DISCUSSION ON PROFESSOR HOTELLING'S PAPER

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THE SITUATION OF STATISTICS as it exists in the American university, so clearly described in Professor Hotelling's paper, cannot be gainsaid. His ultimate solution of the problem with respect to teaching, research, and consultation is a goal not easily attainable, but certainly is one toward which every self-respecting university should strive, in spite of administrative difficulties.

As one interested in the applications of statistical method in the fields of biology, medicine, and public health, I should like to present my concept of statistics for your consideration, as a plausible approach to Professor Hotelling's solution.

Francis Bacon (Novum Organum, Aphorism 95) stated long ago that scientists are either men of experiment or men of dogma. The men of experiment (wrote Bacon) are like ants—they collect and use. The men of dogma resemble spiders, which make cobwebs out of their own substance. But the bee takes a middle course: it gathers its material from the flowers of the garden and transforms and digests that material by a power of its own. Not unlike this process is the true business of philosophy. The philosopher relies not solely on the power of the mind, but takes the matter gathered from natural history in the mechanical experiment and lays it up, not in the memory, whole as he finds it, but in the *understanding*, altered and digested. Therefore from a closer and purer league between these two faculties, the experimental and rational, much may be hoped for.

In seeking to determine the place of statistics in a university, we have become confused because we have lost sight of the fact that the statistic method is a part of the scientific method. Without an appreciation and knowledge of statistics, the experimentalist or social scientist cannot frame his experiment or investigation within appropriate limitation; certainly he cannot adequately test his working hypotheses; and he is incapable of competent generalization.

In other words, experimental and statistical methods are equally important components of the scientific method. But because man is time-bound it seems virtually impossible for one and the same person to be proficient both in theoretical statistics and in one of the physical, biological, or social sciences.

In the occupational summary of what a statistician does, it is implied, although not stated, that the profession of statistics deals with the quantitative techniques of the scientific method. I quote:

> The statistician uses inductive reasoning based on the mathematics of probability, to develop and apply the most effective methods for collecting, tabulating, and interpreting quantitative information. This information may relate to any of a wide variety of fields and may be desired for any of a wide variety of purposes. It may pertain to economic conditions,

biological or physical sciences, manufacturing processes, marketing, transportation, population, or many other subjects. It may be desired as a basis for social or governmental action, action by a manufacturer or other business man, or for the extension of scientific knowledge . . .

Statistical work in all fields depends on the theory of statistics. This theory is in itself a subject of scholarly interest and research, both on account of its applications and because of its relations to philosophy and to mathematics. It is cultivated by many teachers of statistics and also by other people in various occupations.[1]

In the same *Bulletin*, W. Edward Deming writes that the above-quoted statement makes it clear that the statistician goes farther than his colleagues, the experimental and social scientists, who merely collect and interpret data. It is true, indeed, that the latter often fall short in their training and understanding of statistical theory. However, to be competent, they must know a great deal about laboratory and field techniques and understand the difficulties inherent in their data.

Although the statistician has vast knowledge of statistical method he may know little or nothing about the inherent contradictions in special experimental and field projects. However, if the statistician is to become "Bacon's bee," he must be able to subject data to probability analysis without destroying the original intent of the experiment. At the same time, the experimentalist must be wary of misinterpretation of his results because of the application of incorrect statistical techniques.

A university would be an incomplete institution of higher learning if it did not recognize the importance of this duality in the scientific method. In order to increase interest in statistics, a survey course in scientific method for undergraduates could be organized under the joint sponsorship of the departments of philosophy and mathematics, with the collaboration of representatives from the physical, social, and biological sciences. Development of scientific curiosity and integrity of thought, with appreciation of the need for mathematical discipline, could then begin early in the undergraduate's career. This would be in line with Professor Hotelling's suggestion for organizing an elementary introductory course in statistics with only algebra as a prerequisite.

If it could be emphasized that the statistical method is a part of the scientific method, the embryo statistician (undergraduate or graduate) would be encouraged to fulfill at least a minimum of required subject matter in one of the physical, biological, or social sciences, in addition to his extended training in mathematical statistics.

Finally we must consider that challenging group of scientists, university professors, administrators, physicians, and others who are acquainted with the experimental method, whether it entails field sampling or intricate laboratory techniques, and who, of necessity, must collect, analyze, present, and interpret data. A university would not be fulfilling its obligation to scientific research and study if these men were deprived of the opportunity to use some of the basic statistical techniques as applied to particular fields of investigation. These "experimental non-statisticians" (so termed in the *Bulletin* from which quotation was made above) would really be at a great loss. It would be even

[1] *Amer. Stat. Assoc. Bull.*, vol. 5, no. 2 (May, 1945), p. 14.

more difficult for the statistical non-experimentalist. The latter would be required to supervise the routine tasks and uninteresting accepted statistical techniques, although he should be free to search for new statistical devices and possibly disprove the validity of accepted theories.

The solution of the problem, I believe, lies in: (1) improvement in the teaching of arithmetic and mathematics as well as science in the grade and high schools (as suggested by Professor Hotelling); (2) better supervision of curriculum-advising in the high schools; (3) a survey course for college freshmen and sophomores in scientific method, the outline of which would be available to junior colleges; (4) a division of statistics responsible for instruction in theory and method on undergraduate and graduate levels, with candidates for higher degrees in statistical theory being encouraged to fulfill at least a minimum of basic requirements in one of the physical, biological, or social sciences, as at Columbia University (students in these sciences should likewise be encouraged to fulfill minimum requirements in mathematics so that they may enter at least some of the courses in the theory of statistics); and (5) consultative and seminar services involving coöperation between the science departments and the division of statistics so that each group may become more generally acquainted with the other to their mutual benefit.

II. ROBERT J. TRUMPLER

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ASTRONOMY has two quite distinct areas of contact with statistics. The first of these is in the discussion of observational errors; the second lies in the statistical description of our stellar universe, the field of statistical astronomy.

The problems connected with observational errors in astronomy are of course very similar to those in other exact sciences, such as physics or chemistry, except that the use of the method of least squares, which is very old in astronomy, has become standard practice in many problems of practical astronomy. It is therefore necessary that the student in astronomy be introduced early into the statistical theory of errors, as he will need its applications in the basic upper-division courses on astronomy.

During the last fifty years the statistical study of our stellar system has rapidly expanded, so much so that statistical astronomy may now be considered one of the major fields of astronomical investigation. The statistical problems with which the astronomer is confronted in the description of the stellar system are very different from those of ordinary population statistics. There are mainly two difficulties peculiar to stellar statistics.

In the first place, some of the variables suitable for the description of a star relative to its position in space, its physical nature, and its motion are not directly observable. The quantities observable are generally functions of the descriptive variables. For instance, the apparent brightness of a star, which is observed, does not describe the physical nature of a star; it is a function of the luminosity of a star and its distance from the observer. From the statistical distribution of the observed data the distributions and correlations of the descriptive variables must be deduced, which is mainly a problem of solving integral equations.

The second difficulty lies in the fact that all astronomical data used for statistical discussion are incomplete on account of instrumental limitations. Stars which are fainter than a certain apparent brightness are not observable. The stars observed are thus not a random sample but are those selected according to apparent brightness, which is a function of distance and luminosity. To take account of such inevitable effects of selection is one of the major problems in statistical astronomy.

It is clear that a course in statistical astronomy must be taught by an astronomer. It will generally be a graduate course since it requires a thorough understanding of methods of observation. Although statistical astronomy has its peculiar problems and solutions, the latter are developed from the basic principles of mathematical statistics. At present it is necessary to devote a considerable part of this course to an outline of these mathematical foundations.

It would be an advantage for students in astronomy to have a basic course in probability and statistics given by the department of statistics or mathematics, preferably in the junior year after a knowledge of calculus has been acquired. After this general preparation, the astronomy courses would develop the specific applications of least squares to astronomical problems. The graduate course in statistical astronomy could then be built up on a broad mathematical foundation.

III. HAROLD D. CARTER

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A DISCUSSION of the place of statistics in the university may be divided conveniently into two parts. The first and more fundamental question concerns the specific values inherent in the courses in statistics as taught in various departments. The importance of statistics in general is already established in the minds of intelligent and informed persons. However, extended and detailed discussion of particular branches, especially the applied ones, would help to clarify some issues relative to the placement of courses in various departments.

The second question is involved in the usual approach to the problem of administering instruction in statistics within the university. Here the basic question is dealt with largely as a matter of administrative detail, which should be worked out so that the instruction can be economical, efficient, and in harmony with the convictions of the representatives of the various departments.

In the matter of economy and efficiency, we may note of course that the question has been considered at length before. There appears to be little or no economy in over-centralization, and certainly none in too much decentralization. The size of classes is such that little or no economy would result from having the courses all taught in one department. It seems more efficient and more economical to have a few qualified persons specializing in particular fields than to have great numbers of students taking general training which they do not appreciate.

One plan, favored by many, is to have basic and also specialized courses offered in the mathematics department, and to have, in addition, specialized and applied courses offered in the several departments in which a need for such courses is evident. This plan appeals to me as most efficient and most in harmony with the convictions of the representatives of various departments. Certain variations on the plan may be acceptable; for example, the solid training in mathematical statistics might be offered, along with somewhat specialized courses, in a department of statistics operating independently of the mathematics department.

In the departments in which specialized and applied courses are offered, for example in education, many persons take the courses. The instruction tends to be elementary from a mathematical point of view, but sometimes rather advanced from the point of view of knowledge of specialized subject matter. To require the students to take the courses as offered in some other department, with a difference in emphasis, would in part destroy the value of the courses for the persons most concerned. The change would no doubt also lead to new problems of an old type, namely, those resulting from setting up additional course requirements and further prerequisites. My conviction is that the courses offered in the various departments should be continued, and that such courses should be taught by persons trained to some degree in mathematics and mathematical statistics, as well as in the special subject fields.

IV. EVERETT R. DEMPSTER

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IT CERTAINLY IS POSSIBLE to paint a somewhat doleful picture of the present status of the teaching of statistics to non-mathematicians and of their use of the knowledge thereby acquired. It also is probable that some increased use of mathematical statisticians as instructors would be highly salutary. I should like to suggest, however, that the benefits of this procedure tend to be overrated, that its indefinite extension would be harmful, and that there is at least one other possible remedy of equal or greater effectiveness.

The promulgation and perpetuation of wrong ideas of statistical method cannot be attributed entirely to persons ignorant of mathematics. Were there time I could cite a number of loose and misleading statements appearing in textbooks written by eminent mathematical statisticians. The harm resulting does not consist merely in the occasional use of a wrong method or the adoption of an unwarranted conclusion by persons relying on the advice given. The seriousness of these misstatements lies rather in the confusion created with respect to any over-all philosophical basis for statistical testing and estimating, if not its actual negation. And certainly it is confusion concerning a logical framework that constitutes the basis of poor teaching, of the use of methods that are not applicable, and of the interpretation of results to mean what cannot be known.

Much can be said in praise of a number of statistics textbooks that have been published for the use of biologists. But not one of them, it appears to me, succeeds in establishing in a clear-cut fashion those fundamental concepts on which all statistical methods are based.

These considerations lead to the following suggestion: the most effective present aid to the teaching of application of statistical method would be a suitable textbook. Such a book must provide a clear and logical basis for the use of statistics; it must explain and interpret specific formulas and methods in terms of this framework; and the style and level of presentation must be such that an intelligent non-mathematician can, in ordinary instances, choose a suitable method and understand the real meaning of the answer obtained. Such a textbook could not possibly be written by a person not conversant with the mathematics of statistical method; perhaps it could be written only by a mathematician in collaboration with one or a number of men in other fields who possess unusual facility and considerable experience in the applications of statistics to their own fields of research.

Are mathematical statisticians as a class the best teachers of statistical method? This question, in my opinion, possesses only qualified answers. The answer is "yes" with respect to students intending, for example, to specialize in practical breeding work with plants or animals. Such students can advantageously allow perhaps from twelve to twenty units for the study of statistics. It is worth their time to learn how to compute transformations and how to devise special procedures when they are needed.

But there is a much larger group of students with a real but minor interest in statistics who can afford to spend perhaps three units on the subject, or six at a maximum. Am I not right in suggesting that mathematicians do not enjoy teaching of the character necessary to meet the exigencies of this situation? And if they do not enjoy it, are they likely to do it effectively? Do they not feel faced with a distasteful dilemma, namely, to insist on rigor and cover an entirely inadequate amount of material, or to avoid all proofs and thereby forego the exercise of their special virtue as mathematicians?

The proper way to teach students of the type just mentioned is no doubt more easily described than attained. It consists, I believe, in presenting the material, including proofs, in such a way that its essense can be grasped in the so-called "intuitive" manner. This does not preclude a careful distinction between actual proof and "intuitive" proof. "Intuition" is a useful tool for the mathematician himself, but for the non-mathematician its use must predominate, and its exercise should be systematically encouraged and improved. But such encouragement, unbuttressed by rigor, runs counter to the nature of mathematicians; perhaps it consumes their mathematical integrity. It is an activity for which they are unfitted by training, if not by congenital preference.

We must then, I believe, continue to abandon that great mass of students having genuine, though limited, stakes in statistics to the uncertain mercies of non-mathematicians. But need this abandonment be, after all, so very bad?

I believe it need not be. In fact there are, I am sure, many non-mathematicians who are capable of grasping the basic theory of statistical method at least as clearly as perhaps the lower 25 percentile of mathematical statisticians. And these men, as teachers, will have the great advantage of knowledge of the respective fields in which the methods are to be applied. This advantage, admittedly, is called on to compensate for numerous faults.

But what these teachers or prospective teachers rather desperately need is a really excellent textbook, both for their own and their students' guidance. Cannot such a text be soon forthcoming?

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PROFESSOR HOTELLING is to be warmly congratulated on his treatment of the problem of the proper organizational setup of statistics in a university. By his research work, Professor Hotelling has made an ineffaceable imprint on the history of our science. Now he appears as a leader in the movement to give statistics in the universities facilities for development and instruction comparable to those enjoyed by other branches of knowledge. Incidentally, this is the second of Dr. Hotelling's papers on the same general subject that I have read. The first was presented at the meeting of the Institute of Mathematical Statistics in 1940, was printed in the *Annals of Mathematical Statistics*,[1] and, by unanimous decision of the Institute, was given a wide circulation in reprints.

Although endorsing the general ideas of Professor Hotelling, I wish to mention certain specific difficulties.

1. In many experimental sciences, statistics plays an important but nevertheless auxiliary role. Since students specializing in these sciences are not mathematically minded, and since their time is limited, it seems unavoidable that statistical instruction given to them must be brief, without mathematical background, and filled with examples taken from their own special fields. At the same time they must be made to understand the statistical methods presented to them. The question that arises is whether this is at all possible.

I think that the answer is in the affirmative. However, the method of presentation should be specifically adjusted to the audience. The method we use is to substitute sampling experiments for the mathematical proofs that are too difficult for these students. Of course, the teaching of statistics on this level, just as on a mathematical level, is satisfactory only if the teacher knows his subject and is interested in it. The only effective criterion for deciding whether or not a given person belongs in this category is his published research.

2. The second difficulty I wish to mention, and this is the more important one, consists of the current maladjustment in the instruction of future specialists in mathematical statistics, perhaps future university teachers. This maladjustment may be described under two headings.

a) *Maladjustment with mathematical courses.*—The current teaching of mathematics in an average university, especially in the first few years, is

[1] Vol. 11 (1940), pp. 457–470.

geared to satisfy two important needs: (i) to cover the deficiencies in the mathematical instruction in the high schools, and (ii) to satisfy the needs of the school of engineering. As a result of this second circumstance especially, the instruction is hasty, with the emphasis on machinery of calculation of derivatives and on similar problems, and ignores certain branches of mathematics which are essential to future studies in statistics. In fact, in some institutions, certain simple mathematical concepts needed for the theory of statistics are made available to students in graduate courses only.

It may be hoped that the current more or less general trend toward fundamental research and fundamental instruction will result in the reform of mathematical programs which will give students specializing in mathematical subjects an opportunity to learn early in their academic life the elements of what might be called "conceptual mathematics." This is of great importance in the study of mathematical statistics. I believe, moreover, that a reform of this kind would be beneficial to future mathematicians specializing in other branches of mathematics.

b) Maladjustments with instruction in experimental sciences.—The future mathematical statistician needs early contacts with experimental sciences. He needs them because, at this stage of the development of statistics, the experimental sciences are sources of theoretical problems. Also, he needs them because in almost any imaginable job which he may get after graduation he will be called upon to apply his theory to experimental or observational problems.

The term "experimental science" as used here is meant to be very broad and to cover such branches of knowledge as genetics, plant and animal breeding, home economics, medicine, public health, physics, astronomy, economics, and engineering—and even this list is not complete. It is plainly impossible to cover even a fraction of this program in anything like a systematic way. Since, however, contacts with the fields of application are very important indeed, one is forced to think of means to provide such contacts, and the answer seems to be a cycle of courses of experimental sciences especially designed for statisticians. Let me explain this by an example.

Ordinarily, courses in genetics are given for students who intend to specialize in some biological science. It is essential that such students learn certain details which are of no importance whatever to future statisticians. For example, students in biology studying genetics should and do learn to identify external evidence of the presence of specific genes in a number of organisms. This requires that they have previous instruction in botany and zoölogy. Should a future statistician embark on this program, he would have no time for statistics and would be compelled to learn a large number of facts which he would never be able to use. However, to learn the items in genetics which would be really useful to him, the items relating to methods of research and to the machinery of inheritance, which is frequently complex, he would need to follow the classical instruction to the graduate level.

It appears to me that a shorter and more effective way of giving the future statistician the opportunity to acquaint himself with genetics could be made available. This is through a one-semester course, perhaps given once every two or three years, especially designed for future statisticians. Such a course should concentrate on the machinery of inheritance, including mutation, or evolution, and on the methods of research. It would be superfluous to insist that the students learn, for example, that a specific gene A in a specific organism X is linked with some other gene B. However, it would be essential to let the students know that linkage does occur and that it is explainable by means of a certain machinery. Such courses could best be given by geneticists interested in applications of statistics but should not be courses on statistical methods as applied to genetics. Future statisticians will get instruction in statistical methods in courses on the theory of statistics. In order to use this theory properly and to be able to develop new theories suitable for applications, they need instruction in experimental sciences as such.

Apart from greatly benefiting future statisticians, the suggested courses are likely to attract a number of other students who would attend them to broaden their intellectual horizons.