

terms. This notion, which is too restricted, was suggested by the example of the syllogism, in which the conclusion results from the elimination of the middle term, and which for a long time was wrongly considered as the only type of mediate deduction.¹

However this may be, BOOLE and SCHRÖDER have exaggerated the analogy between the algebra of logic and ordinary algebra. In logic, the distinction of known and unknown terms is artificial and almost useless. All the terms are—in principle at least—known, and it is simply a question, certain relations between them being given, of deducing new relations (unknown or not explicitly known) from these known relations. This is the purpose of PORETSKY's method which we shall now expound. It may be summed up in three laws, the *law of forms*, the *law of consequences* and the *law of causes*.

43. The Law of Forms.—This law answers the following problem: An equality being given, to find for any term (simple or complex) a determination equivalent to this equality. In other words, the question is to find all the *forms* equivalent to this equality, any term at all being given as its first member.

We know that any equality can be reduced to a form in which the second member is 0 or 1; *i. e.*, to one of the two equivalent forms

$$N = 0, \quad N' = 1.$$

The function N is what PORETSKY calls the *logical zero* of the given equality; N' is its *logical whole*.²

¹ In fact, the fundamental formula of elimination

$$(ax + bx' = 0) < (ab = 0)$$

is, as we have seen, only another form and a consequence of the principle of the syllogism

$$(b < x < a') < (b < a').$$

² They are called "logical" to distinguish them from the identical *zero* and *whole*, *i. e.*, to indicate that these two terms are not equal to 0 and 1 respectively except by virtue of the data of the problem.

Let U be any term; then the determination of U :

$$U = N' U + N U'$$

is equivalent to the proposed equality; for we know it is equivalent to the equality

$$(N U + N U' = 0) = (N = 0).$$

Let us recall the signification of the determination

$$U = N' U + N U'.$$

It denotes that the term U is contained in N' and contains N . This is easily understood, since, by hypothesis, N is equal to 0 and N' to 1. Therefore we can formulate the *law of forms* in the following way:

To obtain all the forms equivalent to a given equality, it is sufficient to express that any term contains the logical zero of this equality and is contained in its logical whole.

The number of forms of a given equality is unlimited; for any term gives rise to a form, and to a form different from the others, since it has a different first member. But if we are limited to the universe of discourse determined by n simple terms, the number of forms becomes finite and determinate. For, in this limited universe, there are 2^n constituents. Now, all the terms in this universe that can be conceived and defined are sums of some of these constituents. Their number is, therefore, equal to the number of combinations that can be made with 2^n constituents, namely 2^{2^n} (including 0, the combination of 0 constituent, and 1, the combination of all the constituents). This will also be the number of different forms of any equality in the universe in question.

44. The Law of Consequences.—We shall now pass to the law of consequences. Generalizing the conception of BOOLE, who made deduction consist in the elimination of middle terms, PORETSKY makes it consist in the elimination of known terms (*connaissances*). This conception is explained and justified as follows.