Let U be any term; then the determination of U:

$$U = N' U + N U'$$

is equivalent to the proposed equality; for we know it is equivalent to the equality

$$(NU + NU' = 0) = (N = 0).$$

Let us recall the signification of the determination

$$U = N' U + N U'.$$

It denotes that the term U is contained in N' and contains N. This is easily understood, since, by hypothesis, N is equal to 0 and N' to 1. Therefore we can formulate the *law of forms* in the following way:

To obtain all the forms equivalent to a given equality, it is sufficient to express that any term contains the logical zero of this equality and is contained in its logical whole.

The number of forms of a given equality is unlimited; for any term gives rise to a form, and to a form different from the others, since it has a different first member. But if we are limited to the universe of discourse determined by n simple terms, the number of forms becomes finite and determinate. For, in this limited universe, there are 2^n constituents. Now, all the terms in this universe that can be conceived and defined are sums of some of these constituents. Their number is, therefore, equal to the number of combinations that can be made with 2^n constituents, namely 2^{2^n} (including 0, the combination of 0 constituent, and 1, the combination of all the constituents). This will also be the number of different forms of any equality in the universe in question.

44. The Law of Consequences.—We shall now pass to the law of consequences. Generalizing the conception of BOOLE, who made deduction consist in the elimination of middle terms, PORETSKY makes it consist in the elimination of known terms (*connaissances*). This conception is explained and justified as follows.

All problems in which the data are expressed by logical equalities or inclusions can be reduced to a single logical equality by means of the formula [1]

$$(A = 0)\ (B = 0)\ (C = 0) \ldots = (A + B + C \ldots = 0).$$

In this logical equality, which sums up all the data of the problem, we develop the first member with respect to all the simple terms which appear in it (and not with respect to the unknown quantities). Let n be the number of simple terms; then the number of the constituents of the development of 1 is 2^n. Let m ($\leq 2^n$) be the number of those constituents appearing in the first member of the equality. All possible consequences of this equality (in the universe of the n terms in question) may be obtained by forming all the additive combinations of these m constituents, and equating them to 0; and this is done in virtue of the formula

$$(A + B = 0) < (A = 0).$$

We see that we pass from the equality to any one of its consequences by suppressing some of the constituents in its first member, which correspond to as many elementary equalities (having 0 for second member), *i. e.*, as many as there are data in the problem. This is what is meant by "eliminating the known terms".

The number of consequences that can be derived from an equality (in the universe of n terms with respect to which it is developed) is equal to the number of additive combinations that may be formed with its m constituents; *i. e.*, 2^m. This number includes the combination of 0 constituents, which gives rise to the identity $0 = 0$, and the combination of the m constituents, which reproduces the given equality.

Let us apply this method to the equation with one unknown quantity

$$ax + bx' = 0.$$

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[1] We employ capitals to denote complex terms (logical functions) in contrast to simple terms denoted by small letters (a, b, c, . . .)

Developing it with respect to the *three* terms a, b, x:

$$(abx + ab'x + abx' + a'bx' = 0)$$
$$= [ab\,(x + x') + ab'x + a'bx' = 0]$$
$$= (ab = 0)\ (ab'x = 0)\ (a'bx' = 0).$$

Thus we find, on the one hand, the resultant $ab = 0$, and, on the other hand, two equalities which may be transformed into the inclusions

$$x < a' + b, \qquad a'b < x.$$

But by the resultant which is equivalent to $b < a$, we have

$$a' + b = a', \qquad a'b = b.$$

This consequence may therefore be reduced to the double inclusion

$$x < a', \qquad b < x,$$

that is, to the known solution.

Let us apply the same method to the premises of the syllogism

$$(a < b)\ (b < c).$$

Reduce them to a single equality

$$(a < b) = (ab' = 0), \quad (b < c) = (bc' = 0), \quad (ab' + bc' = 0),$$

and seek all of its consequences.

Developing with respect to the three terms a, b, c:

$$abc' + ab'c + ab'c' + a'bc' = 0.$$

The consequences of this equality, which contains four constituents, are 16 (2^4) in number as follows:

1. $(abc' = 0) = (ab < c)$;
2. $(ab'c = 0) = (ac < b)$;
3. $(ab'c' = 0) = (a < b + c)$;
4. $(a'bc' = 0) = (b < a + c)$;
5. $(abc' + ab'c = 0) = (a < bc + b'c')$;
6. $(abc' + ab'c' = 0) = (ac' = 0) = (a < c)$.

This is the traditional conclusion of the syllogism.[1]

7. $(ab\dot{c}' + a'bc' = 0) = (bc' = 0) = (b < c)$.

This is the second premise.

8. $(ab'c + ab'c' = 0) = (ab' = 0) = (a < b)$.

This is the first premise.

9. $(ab'c + a'bc' = 0) = (ac < b < a + c)$;

10. $(ab'c' + a'bc' = 0) = (ab' + a'b < c)$;

11. $(abc' + ab'c + ab'c' = 0) = (ab' + ac' = 0) = (a < bc)$;

12. $(abc' + ab'c + a'bc' = 0) = (ab'c + bc' = 0)$
$= (ac < b < c)$;

13. $(abc' + ab'c' + a'bc' = 0) = (ac' + bc' = 0)$
$= (a + b < c)$;

14. $(ab'c + ab'c' + a'bc' = 0) = (ab' + a'bc' = 0)$
$= (a < b < a + c)$.

The last two consequences (15 and 16) are those obtained by combining 0 constituent and by combining all; the first is the identity

15. $0 = 0$,

which confirms the paradoxical proposition that the true (identity) is implied by any proposition (is a consequence of it); the second is the given equality itself

16. $ab' + bc' = 0$,

which is, in fact, its own consequence by virtue of the principle of identity. These two consequences may be called the "extreme consequences" of the proposed equality. If we wish to exclude them, we must say that the number of the consequences properly so called of an equality of m constituents is $2^m - 2$.

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[1] It will be observed that this is the only consequence (except the two extreme consequences [see the text below]) independent of b; therefore it is the resultant of the elimination of that middle term.