Contents

1 Introduction **1**

2 Preliminaries **9**

2.1 Notation 9

2.2 Function spaces 11

2.3 The Fourier transform and distributions 15

2.4 The Sobolev spaces 21

2.5 Functions with values in Banach spaces 25

2.6 The Radon transform 27

Notes 29

3 Linear Wave Equations **30**

3.1 The explicit formula for solutions 30

3.2 The energy inequality 39

3.3 Solutions in the Sobolev spaces 48

3.4 Basic decay property of solutions 52

3.5 The Friedlander radiation fields 57

Notes 63

4 Local Existence for Nonlinear Wave Equations **65**

4.1 Preliminary estimates 65

4.2 Local existence theorems 67

4.3 Uniqueness of C^2-solutions 74

Notes 77

5 Vector Fields Associated with Wave Equations **79**

5.1 Introduction 79

5.2 Basic properties 80

5.3 The Klainerman–Sobolev inequality 84

5.4 L^1–L^∞ decay estimates 89

5.5 Basic estimates for nonlinear terms 104

Notes 107

6 Global Existence and Blow-Up for Small Data **108**

6.1 Global existence results 108

6.2 Blow-up in finite time 113

6.3 Asymptotically free solutions 118

Notes 125

7 Wave Equations with More General Nonlinearity **126**
7.1 Decay estimates of the solution in three space dimensions 126
7.2 John's global existence result 129
7.3 L^2-estimates 136
7.4 The Case in Three Space Dimensions 141
Notes 148

8 Systems of Reduced Equations **153**
8.1 Basic estimates 153
8.2 The general quasilinear case 155
8.3 The semilinear case 157
8.4 Detailed lower bounds of the lifespan 163

9 The Null Condition **175**
9.1 Estimates for the null forms 175
9.2 Three space dimensional case 180
9.3 Two space dimensional case 183
Notes 190

10 Weaker Conditions for Global Existence **196**
10.1 The weak null condition 196
10.2 Global existence under a weaker condition 199
10.3 Asymptotic behavior of global solutions 214
Notes 242

11 Restricted Usage of Vector Fields **245**
11.1 Systems with Multiple Propagation Speeds 245
11.2 Systems of semilinear wave and Klein–Gordon equations 252
Notes 265

Appendix A The Riemann integral in Banach spaces **271**

Appendix B Distribution-valued functions **274**

Appendix C The proof of Theorem 11.6 **277**