SINGULAR AND QUASI-BOUNDED FUNCTIONS ASSOCIATED WITH
THE HEAT EQUATION

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A theory of singular and quasi-bounded temperatures associated with the heat equation is developed. Guided by the harmonic case [1] an operator S is defined on the set M of non-negative functions on an open set of R^{n+1} which admit superthermic majorants. Quasi-bounded and singular functions in M are defined from the operator S. The basic properties of S are discussed. Non-negative temperatures can be uniquely decomposed as a sum of a quasi-bounded temperature and a singular temperature. Explicit formulae for the operator S are developed in the case of a half space, an unbounded strip, and a bounded Lipschitz domain. These constitute the main results of the work. No corresponding results in the harmonic case, to our knowledge, have been published elsewhere, although it is now obvious that similar explicit formulae for the operator S in the harmonic case can be proved without difficulty. The decomposition of a non-negative temperature u in the half space as a sum of a singular and a quasi-bounded temperature corresponds to taking the convolution with the fundamental solution of the singular and the absolutely continuous parts, respectively, of the Borel measure ν which, from Widder's Theorem, represents u.

As a result of the well-known differences between the potential theories for the harmonic and thermic cases not all the results of [1] carry over to the thermic case, and for some of the results which do carry over more care is required in the proofs.

REFERENCES

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