SOME POSITIVE EIGENFUNCTIONS FOR ELLIPTIC OPERATORS WITH OBLIQUE DERIVATIVE BOUNDARY CONDITIONS AND CONSEQUENCES FOR THE STATIONARY DENSITIES OF REFLECTED BROWNIAN MOTIONS

Ruth J. Williams

ABSTRACT: Positive eigenfunctions for elliptic operators with oblique derivative boundary conditions arise as the stationary densities of reflected Brownian motions, briefly RBM's. An RBM is a diffusion process that behaves like Brownian motion with a constant drift velocity μ in the interior of a d-dimensional domain and is instantaneously reflected at the boundary in a direction specified by a non-tangential vector field v on the boundary. When the domain is bounded and smooth and the vector field is smooth, it is shown that the stationary density is of a simple exponential form for all μ if and only if the vector field v satisfies a certain skew-symmetry condition. A formal analogue of this result for polyhedral domains, where v is constant on each face, will also be given. Consequences for the existence and uniqueness of an RBM with such non-smooth data will be drawn from this.

Details will appear elsewhere.

Department of Mathematics
University of California, San Diego
La Jolla, CA 92093
U.S.A.