

Notation Index

Introduction

$\{\dots\}$	set of elements 3
$\{\dots \dots\}$	set defined by property 3
\in	is an element of 3
\notin	is not an element of 3
\subseteq	is a subset of 3
$\not\subseteq$	is not a subset of 3
\subset	is a proper subset of 3
$\not\subset$	is not a proper subset of 3
\cup	union 3
\cap	intersection 3
$-$	difference 3
Δ	symmetric difference 3
$\max(A)$	maximum of A 3
$\min(A)$	minimum of A 3
\times	cartesian product 3
$\langle \dots \rangle$	tuple 3
A^k	k th power of A 3
$A^{<\omega}$	set of finite sequences from A 3
$\bar{x}^{[k]}$	k th coordinate of x 3
$S^{[k]}$	k th column of S 3
\emptyset	empty set 3
N	set of natural numbers 3
\oplus	direct sum 3
$ A $	cardinality of A 3
\aleph_0, \aleph_1	cardinal numbers 3
2^{\aleph_0}	cardinality of continuum 3
$\varphi: A \rightarrow B$	function notation 3
\downarrow	converges 3
\uparrow	diverges 3
$x \mapsto \varphi(x)$	x is mapped to $\varphi(x)$ 3
$\text{dom}(\varphi)$	domain of φ 3
$\text{rng}(\varphi)$	range of φ 3
χ_S	characteristic function of S 3
\uparrow	restriction 3
\lim_s	limit 4
$\lim \sup_s$	limit supremum 4
$\lim \inf_s$	limit infimum 4
$\lambda x f(x, y)$	lambda notation 4
\subseteq	extension for functions 4
2^S	power set of S 4
$[\dots]$	interval notation 4
(\dots)	interval notation 4
$[\dots)$	interval notation 4

$(\dots]$	interval notation 4
(a, ∞)	interval notation 4
$(-\infty, a]$	interval notation 4
$\&$	and 4
\vee	or 4
\sim, \neg	not 4
\rightarrow, \Rightarrow	implies 4
$\leftrightarrow, \Leftrightarrow$	if and only if 4
\exists	existential quantifier 4
\forall	universal quantifier 4
$\bigwedge_{i=0}^n \sigma_i$	finite conjunction 4
$\bigvee_{i=0}^n \sigma_i$	finite disjunction 4

Chapter I

μ	least number operator 7
\mathcal{R}	class of recursive functions 7
\mathcal{P}	class of partial recursive functions 9
\mathcal{S}	space of strings 10
\mathcal{S}_f	space of f -valued strings 10
\mathcal{S}_c	space of strings with values $\leq c$ 10
\mathcal{S}_2	space of binary strings 10
\sqsubseteq	extension for strings 10
$\text{lh}(\sigma)$	length of string 10
$\sigma * \tau$	concatenation of strings 10
\mathcal{R}_f	functions recursive in f 11
\leq_T	Turing reducibility 11
χ_R	characteristic function of relation 11
φ	enumeration function 12
$\Phi^0, \Phi^0_e, \Phi^A, \Phi^A_e$	enumeration functionals 13

Chapter II

\mathbf{f}, \mathbf{A}	degree of f, A 15
\equiv_T	Turing equivalence 15
\mathbf{D}	degrees of unsolvability 15
\leq	partial ordering on \mathbf{D} 15
\cup	join operation on \mathbf{D} 15
\mathcal{D}	poset of degrees 15
\mathcal{DU}	usl of degrees 15
$\mathbf{0}$	degree of recursive functions 15
$\cup\{\mathbf{a}_i\}$	lub of finite set of degrees 15
$\cap\{\mathbf{a}_i\}$	glb of finite set of degrees 15
$\mathbf{a} \mid \mathbf{b}$	incomparability for degrees 17
$p < q$	p refines q 19
$p \perp q$	incompatibility for forcing conditions 19
\models	satisfies 21
\Vdash	forces 21
\hookrightarrow	embedding 23
\cong	isomorphism 23
$\theta^{(F)}$	union of columns of θ 24
$\theta^{(I)}$	union of all but one column of θ 24

\exists_n, \forall_n	classes of formulas of bounded complexity	25
$\text{Th}(\mathcal{D})$	elementary theory of \mathcal{D}	25
\subseteq	substructure	27

Chapter III

$\Sigma_n^0, \Pi_n^0, \Delta_n^0$	levels of arithmetical hierarchy	35
f'	completion of f	39
\mathbf{f}'	jump of f	39
f^*	set recursively isomorphic to f'	39
$f^{(n)}$	n th completion of f	39
$\mathbf{f}^{(n)}$	n th jump of f	39
$\mathbf{D}[a, b]$	interval notation for \mathbf{D}	41
$\mathcal{D}[a, b]$	interval notation for \mathcal{D}	41
$\mathbf{J}[0, 0']$	range of jump operator on $\mathbf{D}[0, 0']$	46
$\mathcal{D}\mathcal{U}'$	usl of degrees with jump	50
\mathcal{D}'	poset of degrees with jump	50
α^{-1}	inverse of α	61
$\mathbf{L}_n, \mathbf{H}_n, \mathbf{I}$	levels of high/low hierarchy	62, 63

Chapter IV

$\text{high}_n, \text{low}_n$	classification in high/low hierarchy	75
$\mathbf{a}\text{-high}_n, \mathbf{a}\text{-low}_n$	classification in relativized high/low hierarchy	75
$\mathbf{L}_n(\mathbf{a}), \mathbf{H}_n(\mathbf{a}), \mathbf{I}(\mathbf{a})$	levels of relativized high/low hierarchy	75
$\mathbf{GL}_n, \mathbf{GH}_n, \mathbf{GI}$	levels of generalized high/low hierarchy	77
$\mathbf{GL}_n(\mathbf{a}), \mathbf{GH}_n(\mathbf{a}), \mathbf{GI}(\mathbf{a})$	levels of relativized generalized high/low hierarchy	77
Tot	index set of total recursive functions	84
Tot(f)	index set of total functions computable from f	84

Chapter V – XII

$\sigma \upharpoonright \tau$	incompatibility for strings	101
Id	identity tree	102, 126, 150, 199
Ext	extension subtree	102, 126, 150, 230
Sp	splitting subtree	106, 135, 155, 235
Tot	e -total subtree	109, 128, 151
Nar	narrow subtree	111
Pt	pointed subtree	112, 151
$\mathbf{D}_{\text{arith}}$	arithmetical degrees	113
\equiv	elementary equivalence	116
$\mathbf{0}^{(\omega)}$	ω -jump of $\mathbf{0}$	117
\equiv_j	congruence for n -tuples	122, 124
\subseteq_u	uniform subtree	126
$\sigma^{(i)}$	i th coordinate of string of tuples	127
$\text{tr}(\sigma \rightarrow \tau; \rho)$	transfer strings	127
Diff	differentiating subtree	127, 151, 231
Tr	transfer tree	140
Exp	expansion tree	163
$\mathcal{L}_b, \mathcal{L}_l, \mathcal{L}_a$	languages	168
\mathcal{N}	second order arithmetic	168
D_d^*	integers in model of second order arithmetic	170
$\mathbf{0}^{(\omega \rightarrow \omega)}$	iterate of jump operation	172
PEExt	partial extension tree	184, 204

PDiff	partial differentiating tree 184, 204
PSp	partial splitting tree 185, 204, 213
PTot	partial e -total tree 185
$E(\lambda)$	e -state function 215
0_s	string of s consecutive zeroes 219
$I_{\sigma,\tau}$	interval notation 222
$\text{ht}(T)$	height of T 223
$s(\xi), p(\xi)$	successor, predecessor of ξ 229
Init	initial tree 229

Appendices

\subseteq_a	admissible extension 282
\equiv_0	agree on coordinate 0 283