the definition. We will prove that they are equal. Since, by hypothesis,

$$aa'_1 = 0, \quad a + a'_1 = 1,$$
$$aa'_2 = 0, \quad a + a'_2 = 1,$$

we have

$$aa'_1 = aa'_2, \quad a + a'_1 = a + a'_2;$$

whence we conclude, by the preceding lemma, that

$$a'_1 = a'_2.$$

We can now speak of *the* negative of a term as of a unique and well-defined term.

The *uniformity* of the operation of negation may be expressed in the following manner:

If $a = b$, then also $a' = b'$. By this proposition, both members of an equality in the logical calculus may be "denied".

16. The Principles of Contradiction and of Excluded Middle.—By definition, a term and its negative verify the two formulas

$$aa' = 0, \quad a + a' = 1,$$

which represent respectively the *principle of contradiction* and the *principle of excluded middle.*[1]

C. I.: 1. The classes a and a' have nothing in common; in other words, no element can be at the same time both a and not-a.

2. The classes a and a' combined form the whole; in other words, every element is either a or not-a.

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[1] As Mrs. LADD-FRANKLIN has truly remarked (BALDWIN, *Dictionary of Philosophy and Psychology*, article "Laws of Thought"), the principle of *contradiction* is not sufficient to define *contradictories;* the principle of excluded middle must be added which equally deserves the name of principle of contradiction. This is why Mrs. LADD-FRANKLIN proposes to call them respectively the *principle of exclusion* and the *principle of exhaustion,* inasmuch as, according to the first, two contradictory terms are *exclusive* (the one of the other); and, according to the second, they are *exhaustive* (of the universe of discourse).

P. I.: 1. The simultaneous affirmation of the propositions a and not-a is false; in other words, these two propositions cannot both be true at the same time.

2. The alternative affirmation of the propositions a and not-a is true; in other words, one of these two propositions must be true.

Two propositions are said to be *contradictory* when one is the negative of the other; they cannot both be true or false at the same time. If one is true the other is false; if one is false the other is true.

This is in agreement with the fact that the terms 0 and 1 are the negatives of each other; thus we have

$$0 \times 1 = 0, \quad 0 + 1 = 1.$$

Generally speaking, we say that two terms are *contradictory* when one is the negative of the other.

17. Law of Double Negation.—Moreover this reciprocity is general: if a term b is the negative of the term a, then the term a is the negative of the term b. These two statements are expressed by the same formulas

$$ab = 0, \quad a + b = 1,$$

and, while they unequivocally determine b in terms of a, they likewise determine a in terms of b. This is due to the symmetry of these relations, that is to say, to the commutativity of multiplication and addition. This reciprocity is expressed by the *law of double negation*

$$(a')' = a,$$

which may be formally proved as follows: a' being by hypothesis the negative of a, we have

$$aa' = 0, \quad a + a' = 1.$$

On the other hand, let a'' be the negative of a'; we have, in the same way,

$$a'a'' = 0, \quad a' + a'' = 1.$$

But, by the preceding lemma, these four equalities involve the equality

$$a = a''. \qquad \text{Q. E. D.}$$