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Further decay results on the system of NLS equations in lower order Sobolev spaces

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Abstract.

The initial value problem of a system of nonlinear Schrödinger equations with quadratic nonlinearities in two space dimensions is studied. We show there exists a unique global solution for this initial value problem which decays like t^{-1} as $t \to +\infty$ in $\mathbf{L}^{\infty}(\mathbb{R}^2)$ for small initial data in lower order Sobolev spaces.

§1. Introduction and main results

We consider global existence of solutions and time decay of the solutions to the following system of nonlinear Schrödinger equations

$$(1) \qquad \begin{cases} i\partial_t v_j + \frac{1}{2m_j}\Delta v_j = F_j(v_1, \cdots, v_l), t \in \mathbb{R}, x \in \mathbb{R}^2, \\ v_j(0,x) = \phi_j(x), x \in \mathbb{R}^2, \end{cases}$$

for $1 \leq j \leq l$, where $\overline{v_j}$ is the complex conjugate of v_j, m_j is a mass of a particle and quadratic nonlinearity has the form

$$F_j(v_1, \cdots, v_l) = \sum_{1 \leq m \leq k \leq 2l} \lambda^j_{m,k} v_m v_k,$$

with

$$v_m, v_k \in \{v_1, \cdots, v_l, \overline{v_1}, \cdots, \overline{v_l}\} = \{v_1, \cdots, v_l, v_{l+1}, \cdots, v_{2l}\}, \lambda^j_{m,k} \in \mathbb{C}.$$

The special system

$$(2) \qquad \begin{cases} i\partial_t v_1 + \frac{1}{2m_1}\Delta v_1 = \gamma \overline{v_1} v_2, \\ i\partial_t v_2 + \frac{1}{2m_2}\Delta v_2 = v_1^2, \end{cases}$$

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in $(t,x) \in \mathbb{R} \times \mathbb{R}^2$, where m_1 and m_2 are masses of particles and γ is a complex constant, is studied in [2] and [3]. In [2], we showed global existence in time of small solutions and time decay estimates to the solutions in the Sobolev space $\mathbf{H}^{2,2}(\mathbb{R}^2)$. We also proved nonexistence of scattering states in $\mathbf{L}^2(\mathbb{R}^2)$. We constructed the modified wave operators of System (2) for suitable given data in [3]. In the case of higher dimensions, existence of the wave operators of System (2) was studied in [5]. In [13], we considered the generalized system (1). We proved $\mathbf{L}^\infty(\mathbb{R}^2)$-time decay estimates of small solutions in the Sobolev space $\mathbf{H}^{2,2}(\mathbb{R}^2)$. We also discussed existence and nonexistence of wave operators. The purpose in this paper is to show that $\mathbf{L}^\infty(\mathbb{R}^2)$-time decay of small solutions in lower order Sobolev spaces $\mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2)$, where $1 < \beta < 2$.

We make the following assumptions on quadratic terms F_j for $1 \le j \le l$.

(A_1) There exist positive constants c_j for $1 \le j \le l$ such that

$$\operatorname{Im}\sum_{j=1}^{l} c_j F_j \overline{v_j} = 0.$$

(A_2)

$$F_j(v_1,\cdots,v_l) = e^{im_j\theta} F_j\left(e^{-im_1\theta}v_1,\cdots,e^{-im_l\theta}v_l\right)$$

for any $\theta \in \mathbb{R}$.

Under the mass condition $2m_1 = m_2$ and $\gamma = 1$, System (2) obeys these two assumptions. We may find another physical example satisfying these assumptions in [1].

Condition (A_1) is a sufficient condition under which System (1) satisfies $\mathbf{L}^2(\mathbb{R}^2)$ conservation law $\partial_t \sum_{j=1}^{l} c_j \|v_j\|^2_{\mathbf{L}^2(\mathbb{R}^2)} = 0$, where $c_j > 0$ for $1 \le j \le l$. In fact, System (1) can be regarded as the nonrelativistic version of a system of nonlinear Klein–Gordon equations

$$\text{(3)}\quad \frac{1}{2c^2 m_j}\partial_t^2 u_j - \frac{1}{2m_j}\Delta u_j + \frac{m_j c^2}{2} u_j = -F_j(u_1,\cdots,u_l), \quad j = 1,\cdots,l,$$

under Condition (A_2), where c is the speed of light. The related systems of Klein–Gordon equations were considered in [7], [8] and [10].

In what follows, we use the same notations both for the vector function spaces and the scalar ones. For $m,s \in \mathbb{R}$, weighted Sobolev space $\mathbf{H}^{m,s}(\mathbb{R}^2)$ is defined by

$$\mathbf{H}^{m,s}(\mathbb{R}^2) = \left\{ \begin{array}{l} f = (f_1,\ldots,f_l) \in \mathbf{L}^2(\mathbb{R}^2); \\ \|f\|_{\mathbf{H}^{m,s}(\mathbb{R}^2)} = \sum_{j=1}^{l} \|f_j\|_{\mathbf{H}^{m,s}(\mathbb{R}^2)} < \infty \end{array} \right\},$$

where $\|f_j\|_{\mathbf{H}^{m,s}(\mathbb{R}^2)} = \left\|(1-\Delta)^{\frac{m}{2}}(1+|x|^2)^{\frac{s}{2}} f_j\right\|_{\mathbf{L}^2(\mathbb{R}^2)}$. We write $\|f_j\|_{\mathbf{L}^2(\mathbb{R}^2)} = \|f_j\|$ and $\mathbf{H}^m(\mathbb{R}^2) = \mathbf{H}^{m,0}(\mathbb{R}^2)$ for simplicity. We denote by the same letter C various positive constants.

Our main theorem is stated as follows :

Theorem 1. *Assume that* (A_1) *and* (A_2) *hold. We also assume that* $\phi = (\phi_1, \cdots, \phi_l) \in \mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2)$, *where* $1 < \beta < 2$. *Then for some* $\varepsilon > 0$ *there exists a unique global solution* $v = (v_1, \cdots, v_l)$ *to System (1) such that* $v = (v_1, \cdots, v_l) \in \mathbf{C}(\mathbb{R}; \mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2))$ *and*

$$\|v(t)\|_{\mathbf{L}^\infty(\mathbb{R}^2)} = \sum_{i=1}^{l} \|v_i(t)\|_{\mathbf{L}^\infty(\mathbb{R}^2)} \le C(1+|t|)^{-1}$$

for any $\phi = (\phi_1, \cdots, \phi_l)$ *satisfying*

$$\|\phi\|_{\mathbf{H}^{\beta,0}(\mathbb{R}^2)} + \|\phi\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} = \sum_{i=1}^{l} \left(\|\phi_i\|_{\mathbf{H}^{\beta,0}(\mathbb{R}^2)} + \|\phi_i\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \right) \le \varepsilon.$$

The global existence result of System (1) can be obtained by using the method of [11] and [9]. $\mathbf{L}^\infty(\mathbb{R}^2)$-time decay of small solutions for System (1) in $\mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2)$, where $1 < \beta < 2$, is our main result and will be proved by showing a priori estimates of local in time of solutions. This idea was used in [4] and [15].

Remark 1. By the same method, we may obtain the similar time decay results to Theorem 1 in the case of $\beta > 2$.

§2. A priori estimates of solutions

For any $\phi = (\phi_1, \cdots, \phi_l) \in \mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2)$, where $1 < \beta < 2$, we let $T > 0$ and $v = (v_1, \cdots, v_l)$ be a solution of System (1) in Space $X_T = \{\mathbf{C}([0,T]; \mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2)); \|v\|_{X_T} < \infty\}$ with norm

$$\|v\|_{X_T} = \sum_{j=1}^{l} \|v_j\|_{X_T} = \sup_{t \in [0,T]} \sum_{j=1}^{l} (1+t)^\delta \left\| U_{\frac{1}{m_j}}(-t) v_j \right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)},$$

where $0 < \delta < \frac{1}{4}(\beta - 1)$. Existence of local in time of solutions can be obtained by contraction mapping principle. We give it without proof (See [14]).

Theorem 2. *Let* $T > 1$, *then there exists a small* $\varepsilon > 0$ *such that for any* $\phi = (\phi_1, \cdots, \phi_l) \in \mathbf{H}^{\beta,0}(\mathbb{R}^2) \cap \mathbf{H}^{0,\beta}(\mathbb{R}^2)$ *with* $\|\phi\|_{\mathbf{H}^{\beta,0}(\mathbb{R}^2)} +$

$\|\phi\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \leq \varepsilon$, *where* $1 < \beta < 2$. *System (1) has a unique pair of solutions* $v = (v_1, \cdots, v_l) \in X_T$ *such that* $\|v\|_{X_T} \leq 2\varepsilon$.

Let $U_\delta(t)$ be the Schrödinger evolution group defined by $U_\delta(t) = \mathcal{F}^{-1}E^\delta\mathcal{F}$ with $\delta \neq 0, E = e^{-\frac{i}{2}t|\xi|^2}$ for $t \neq 0$. In what follows we let v be a solution given by the above theorem. We define the dilation operator by $(D_\delta\phi)(x) = \frac{1}{(i\delta)}\phi\left(\frac{x}{\delta}\right)$ for $\delta \neq 0$ and define $E = e^{-\frac{i}{2}t|\xi|^2}$, $M = e^{-\frac{i}{2t}|x|^2}$ for $t \neq 0$. Evolution operator $U_\delta(t)$ for $t \neq 0$ is written as

$$(U_\delta(t)\phi)(x) = M^{-\frac{1}{\delta}}(x) D_{\delta t}\left(\left(\mathcal{F}\left(M^{-\frac{1}{\delta}}(y)\phi(y)\right)\right)(\xi)\right)(x).$$

We have

$$U_\delta(-t)\phi(x) = -M^{\frac{1}{\delta}}\left(\mathcal{F}^{-1}E^\delta D_{\frac{1}{\delta t}}\phi\right)(x).$$

Then the free evolution group is factorized as $U_\delta(t)\mathcal{F}^{-1} = M^{-\frac{1}{\delta}}D_{\delta t}\mathcal{M}_{-\frac{1}{\delta}}$, where $\mathcal{M}_{-\frac{1}{\delta}} = \mathcal{F}M^{-\frac{1}{\delta}}\mathcal{F}^{-1}$. Moreover we have $\mathcal{F}U_\delta(-t) = -\mathcal{M}_{\frac{1}{\delta}}E^\delta D_{\frac{1}{\delta t}}$. These formulas were used in [6] first.

We estimate difference between the free Schrödinger solution and its main term. Lemma 1 is obtained in [4].

Lemma 1. *Let* $f \in \mathbf{H}^{0,\beta}(\mathbb{R}^2), \delta \neq 0$. *Then*

$$\left\|f - M^{-\frac{1}{\delta}}D_{\delta t}\mathcal{F}U_\delta(-t)f\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)} \leq C|t|^{-1-\alpha}\|U_\delta(-t)f\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)},$$

for $|t| \geq 1$, *where* $0 < \alpha < 1$ *and* $\beta > 1 + 2\alpha$.

If we multiply both sides of (1) by $\mathcal{F}U_{\frac{1}{m_j}}(-t)$, then we can divide the nonlinear term into the main term and the remainder term under the gauge condition (A_1). Detailed calculations can be seen in [13].

We define

$$\begin{aligned} & R_{1,j} \\ = \;& i\left(\mathcal{M}_{m_j} - 1\right)\frac{m_j}{t}F_j\left(-D_{\frac{m_j}{m_1}}\mathcal{M}_{m_1}^{-1}\mathcal{F}U_{\frac{1}{m_1}}(-t)v_1, \cdots,\right. \\ & \left. -D_{\frac{m_j}{m_l}}\mathcal{M}_{m_l}^{-1}\mathcal{F}U_{\frac{1}{m_l}}(-t)v_l\right) \end{aligned}$$

and

$$\begin{aligned} & R_{2,j} \\ = \;& i\frac{m_j}{t}F_j\left(-D_{\frac{m_j}{m_1}}\mathcal{M}_{m_1}^{-1}\mathcal{F}U_{\frac{1}{m_1}}(-t)v_1, \cdots, -D_{\frac{m_j}{m_l}}\mathcal{M}_{m_l}^{-1}\mathcal{F}U_{\frac{1}{m_l}}(-t)v_l\right) \\ & -i\frac{m_j}{t}F_j\left(-D_{\frac{m_j}{m_1}}\mathcal{F}U_{\frac{1}{m_1}}(-t)v_1, \cdots, -D_{\frac{m_j}{m_l}}\mathcal{F}U_{\frac{1}{m_l}}(-t)v_l\right). \end{aligned}$$

Then the nonlinear term can be divided into two parts such that

$$i\partial_t u_j = \frac{1}{t} F_j\left(u_1, \cdots, u_l\right) + D_{\frac{1}{m_j}} \sum_{i=1}^{2} R_{i,j}, \tag{4}$$

where

$$u_j = D_{\frac{1}{m_j}} \mathcal{F} U_{\frac{1}{m_j}}\left(-t\right) v_j.$$

We multiply both sides of (4) by $c_j \overline{u}_j$, take the imaginary part and use Condition (A_1) to obtain

$$\partial_t \left(\sum_{j=1}^{l} c_j \left|u_j\right|^2 \right) = 2\mathrm{Im} \left(\sum_{j=1}^{l} c_j \left(D_{\frac{1}{m_j}} \sum_{i=1}^{2} R_{i,j} \right) \overline{u_j} \right), \tag{5}$$

where $c_j > 0$ for $1 \leq j \leq l$. We prove the second term of the right hand side of (4) is a remainder term.

Lemma 2. *We have*

$$\sum_{j=1}^{l} \sum_{i=1}^{2} \|R_{i,j}\|_{\mathbf{L}^\infty(\mathbb{R}^2)} \leq C|t|^{-1-\alpha} \left\| U_{\frac{1}{m}}\left(-t\right) v \right\|^2_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)},$$

for $|t| \geq 1$, *where*

$$\left\| U_{\frac{1}{m}}\left(-t\right) v \right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} = \sum_{j=1}^{l} \left\| U_{\frac{1}{m_j}}\left(-t\right) v_j \right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)},$$

$0 < \alpha < 1$ *and* $\beta > 1 + 2\alpha$.

Proof. By Schwarz inequality and Lemma $X4$ in [12], we have

$$\begin{aligned}
&\|R_{1,j}\|_{\mathbf{L}^\infty(\mathbb{R}^2)} \\
\leq\ & C|t|^{-1-\alpha} \left\| F_j \left(-D_{\frac{m_j}{m_1}} \mathcal{F} M^{-m_1} U_{\frac{1}{m_1}}\left(-t\right) v_1, \cdots, \right.\right. \\
&\qquad\qquad \left.\left. -D_{\frac{m_j}{m_l}} \mathcal{F} M^{-m_l} U_{\frac{1}{m_l}}\left(-t\right) v_l \right) \right\|_{\mathbf{H}^{\beta,0}(\mathbb{R}^2)} \\
\leq\ & C|t|^{-1-\alpha} \sum_{p,q=1}^{l} \left\| U_{\frac{1}{m_p}}\left(-t\right) v_p \right\|_{\mathbf{L}^1(\mathbb{R}^2)} \left\| U_{\frac{1}{m_q}}\left(-t\right) v_q \right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \\
\leq\ & C|t|^{-1-\alpha} \sum_{p,q=1}^{l} \left\| U_{\frac{1}{m_p}}\left(-t\right) v_p \right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \left\| U_{\frac{1}{m_q}}\left(-t\right) v_q \right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \\
\leq\ & C|t|^{-1-\alpha} \sum_{j=1}^{l} \left\| U_{\frac{1}{m_j}}\left(-t\right) v_j \right\|^2_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)},
\end{aligned}$$

where $0<\alpha<1$ and $\beta>1+2\alpha$.
We can estimate $\|R_{2,j}\|_{\mathbf{L}^\infty(\mathbb{R}^2)}$ by the same method. Q.E.D.

We define

$$\left|J_{\frac{1}{m_j}}\right|^s = U_{\frac{1}{m_j}}(t)\,|x|^s\,U_{\frac{1}{m_j}}(-t), \tag{6}$$

where $s>0$. Then (6) can be presented as (see [4])

$$\left|J_{\frac{1}{m_j}}\right|^s = \overline{M^{m_j}}\left(-\frac{t^2}{m_j^2}\Delta\right)^{\frac{s}{2}} M^{m_j}.$$

Moreover we have commutation relations with $\left|J_{\frac{1}{m_j}}\right|^s$ and $L_{\frac{1}{m_j}} = i\partial_t + \frac{1}{2m_j}\Delta$ such that

$$\left[L_{\frac{1}{m_j}}, \left|J_{\frac{1}{m_j}}\right|^s\right] = 0.$$

We evaluate the derivative of $\left\|U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)}$ with respect to t. Then we have

Lemma 3. *We have*

$$\begin{aligned} &\frac{d}{dt}\left\|U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \\ \le\ & Ct^{-1}\left\|U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)}\left\|\mathcal{F}U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)} \\ +\ & Ct^{-1-\alpha}\left\|U_{\frac{1}{m}}(-t)\,v\right\|^2_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)} \end{aligned}$$

for any $t\in[1,T]$, *where* $0<\alpha<1$, $2>\beta>1+2\alpha$ *and*

$$\left\|\mathcal{F}U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)} = \sum_{j=1}^{l}\left\|\mathcal{F}U_{\frac{1}{m_j}}(-t)\,v\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)}.$$

By Lemma 3, we have the following desired a priori estimates of local solutions.

Lemma 4. *There exist small* $\varepsilon>0$ *and* δ *with* $\varepsilon^{\frac{1}{2}}<\delta<\frac{\alpha}{2}$ (α *is mentioned in Lemma* 3.) *such that*

$$\left\|U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)}(1+t)^{-\delta} + \left\|\mathcal{F}U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)} < \varepsilon^{\frac{1}{2}},$$

and

$$\sum_{j=1}^{l}\left(\|\phi_j\|_{\mathbf{H}^{\beta,0}(\mathbb{R}^2)}+\|\phi_j\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)}\right)\leq\varepsilon$$

for any $t\in[1,T]$, *where* $2>\beta>1$.

The proofs of Lemma 3 and Lemma 4 are similar to the proofs in [13]. Because of the limitation of length, we omit the proofs of them here.

§3. Proof of Theorem 1.

Proof. We consider the case of $t\geq1$. From Lemma 1 we have

$$\|v_j\|_{\mathbf{L}^\infty(\mathbb{R}^2)}\leq Ct^{-1}\left\|\mathcal{F}U_{\frac{1}{m_j}}(-t)\,v_j\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)}+Ct^{-1-\alpha}\left\|U_{\frac{1}{m_j}}(-t)\,v_j\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)},$$

where $0<\alpha<1$ and $\beta>1+2\alpha$. By the standard continuation argument we have a unique time global solution such that

$$\left\|U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)}\leq\varepsilon^{\frac{1}{2}}(1+t)^{\delta},$$
$$\left\|\mathcal{F}U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)}\leq\varepsilon^{\frac{1}{2}}$$

for any $t\geq1$, where $\varepsilon^{\frac{1}{2}}<\delta<\frac{\alpha}{2}$ and $2>\beta>1+2\alpha$.

Therefore we get the time decay estimates

$$\begin{aligned}\|v\|_{\mathbf{L}^\infty(\mathbb{R}^2)} &= \sum_{j=1}^{l}\|v_j\|_{\mathbf{L}^\infty(\mathbb{R}^2)}\\ &\leq Ct^{-1}\left\|\mathcal{F}U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{L}^\infty(\mathbb{R}^2)}+Ct^{-1-\alpha}\left\|U_{\frac{1}{m}}(-t)\,v\right\|_{\mathbf{H}^{0,\beta}(\mathbb{R}^2)}\\ &\leq C\left(\varepsilon^{\frac{1}{2}}t^{-1}+t^{-1-\alpha+\delta}\right)\leq Ct^{-1}\end{aligned}$$

for $t\geq1$. If $t\in[0,1]$, we have $\|v\|_{\mathbf{L}^\infty(\mathbb{R}^2)}\leq C\varepsilon$ by $\|v(0)\|_{\mathbf{H}^{\beta,0}(\mathbb{R}^2)}\leq\varepsilon$ for $2>\beta>1$. In the case of $t\leq0$, the theorem follows by the same method. This completes the proof of the theorem. Q.E.D.

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