List of Notations

$\overline{a}$	the characteristic element (if $\alpha = 0$) of the root invariant $R(X,\theta)$	41
a	one of main invariants: $(S^*/S) \cong (\mathbb{Z}/2\mathbb{Z})^a$, of $Z, Y, (X,\theta)$	30
A_n	Dynkin diagram, Du Val singularity, root system of type A_n	5
b	a finite symmetric bilinear form	117
$b_\theta^{(p)}(p^k)$	one of elementary finite symmetric bilinear forms	118
b_M	the discriminant bilinear form of a lattice M	118
$\det(M)$	the determinant of a lattice M	118
$\mathrm{Duv}(\cdot)$	the Du Val part of $(\cdot)$	61
D_n	Dynkin diagram, Du Val singularity, root system of type D_n	5
E_n	Dynkin diagram, Du Val singularity, root system of type E_n	5
g	genus of a non-singular curve in $\lvert -2K_Z\rvert$ or of moving part of $\lvert -2K_Y\rvert$, maximal genus g of a component of X^θ, $g = (22-r-a)/2$	31
H	a part (kernel) of the root invariant $R(X,\theta)$	41
$\mathcal{H}_\beta$	hyperplane orthogonal to β of hyperbolic space	24
$\mathcal{H}_\beta^+$	half-space orthogonal to β of hyperbolic space	24
k	number of exceptional -4 curves on Y; number of double transparent vertices of the graph Γ; number of genus 0 curves of X^θ; $k = (r-a)/2$	31
(k, g, δ)	alternative main invariants of $Z, Y, (X,\theta)$	32
K	a part (the root lattice K or $K(2)$) of the root invariant $R(X,\theta)$	40
K_H	the overlattice of K related to the root invariant $R(X,\theta)$	44
K_n	log terminal singularities of index 2	18
$K_\theta^{(p)}(p^k)$	one of (with rank 1) elementary p-adic lattices	118
$\mathrm{Log}(\cdot)$	the logarithmic part of $(\cdot)$	61
$l(\mathfrak{A})$	the minimal number of generators of a finite Abelian group $\mathfrak{A}$	45

L_{K3}	the abstract lattice isomorphic to $H^2(X,\mathbb{Z})$ for a K3 surface X	26
$\mathcal{L}(X)$	$\mathcal{L}(X) = V^+(S_X)/\mathbb{R}^+$, the hyperbolic space of a surface X	24
$M(a)$	multiplication by $a \in \mathbb{Q}$ of the form of a lattice M	117
M	an arbitrary lattice usually; in Sect. 2.7, the lattice related to the root invariant and defined in (57)	117
$\mathcal{M}(X)$	$\mathcal{M}(X) = \mathrm{NEF}(X)/\mathbb{R}^+ \subset \mathcal{L}(X)$, the projectivization of the nef cone of a surface X	24
$\mathcal{M}^{(2)}$	a fundamental chamber of $W^{(2)}(S)$ in $\mathcal{L}(S)$	36
$\mathcal{M}^{(2,4)}$	a fundamental chamber of $W^{(2,4)}(S)$ in $\mathcal{L}(S)$	36
$\mathcal{M}^{(2,4)}_+$	a fundamental chamber of $W^{(2,4)}_+$ in $\mathcal{L}(S)$	37
Mod_M	moduli space of K3 with condition M on Picard lattice	29
$\mathrm{Mod}_{M\subset L_{K3}}$	moduli space of K3 with condition $M \subset L_{K3}$ on Picard lattice	28
$\mathrm{Mod}_{(r,a,\delta)}$	moduli space of (X,θ) with the main invariants (r,a,δ)	30
Mod'_S	moduli space of (X,θ) with the main invariant S	30
N	labels main invariants (r,a,δ) or (k,g,δ) of Z, and Y, (X,θ) of elliptic type	52
$\overline{\mathrm{NE}}(Z)$	Kleiman–Mori cone of a surface Z	12
$\mathrm{NEF}(X)$	nef cone of a surface X	23
$P(X)$	set of all exceptional curves (or their classes) on a surface X	23
$P(X)_+$	the set of exceptional classes of (X,θ)	33
$P(X)_{+I}$	the subset of exceptional classes of (X,θ)	33
$P(X)_{+IIa}$	the subset of exceptional classes of (X,θ)	33
$P(X)_{+IIb}$	the subset of exceptional classes of (X,θ)	33
$P(X)_{+III}$	the subset of exceptional classes of (X,θ)	33
$P^{(2)}(X)_+$	the subset of exceptional classes of (X,θ) with square -2	34
$P^{(4)}(X)_+$	the subset of exceptional classes of (X,θ) with square -4	34
$P(Y)_I$	the subset of exceptional curves of Y	33
$P(Y)_{IIa}$	the subset of exceptional curves of Y	33
$P(Y)_{IIb}$	the subset of exceptional curves of Y	33
$P(Y)_{III}$	the subset of exceptional curves of Y	33
$P(\mathcal{M}^{(2)})$	all orthogonal primitive roots (they are all (-2) roots) to $\mathcal{M}^{(2)}$	36
$P^{(2)}(\mathcal{M}^{(2,4)})$	all (-2)-roots orthogonal to $\mathcal{M}^{(2,4)}$	36
$P^{(2)}(\mathcal{M}^{(2,4)}_+)$	all (-2)-roots orthogonal to $\mathcal{M}^{(2,4)}_+$	37
$P^{(4)}(\mathcal{M}^{(2,4)})$	all (-4)-roots orthogonal to $\mathcal{M}^{(2,4)}$	36
$P^{(4)}(\mathcal{M}^{(2,4)}_+)$	all (-4)-roots orthogonal to $\mathcal{M}^{(2,4)}_+$	37
q	a finite quadratic form	118
$q^{(p)}_\theta(p^k)$	one of elementary finite quadratic forms	118
q_M	the discriminant quadratic form of an even lattice M	118
$Q_\pm$	the ± 1 eigenspaces of an involution θ on a module Q	29

r	one of main invariants: the Picard number $r = \operatorname{rk} S_Y$; $r = \operatorname{rk} S$	29
$\widetilde{r}$	the Picard number of a log del Pezzo surface Z	106
(r, a, δ)	main invariants of Z, Y, (X, θ)	30
$R(X, \theta)$	the root invariant of (X, θ)	40
$R_{gen}(X, \theta)$	the generalized root invariant of (X, θ)	42
S	$S = (S_X)_+ = H^2(X, \mathbb{Z})_+$, the main invariant of Z, Y, (X, θ)	29
S_X	the Picard lattice (modulo torsion) of a surface X	23
$U^{(2)}(2^k)$	one of elementary (2-dimensional) 2-adic lattices	118
$u_-^{(2)}(2^k)$	one of elementary finite symmetric bilinear forms	119
$u_+^{(2)}(2^k)$	one of elementary finite quadratic forms	118
$\operatorname{Var}(\cdot)$	the varying part of $(\cdot)$	62
$V^{(2)}(2^k)$	one of elementary (2-dimensional) 2-adic lattices	118
$v_-^{(2)}(2^k)$	one of elementary finite symmetric bilinear forms	119
$v_+^{(2)}(2^k)$	one of elementary finite quadratic forms	118
$V(M)$	the light cone $V(M) = \{x \in M \otimes \mathbb{R} \mid x^2 > 0\}$ of a hyperbolic lattice M	26
$V^+(X)$	the half containing polarization of the light cone $V(S_X)$ of a surface X	23
$W^{(2)}(M)$	the group generated by reflections in all $f \in M$ with $f^2 = -2$	23
$W^{(4)}(M)$	the group generated by reflections in all (-4) roots of M	36
$W^{(2,4)}(M)$	the group generated by reflections in all (-2) and (-4) roots of M	36
$W_+^{(2,4)}$	the subgroup of $W^{(2,4)}(S)$ generated by reflections in $\Delta_+^{(2,4)} \subset \Delta^{(2,4)}(S)$	35
$W^{(4)}(\mathcal{M}^{(2)})$	see Proposition 2.2	36
$W_+^{(4)}(\mathcal{M}^{(2)})$	see (50)	37
$W(R)$	the Weyl group of a finite root system R	2
X	K3 surface X	22
(X, θ)	K3 surface X with a non-symplectic involution θ	22
X^θ	fixed points of involution θ on X	31
Y	DPN surface; also $Y = X/\{1, \theta\}$, also right resolution of Z	20
(Y, C)	a DPN pair; also $(Y = X/\{1, \theta\}, C = X^\theta)$	20
Z	log del Pezzo surface Z of index ≤ 2	18
α	the invariant (0 or 1) of the root invariant $R(X, \theta)$	41
δ	0 or 1; zero 0 iff $X^\theta \sim 0 \mod 2$ in $H_2(X, \mathbb{Z})$; zero 0 iff $\delta_S = 0$ for the main invariant S of Z, Y or (X, θ)	30
δ_M	0 or 1; one of invariants of a 2-elementary lattice M, zero 0 iff $(m^*)^2 \in \mathbb{Z}$ for any $m^* \in M^*$	30

$\Delta_+^{(2)}$	the set $\Delta^{(2)}(S)$ of all (-2)-roots of $S = (S_X)_+$	34
$\Delta_{+t}^{(2)}$	the subset of (-2)-roots of $S = (S_X)_+$	34
$\Delta_\pm^{(4)}$	the subsets of (-4)-roots of $(S_X)_\pm$	34
$\Delta_+^{(2,4)}$	it is $\Delta_+^{(2)} \cup \Delta_+^{(4)}$	35
$\Delta_-^{(6)}$	the subset of (-6)-elements of $(S_X)_-$	34
$\Delta^{(2)}(S)$	the set of all (-2)-roots of a lattice S	34
$\Delta^{(4)}(S)$	the set of all (-4)-roots of a lattice S	36
$\Delta^{(4)}(\mathcal{M}^{(2)})$	see Proposition 2.2	36
$\Delta_+^{(4)}(\mathcal{M}^{(2)})$	see (51)	37
$\Gamma(\cdot)$	the Dynkin diagram of $(\cdot)$; equivalent to Gram matrix of $(\cdot)$; the dual graph of exceptional curves of $(\cdot)$	50
$\widetilde{\Omega}_{M\subset L_{K3}}$	period domain of K3 surfaces with condition $M \subset L_{K3}$ on Picard lattice	27
$\Omega_{M\subset L_{K3}}$	period domain of K3 surfaces with condition $M \subset L_{K3}$ on Picard lattice with forgetting $\mathcal{M}$	27
ξ	a part (the homomorphism) of the root invariant $R(X,\theta)$	40
$\mathfrak{A}_M$	the discriminant group $\mathfrak{A}_M = M^*/M$ of a lattice M	45
$\oplus$	the orthogonal sum of lattices (with very few exceptions when it is used to denote the direct sum of modules)	117