ON THE DETERMINATION OF THE ELECTRIC CONDUCTIVITY OF THE EARTH'S INTERIOR FROM GEOMAGNETIC DATA

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1. BACKGROUND

In the study of electrical conductivity as a function of depth within the Earth, the structure of the external and internal components of the Earth's transient magnetic field plays a crucial role for two reasons. On the one hand, the relationship between the external and internal components is determined solely by the electrical conductivity, and, on the other, estimates of the relationship can be determined from observational data. The form of the estimates of the external and internal components derived from observational data depends heavily on the subsequent use to be made of them. In fact, when information is required about the average spherically symmetric structure of the Earth's electrical conductivity, it is necessary to use global features of the Earth's transient magnetic field. In this paper, we discuss how to reconstruct the external and internal components of the P_1° - field for the period 1964-1965. Because missing data techniques have been applied so that observatories, which had less than 1% of their hourly values missing for the period 1964-65, could be included in the analysis and because more than the normal number of observatories were operational in the International Geophysical Year period 1964-65, the estimates for the P_1° - field will be based on the most comprehensive data-base so far acquired. We will present some preliminary results on the distribution of the electric conductivity in the interior of a spherical model Earth.

We start by recalling that, for the P_1°-field, the H and Z components of the Earth's transient magnetic field at geomagnetic colatitude θ and time t can be written as

(1.1) $$H = -A_H(t)\sin\theta \ , \quad A_H(t) = e_1^\circ(t) + i_1^\circ(t) \ ,$$

and

(1.2) $$Z = A_Z(t)\cos\theta \ , \quad A_Z(t) = e_1^\circ(t) - 2i_1^\circ(t) \ ,$$

where $e_1^\circ(t)$ and $i_1^\circ(t)$ denote the external and internal components of the P_1°-field at the surface of the Earth which varies with time t . Thus, for an observatory with geomagnetic colatitude θ_j , the observations (hourly value) of H and Z yield the following time series

(1.3) $$H_j(t_i) = -A_H(t_i)\sin\theta_j + \varepsilon_{ij} \ , \quad i = 1,2,\ldots,n \ ,$$

and

(1.4) $$Z_j(t_i) = A_Z(t_i)\cos\theta_j + \varepsilon_{ij} \ , \quad i = 1,2,\ldots,n \ ,$$

where the ε_{ij} denote observational errors.

If these data are used to infer information about the global features of the electrical conductivity of the Earth, then there are two ways in which they can be used to construct a transfer function between the external and internal components of the P_1°-field or equivalently a transfer function between the H and Z components of the P_1°-field. Because theory is available as to how the electrical conductivity of a spherically symmetric Earth is related to these transfer functions, either can be used. See the referenced papers by Anderssen et al and Banks. We will concentrate on the second of these two alternatives (i.e. the transfer function between the H and Z components of the P_1°-field), because its analysis is simpler than the first alternative. However, we comment on the first alternative as the discussion unfolds.

The two alternatives for constructing the transfer function are:

Method 1 For each time point t_i, apply spherical harmonic analysis to the data $H_j(t_i)$ and $Z_j(t_i)$ to find the estimates $\hat{A}_H(t_i)$ and $\hat{A}_Z(t_i)$ (cf. Anderssen and Seneta (1969)) of the amplitudes $A_H(t_i)$ and $A_Z(t_i)$ of (1.1) and (1.2); and then Fourier analyse the multiple time series $\hat{A}_H(t_i)$, $\hat{A}_Z(t_i)$ to determine the Fourier decomposition of $\hat{A}_H(t_i)$ and of the corresponding coherent component of $\hat{A}_Z(t_i)$ (cf. Banks (1969) and Anderssen and Gustafson (1982)).

Method 2 For each observatory j, Fourier analyse the multiple time series $\{H_j(t_i),Z_j(t_i)\}$ to determine the Fourier decomposition of $H_j(t_i)$ and of the corresponding coherent component of $Z_j(t_i)$

$$\hat{H}_j(t_i) = -A_H(\omega_i)\exp(i\theta_H(\omega_i)\sin\theta_j \ , \tag{1.5}$$

and

$$\hat{Z}_j(t_i) = A_Z(\omega_i)\exp(i\theta_Z(\omega_i))\cos\theta_j \ , \tag{1.6}$$

where $A_H(\omega_j)$ and $A_Z(\omega_j)$ denote the amplitudes of the H and Z components of the P_1°-field at frequency ω_i and $\theta_H(\omega_j)$ and $\theta_Z(\omega_j)$ the corresponding phases; and then apply spherical harmonic analysis to recover estimates of the Fourier terms in (1.5) and (1.6).

Because of the linearity of the equations and operations applied, these two approaches are equivalent mathematically. Computationally, they are not equivalent and, in part, the purpose of this paper is to exploit this for data analysis purposes.

FIGURE 1

DIAGRAMATIC REPRESENTATION OF METHODS 1 AND 2

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\{H_j(t_i), Z_j(t_i), \theta_j\} \arrow[r, "\text{Fast Fourier Transform}", "\text{(observatory by observatory)}"'] \arrow[d, "\begin{array}{c}\text{Spherical Harmonic}\\ \text{Analysis}\\ \text{(hour by hour)}\end{array}"] & \{\hat{H}_j(f_i), \hat{Z}_j(f_i), \theta_j\} \arrow[d, "\begin{array}{c}\text{Spherical Harmonic}\\ \text{Analysis}\\ \text{(frequency by frequency)}\end{array}"'] \\
\{A_H(t_i), A_Z(t_i)\} \arrow[r, "\text{Fast Fourier Transform}", "\text{(a single transform)}"'] & \{\hat{A}_H(f_i), \hat{A}_Z(f_i)\}
\end{tikzcd}
$$

If the transfer function between the external and internal components of the P_1^o - field is required, then the relationship between $A_H(t)$, $A_Z(t)$ and $e_1^o(t)$, $i_1^o(t)$ given in (1.1) and (1.2) must be invoked after applying the spherical harmonic analysis.

The relationship between Methods 1 and 2 is presented in Figure 1. We note that the spherical harmonic analysis only involves the calculation of a weighted average of a particular hourly value for each observatory. Method 1 requires only two Fourier transformations (one for each of the A_H and A_Z time series), while Method 2 calls for two Fourier transformations for each observatory. Thus, Method 1 is computationally less laborious than Method 2. In the next section, we shall show that Method 1 has further advantages over Method 2.

2. COMPARISON OF THE TWO ALTERNATIVES FOR TRANSFER FUNCTION MODELLING

For a study of the global structure of the Earth's electrical conductivity, observations are required of the Earth's transient magnetic field which span a long duration in time and come from a suitable distribution of observatories. The former ensures that the transfer function will have been estimated for long period disturbances which will have penetrated deep into the interior of the Earth; while the latter guarantees an accurate spherical harmonic analysis. The difficulty associated with achieving the former is missing data. The longer the period over which an analysis is made, the fewer will be the number of observatories which have an unbroken sequence of data (hourly values). The difficulty with the latter is the poor geographical location of observatories around the Earth.

If attention is restricted to the P_1^o - field, the last-mentioned difficulty is greatly diminished since its representation is longitude independent. The distribution of observatories is such that virtually no bias will be introduced due to the colatitude distribution of stations, since the Southern Hemisphere can be seen as a reflection of the Northern. However, because of the preponderance of European stations, the longitude distribution is such that the daily variation Sq could cause a bias. For this reason, Buys-Ballot filters were constructed from the data of each station to remove that station's Sq.

The two alternative procedures discussed above can be modified to cope with (a small amount of) missing data. When the Fourier analysis is done first, it is necessary to utilise the standard techniques discussed in the time series literature (cf. Bloomfield (1976), Koopmans (1974)). The situation is much easier when the harmonic analysis is done first. Then it is only necessary to do the spherical harmonic analysis for those observatories for which data are available at the particular hour being analysed. In this way, the number of observatories which contribute to the spherical harmonic analysis changes from hour to hour.

Because of this ease with which it handles missing data, the first alternative has much appeal. It is the method we suggest here because it has other advantages which are:

(i) <u>Saving in Computational Effort</u>. Fewer computer operations are required if the transfer function is computed using Method 1.

(ii) <u>Compact Storage of Information about the P_1° - field</u>. The spherical harmonic analysis of Method 1 isolates from the observational data the essential information about the P_1° - field into just two global time series $\{A_H(t_i), A_Z(t_i)\}$ which represents a compact form in which information about the P_1° - field could be stored for subsequent use.

3. ANALYSIS OF THE DATABASE

The initial data base consisted of all hourly values recorded during the period January 1, 1964 to December 31, 1965. It was compiled by Professor Denis Winch, Department of Applied Mathematics, University of Sydney, and a copy is held at the National Geophysical Data Center, Boulder, Colorado 80302, U.S.A. The data base contained observations from a total of 129 observatories.

The first step was to remove observatories with too many missing hourly values. An unbroken record for any one of the three components of the Earth's magnetic field recorded at an observatory would contain $731 \times 24 = 17544$ observations (hourly values). Observatories were deleted from the data base if they had more than 1% of the measurements missing for any one of the three components. That is, no more than 175 hourly values could be missing from any one of its components for an observatory to be retained in the data set. As a result, only the 69 observatories listed in Table 1 were retained.

The three components recorded for 64 of these observatories were D, H and Z, but 5 (namely, ALE, BLC, MBC, NGK and RES) recorded X, Y and Z. For these observatories, the H component was calculated as

$$H^2 = X^2 + Y^2 .$$

Thus, such H values would be missing if either X or Y was missing. For the transfer function calculations, we worked with a data base of $69 \times 2 \times 17544 \doteqdot 2.4 \times 10^6$ entries consisting of the H and Z components for the 69 observatories listed in Table 1. Of these entries, only 5112 ($\simeq$0.21%) corresponded to missing values. Of the 69 observatories, 25 had unbroken records for the two-year period. The lengths of the data gaps

TABLE 1

#	OBSERVATORY CODE	NAME	GEOGRAPHIC COLAT	GEOGRAPHIC LONG E	GEOMAGNETIC COLAT	GEOMAGNETIC LONG E
1	ABG	ALIBAG	71.37	72.87	80.57	144.34
2	AGN	AGINCOURT	46.22	280.73	34.98	347.99
3	AIA	ARGENTINE ISLANDS	155.25	295.73	143.85	3.89
4	ALE	ALERT	7.50	297.50	4.10	166.63
5	ALM	ALMERIA	53.15	357.53	49.53	76.01
6	AQU	L'ALQUILA	47.62	13.32	47.29	93.62
7	ASH	ASHKABAD	52.05	58.10	59.55	133.77
8	BLC	BAKER LAKE	25.67	263.97	16.17	316.50
9	CCS	CAPE CHELYUKSIN	12.28	104.28	23.68	176.85
10	CMO	COLLEGE	25.13	212.17	25.21	257.32
11	CWE	CAPE WELLEN	23.83	190.17	28.04	237.79
12	DAL	DALLAS	57.02	263.25	47.02	328.64
13	DIK	DIXON ISLAND	16.45	80.57	26.97	162.00
14	DOB	DOMBAS	27.93	9.12	27.88	100.67
15	DOU	DOURBES	39.90	4.60	38.17	88.39
16	ESK	ESKDAILEMUIR	34.68	356.80	31.69	83.57
17	FRD	FREDERICKSBURG	51.80	282.63	40.49	350.78
18	FUR	FURSTENFELDBRUCK	41.83	11.28	41.33	93.99
19	GDH	GODHAVN	20.77	306.48	10.28	33.77
20	HAD	HARTLAND	39.00	355.52	35.52	79.70
21	HER	HERMANUS	124.42	19.20	123.44	81.26
22	HIS	HEISS ISLAND	9.38	58.05	18.70	156.32
23	HON	HONOLULU	68.68	202.00	68.77	267.29
24	IRK	IRKUTSK-PATRONY	37.83	104.45	49.22	175.31
25	KAK	KAKIOKA	53.77	140.18	63.85	206.65
26	KIV	KIEV	39.28	30.30	42.54	112.79
27	KNY	KANOYA	58.58	130.88	69.35	198.75
28	KZN	KAZAN	34.17	48.85	40.74	130.94
29	LER	LERWICK	29.87	358.82	27.62	89.22
30	LNN	LENINGRAD	30.05	30.70	33.86	117.90
31	LOV	LOVO	30.65	17.83	32.06	106.36
32	LVV	LVOV	40.10	23.75	42.11	106.49
33	MBC	MOULD BAY	13.80	240.60	10.74	257.25
34	MBO	MBOUR	75.60	343.03	68.91	55.77
35	MEA	MEANOOK	35.38	246.67	28.08	302.02
36	MFD	MOCA	86.65	8.67	84.44	79.31
37	MIR	MIRNY	156.55	93.00	167.05	148.26
38	MLT	MISALLAT	60.48	30.90	63.19	106.63
39	MMB	MEMAMBETSU	46.10	144.20	55.86	209.10
40	MNK	MINSK	35.90	26.52	38.63	110.98
41	MOS	MOSCOW	34.52	37.32	39.23	121.08
42	NGK	NIEMEGK	37.93	12.68	37.90	97.21
43	NUR	NURMIJARVI	29.48	24.65	32.25	113.14
44	NVL	NOVOLAZAREVSKAYA	160.77	11.82	156.33	54.26
45	ODE	STEPANOVKA	43.22	30.88	46.43	111.74
46	PAG	PANAGJURISCHTE	47.48	24.18	49.28	104.02
47	RES	RESOLUTE BAY	15.30	265.10	6.89	290.94
48	RSV	RUDE SKOV	34.15	12.45	34.29	99.14
49	SIT	SITKA	32.93	224.67	29.86	276.22
50	SJG	SAN JUAN	71.88	293.85	60.47	3.94
51	SOD	SODANKYLA	22.63	26.63	26.34	120.47
52	SSO	SIMOSATO	56.43	135.93	66.85	203.13
53	STO	STONYHURST	36.15	357.53	33.22	83.38
54	SVD	SVERDLOVSK	33.27	61.07	41.55	141.26
55	TAN	TANANARIVE	108.92	47.55	113.82	113.25
56	THL	THULE	12.52	290.83	1.09	6.75
57	TIF	TBILISI	47.92	44.70	53.42	122.72
58	TKT	TASHKENT	48.53	69.20	57.57	144.31
59	TOL	TOLEDO	50.12	355.95	46.29	75.39
60	TOO	TOOLANGI	127.53	145.47	136.51	221.66
61	TRD	TRIVANDRUM	81.52	76.95	91.11	147.11
62	TRO	TROMSO	20.33	18.95	22.98	117.17
63	TUC	TUCSON	57.75	249.17	49.50	313.06
64	VAL	VALENTIA	38.07	349.75	33.52	74.13
65	VIC	VICTORIA	41.50	236.60	35.71	293.91
66	WIK	WIEN-KOBENZL	41.73	16.32	42.23	98.85
67	WIT	WITTEVENN	37.18	6.67	36.00	91.85
68	WNG	WINGST	36.25	9.07	35.60	94.69
69	YAK	YAKUTSK	27.98	129.67	38.93	194.38

varied irregularly and very few gaps consisted of just one missing value. Sometimes, only one or two of the three geomagnetic components were missing.

Before applying Method 1, Buys-Ballot filters (cf. Koopmans (1974)) were used to remove the Sq components from the 69 observatories. Some of the 69 observatories may be strongly influenced by local factors and hence unsuitable for the purpose of determining the phase and magnitude of the response of Earth. Therefore these observatories should be excluded before applying Method 1 above to the data from the remaining ones. Our task is therefore to find these atypical observatories. For this puspose we determined the phase and magnitude based on just one observatory and carried out this calculation for each of the 69 observatories. The data were smoothed as stated in Fig 2c in Anderssen et al (1979). First the mean was subtracted from each time series which comprises 17544 hourly values. The series was subsequently filtered using a 24 hour Buys-Ballot filter. Missing values were replaced by 0 at this state. Then the data were tapered by applying a 5% cosine bell tapering (See e.g. Bloomfield, 1976). Next the time series was padded to 49152 hours before applying the Fast Fourier Transform and calculating periodograms. These were smoothed using in succession 5- , 23- , 51- , 101- , 201- and 301-point Danielle filters. This process was carried out for 2 series from each observatory namely the sequences of values of the H- and the Z-component. It is then possible to calculate the phase and the magnitude of the response for a number of frequencies. Consequently we have for each frequency 69 different estimates for the phase and magnitude of the response. We illustrate the situation by discussing the 69 phase values corresponding to the period 30.118 days. (The conclusions are rather similar for other period lengths as well as for the response estimate). We try to minimise the influence of"wild data" by selecting various subsets of 69 estimates and estimate the global phase by means of a suitable statistic. Our results are collected in Table 2.

TABLE 2

Estimates of the phase of the global response at the period length 30.118 days. The subsets are defined in the main text.

Subset	# observatories	average value	standard deviation	50th percentile
A	69	167°.34	32°.03	160°.70
B	44	154°.99	14°.75	157°.32
C	21	159°.85	9°.42	160°.36
D	17	155°.60	10°.79	154°.97
E	15	153°.48	8°.83	152°.46

The 5 different subsets are defined as follows. Subset A consists of all 69 observatories and Subset D comprises the 17 observatories listed in Table 1 of Anderssen et al, [2]. Subset E arises from D by excluding SJG (San Juan) and THL (Thule-Qanag). This subset was used for estimating the magnitude of the response. We note that the average phase for Subset D coincides with the result reported in [2]. It turned out that the following 25 observatories had at least one phase-value greater than 180°, which is physically unrealistic: ABG, AGN, AIA, BLC, CMO, CWE, DIK, DOB, HER, HON, LER, MBC, MBO, MEA, MFD, MIR, MLT, NVL, RES, SIT, SOD, THL, TRD, TRO, YAK. Subset B is obtained by excluding these observatories from Subset A . We note that most of these stations lie close to the geomagnetic poles or equator. Subset C consists of those observatories satisfying the conditions (i) through (iv) below:

(i) The geomagnetic colatitude is in one of the intervals [30°,85°], [95°,150°].

(ii) The phase-value belongs to the interval [140°,180°] for each of the period lengths 34.113, 30.118, 24.976, 20.078, 15.059, 10.039 and 5.007 days.

(iii) The magnitude of the response is less than 0.6 for the same period lengths as under (ii).

(iv) The depth estimated according to Schmucker (See [2]) increases with period length.

The following 21 observatories belong to Subset C: ALM, AQU, CCS, DAL, DOU, FUR, HAD, KAK, KNY, MNK, NGK, PAG, ODE, RSV, STO, TUC, VAL, WIK, WIT, WNG, TKT.

Either the average value or the 50th percentile may be used to estimate the phase of the global response. The latter is a more robust estimator and it varies less among the 5 subsets A through E than the average value does. If we now take the average value of Subset B as estimator for the global phase we get, with a 95% confidence interval,

$$154^\circ.99 \pm 4^\circ.45 \ .$$

The 50-percentiles of all 5 subsets and the average values of subsets B through E lie inside or very close to the confidence interval. Since Subset B is the largest subset giving an estimator for the global phase with these properties we recommend that this subset is used for estimating the global field.

4. FUTURE WORK

Tables of phase and response for each observatory will be published elsewhere. We also intend to proceed according to Method 1 and calculate time series representing the global field. It could be advantageous to tabulate their Fourier transforms instead of the time series themselves. These tables may then be used for testing various models for determination of the electric conductivity of the interior of Earth.

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