deformations | 195 |
| 14.8. Several variables | 195 |
| 14.9. Other problems | 196 |
| Appendix | 197 |
| Bibliography | 199 |
| Index | 201 |