NEW KALUZA-KLEIN THEORY

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Abstract

We investigate supersymmetric Kaluza–Klein theory for a realistic unification theory. To account for chiral phenomenology of low energy physics, the Kaluza–Klein world must be of even dimensions, and for a theory beyond ten dimensions, we must introduce extra time–like dimension. However, we can avoid massless ghosts in 4–dimensional spacetime after spontaneous compactification of the ground state, and the cosmological constant is zero.

Unification of gravity with all other forces is a long pursuing goal of many scientists. It is well known that the strong interaction is a SU(3) gauge force and the electroweak forces are described by a $SU(2) \times U(1)$ gauge model. A fundamental theory should be able to explain such gauge symmetries in a unified way.

Kaluza–Klein theory can ascribe gauge symmetry to be the geometric property of an assumed internal space. The original Kaluza–Klein theory formulated a 5–dimensional unified theory after the gravitation theory of Einstein. In Einstein theory, the dynamical field is the metric tensor (or the same thing, vierbein field) of the 4–dimensional spacetime, it has 10 degrees of freedom. In the 5–dimensional Kaluza–Klein theory, the metric field has 15 degrees of freedom. One assumes the ground state of the 5–dimensional space M^5 to be the product $M^4 \times S^1$ of the 4–dimensional spacetime with the circle S^1, the radius of which is of the order of the Planck length, and assumes that the metric field of M^5 does not depend on the fifth coordinate θ. After a trivial integration on the circle S^1, the action describes a field theory containing a metric field of M^4, a 4–vector A_μ and a Brans–Dicke scalar field, all counts 15 degrees of freedom. The massless vector field can be identified to be the gauge potential of the electromagnetic $U(1)$ theory. In a word, the 5–dimensional Kaluza–Klein theory appears to be a unifield theory of gravity with electromagnetism and the $U(1)$ gauge symmetry stems from the isometry of the internal space S^1.

Although $U(1)$ is an Abelian group, it is not impossible to unify gravity with non–Abelian gauge forces so long as it is formulated in higher dimensions.(1) Nevertheless, the realistic problem is the fact that there is no fermion in the massless sector of the Kaluza–Klein theory according to Lichnerowiz's theorem.(2) It is the supersymmetry that puts fermions on the same footing with bosons. Therefore, it is rational to convince that the Kaluza–Klein supergravity theory, i.e., supergravity theory in higher dimensions, is the promising unified theory.

E. Cremmer *et al.*(3) proposed a $d = 11$ supergravity stimulating many works in some years hence. But being in odd dimensions, this theory can not account for the chiral phenomenology. L. Castellani *et al.*(4) have shown that in twelve dimensions supergravity theory is inconsistent. However, we have shown(5) that in a $d = 12$ space with an extra time–like dimension a supergravity theory is consistent. Recently, we propose a $d = 20$ supergravity theory(6) with metric signature $(2, 18)$. As for the ghost problem in relation to extra time–like dimensions, it is shown(7) that massless ghosts are absent in the spontaneous compactification if certain conditions are satisfied.

The supermultiplet of the $d = 12$ supergravity contains a graviton $e_M{}^A$, a 3–form A_{MNP}, two scalars A, B, a gravitino ψ_M and two chiral spinors λ and χ. The field equations of the bosonic sector are

$$R_{MN} - \tfrac{1}{2} g_{MN} R = -\tfrac{1}{2} g_{MN}[(\partial A)^2 + (\partial B)^2] + \partial_M A \partial_N A + \partial_M B \partial_N B$$
$$- \tfrac{1}{48} g_{MN} F_{PQRS} F^{PQRS} + \tfrac{1}{6} F_{MPQR} F_N{}^{PQR} \qquad (1)$$

$$\Box A = \Box B = \nabla^M F_{MNPQ} = 0 \qquad (2)$$

where

$$F_{MNPQ} = 4\partial_{[M}A_{NPQ]} \,.$$

We assume the ground state to be

$$g_{MN} = (g_{\mu\nu}, g_{\alpha\beta}, g_{ab}, g_{ss}, g_{ij}) \tag{3}$$

$$\mu,\nu = 1...4\ ,\quad \alpha,\beta = 5,6\ ,\quad a,b = 7,8\ ,\quad s = 9\ ,\quad i,j = 10,11,12$$

with metric signature $(-++\ ...\ ++-)$. And assume

$$F_{\alpha abS} = \xi\ \epsilon_{\alpha abs}$$

$$F_{ijks} = \eta\ \epsilon_{ijks}$$

$$\text{all other fields} = 0 \tag{4}$$

where ξ , η are constants. Then we get

$$R_{\mu\nu} = -\tfrac{3}{10}\,(2\xi^2 - \eta^2)\ g_{\mu\nu}$$

$$R_{\alpha\beta} = -\tfrac{3}{10}\,(2\xi^2 - \eta^2)\ g_{\alpha\beta} + \xi^2 g_{\alpha\beta}$$

$$R_{ab} = -\tfrac{3}{10}\,(2\xi^2 - \eta^2)\ g_{ab} + 2\xi^2 g_{ab}$$

$$R_{ss} = +\tfrac{7}{10}\,(2\xi^2 - \eta^2)\ g_{ss}$$

$$R_{ij} = -\tfrac{3}{10}\,(2\xi^2 - \eta^2)\ g_{ij} - \eta^2 g_{ij} \,. \tag{5}$$

Since $R_{ss} \equiv 0$, so that we get

$$R_{\mu\nu} = 0$$

$$R_{\alpha\beta} = \xi^2 g_{\alpha\beta}$$

$$R_{ab} = 2\xi^2 g_{ab}$$

$$R_{ss} = 0$$

$$R_{ij} = -2\xi^2 g_{ij} \,. \tag{6}$$

The topology of the vaccum is

$$M^{12} = M^4 \times S^2 \times S^2 \times S^1 \times H^3/\Gamma \,. \tag{7}$$

The M^4 is the 4–spacetime, the cosmological constant is zero. The H^3 is a hyperbolic manifold, it is quotiented by a discrete isometry group acting nonfreely, i.e., with fixed points, so that there is no Killing vector in this quotient space. Therefore, there is no gauge field associating with the extra time like dimension.

After spontaneous compactification, this theory describes a super–Yang–Mills $SU(2) \times SU(2) \times U(1)$ field theory coupled with gravitation.

Next we discuss the $d = 20$ supergravity wth metric signature $(-++ \dots ++-)$. The supermultiplet contains a vielbein $e_M{}^A$, two scalars A, B, three completely antisymmetric tensors A_{MNP}, B_{MNP}, B_{MNPQ}, two Majorana chiral spinors λ, χ and a gravitino field ψ_M, where B_{MNP} and B_{MNPQ} have negative parity. For the spontaneous compactification, the relevant bosonic field equations are

$$\begin{aligned} R_{MN} - \tfrac{1}{2} g_{MN} = -\tfrac{1}{2} g_{MN}\{(\partial A)^2 + (\partial B)^2\} + \partial_M A \partial_N A + \partial_M B \partial_N B \\ - \tfrac{1}{48} g_{MN}(G^2 + H^2) + \tfrac{1}{6}(G_{MPQR}G_N{}^{PQR} + H_{MPQR}H_N{}^{PQR}) \\ - \tfrac{1}{240} g_{MN}F^2 + \tfrac{1}{24} F_{MPQRS}F_N{}^{PQRS} \end{aligned}$$

$$\Box A = \Box B = \nabla^M G_{MNPQ} = \nabla^M H_{MNPQ} = \nabla^M F_{MNPQR} = 0 \,, \tag{8}$$

where

$$\begin{aligned} F_{MNPQR} &= 5\partial_{[M}B_{NPQR]} \\ G_{MNPQ} &= 4\partial_{[M}A_{NPQ]} \,, \\ H_{MNPQ} &= 4\partial_{[M}B_{NPQ]} \,. \end{aligned} \tag{9}$$

We assume the ground state metric to be

$$g_{MN} = (g_{\mu\nu}, g_{\alpha\beta}, g_{ab}, g_{ss}, g_{ij})$$

$$\mu,\nu = 1...4, \quad \alpha,\beta = 5...14 , \quad a,b = 15,16 , \quad s = 17 , \quad i,j = 18,19,20 \tag{10}$$

and the ansatz

$$G_{\alpha abs} = \xi \epsilon_{\alpha abs}$$

$$G_{ijks} = \eta \epsilon_{ijks}$$

$$\text{all other fields} = 0 \tag{11}$$

where ξ, η are constants. Eq. (8) turns to be

$$R_{\mu\nu} = -\tfrac{1}{6}(10\xi^2 - \eta^2)\, g_{\mu\nu}$$

$$R_{\alpha\beta} = -\tfrac{1}{6}(10\xi^2 - \eta^2)\, g_{\alpha\beta} + \xi^2 g_{\alpha\beta}$$

$$R_{ab} = -\tfrac{1}{6}(10\xi^2 - \eta^2)\, g_{ab} + 10\xi^2 g_{ab}$$

$$R_{ss} = \tfrac{5}{6}(10\xi^2 - \eta^2)\, g_{ss}$$

$$R_{ij} = -\tfrac{1}{6}(10\xi^2 - \eta^2)\, g_{ij} - \eta^2 g_{ij} \,. \tag{12}$$

From $R_{ss} \equiv 0$ we get

$$R_{\mu\nu} = 0$$

$$R_{\alpha\beta} = \xi^2 g_{\alpha\beta}$$

$$R_{ab} = 10\xi^2 g_{ab}$$

$$R_{ss} = 0$$

$$R_{ij} = -10\xi^2 g_{ij} \,. \tag{13}$$

This solution has the topology

$$M^{20} = M^4 \times S^{10} \times S^2 \times S^1 \times H^3/\Gamma \,. \tag{14}$$

References

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