

INTRODUCTION

These notes grew out of lectures given by the author at the Institut für Angewandte Mathematik, Heidelberg University, and at the Centre for Mathematical Analysis, Australian National University.

A central aim was to give the basic ideas of Geometric Measure Theory in a style readily accessible to analysts. I have tried to keep the notes as brief as possible, subject to the constraint of covering the really important and central ideas. There have of course been omissions; in an expanded version of these notes (which I hope to write in the near future), topics which would obviously have a high priority for inclusion are the theory of flat chains, further applications of G.M.T. to geometric variational problems, P.D.E. aspects of the theory, and boundary regularity theory.

I am indebted to many mathematicians for helpful conversations concerning these notes. In particular C. Gerhardt for his invitation to lecture on this material at Heidelberg, K. Ecker (who read thoroughly an earlier draft of the first few chapters), R. Hardt for many helpful conversations over a number of years. Most especially I want to thank J. Hutchinson for numerous constructive and enlightening conversations.

As far as *content* of these notes is concerned, I have drawn heavily from the standard references Federer [FH1] and Allard [AW1], although the reader will see that the presentation and point of view often differs from these references.

An outline of the notes is as follows. Chapter 1 consists of basic measure theory (from the Caratheodory viewpoint of outer measure). Most of

the results are by now quite classical. For a more extensive treatment of some of the topics covered, and for some bibliographical remarks, the reader is referred to Chapter 2 of Federer's book [FH1], which was in any case the basic source used for most of the material of Chapter 1.

Chapter 2 develops further basic preliminaries from analysis. In preparing the discussion of the area and co-area formulae we found Hardt's Melbourne notes [HR1] particularly useful. There is only a short section on BV functions, but it comfortably suffices for all the later applications. We found Giusti's Canberra notes [G] useful in preparing this material (especially in relation to the later material on sets of locally finite perimeter).

Chapter 3 is the first specialized chapter, and gives a concise treatment of the most important aspects of countably n -rectifiable sets. There are much more general results in Federer's book [FH1], but hopefully the reader will find the discussion here suitable for most applications, and a good starting point for any extensions which might occasionally be needed.

In Chapters 4, 5 we develop the basic theory of rectifiable varifolds and prove Allard's regularity theorem. ([AW1].) Our treatment here is formally much more concrete than Allard's; in fact the entire argument is given in the concrete setting of rectifiable varifolds, considered as countably n -rectifiable sets equipped with a locally \mathcal{H}^n -integrable multiplicity function. Hopefully this will make it easier for the reader to see the important ideas involved in the regularity theorem (and in the preliminary theory involving monotonicity formulae etc.).

Chapter 6 contains the basic theory of currents, including integer multiplicity rectifiable currents, but not including a discussion of flat

chains. The basic references for this chapter are the original paper of Federer and Fleming [FF] and Federer's book [FH1], although in a number of respects our treatment is a little different from these references.

In Chapter 7 there is a discussion of the basic theory of minimizing currents. The theorem 36.4, the proof of which is more or less standard, does not seem to appear elsewhere in the literature. In the last section we develop the regularity theory for codimension 1 minimizing currents. A feature of this section is that we treat the case when the currents in question are actually codimension 1 in some smooth submanifold. (This was of course generally known, but does not explicitly appear elsewhere in the literature.)

Finally in Chapter 8 we describe Allard's theory of general varifolds, which originally appeared in [AW1]. (Important aspects of the theory of varifolds had earlier been developed by Almgren [A3].)

In conclusion I want to express my sincere gratitude to Dorothy Nash, who did such a superb job in typing these lectures from a manuscript that was often messy and which frequently had to be corrected.