General Introduction.

Here, only a general introduction is given. The content of each chapter is summarized in the introduction to each chapter.

The Problems .[5] Let $G = PSL_2(\mathbf{R}) \times PSL_2(k_\mathfrak{p})$, where $\mathbf{R}$ and $k_\mathfrak{p}$ are the real number field and a $\mathfrak{p}$-adic number field with $N\mathfrak{p} = q$ respectively, and $PSL_2 = SL_2/\pm 1$. Let Γ be a torsion-free discrete subgroup of G with compact quotient, having a dense image of projection in each component of G. Our subject is such a discrete subgroup Γ. This study was motivated by the following series of conjectures which were suggested by our previous work **[15]**. Since our group Γ is essentially non-abelian (see Chapter 3, Theorem 2 in §6), the readers will see that, by our conjectures, Γ would describe a "*nonabelian class field theory*" over an algebraic function field of one variable with finite constant field $\mathbf{F}_{q^2}$. We would like to call the problems of determining the validity of these conjectures, *the congruence monodromy problems.*

Conjectures . With each Γ, we can associate an algebraic function field K of one variable with finite constant field $\mathbf{F}_{q^2}$ and with genus $g \geq 2$, and a finite set $\mathfrak{S}(K)$ consisting of $(q-1)(g-1)$ prime divisors of K of degree one over $\mathbf{F}_{q^2}$ satisfying the following properties. Here the elements of $\mathfrak{S}(K)$ are called the *exceptional prime divisors,* while all other prime divisors of K are called the *ordinary prime divisors.*

Conjecture 1. The ordinary prime divisors P of K are in one-to-one correspondence with the pairs $\{\gamma_P\}_\Gamma^\pm$ of mutually inverse primitive elliptic conjugacy classes of Γ (See Chapter 1, §§1-12 for the definitions).

Conjecture 2. The finite unramified extensions K' over K, in which all $(q-1)(g-1)$ exceptional prime divisors of K are decomposed completely, are in one-to-one correspondence with the subgroups Γ' of Γ with finite indices. Moreover, this one-to-one correspondence satisfies the Galois theory.

Conjecture 3. The law of decomposition of ordinary prime divisors P of K in K' is described by the corresponding $\{\gamma_P\}_\Gamma^\pm$ and Γ'. Namely, decompose the Γ-conjugacy class $\{\gamma_P\}_\Gamma$ into a disjoint union of Γ'-conjugacy classes:

$$\{\gamma_P\}_\Gamma = \{\gamma_{P,1}\}_{\Gamma'} \cup \cdots \cup \{\gamma_{P,t}\}_{\Gamma'},$$

and for each i, let f_i be the smallest positive integer such that $\gamma_{P,i}^{f_i}$ is contained in Γ'. Then we have $\sum_{i=1}^{t} f_i = (\Gamma : \Gamma')$, and our conjecture asserts that the decomposition of P in K' is

[5] Here, we shall reproduce a part of the introduction of my paper **[18]**. As for the details, cf. **[18]** §3.

of type

$$P = P'_1 P'_2 \cdots P'_t,$$

where P'_i $(1 \leq i \leq t)$ are prime divisors of K' with relative degrees f_i $(1 \leq i \leq t)$ respectively.

As mentioned in the preface, we shall attack the problems from several different approaches. The results obtained are still far from the solution of the problems, but are very encouraging. For the summary of the contents, the readers are asked to refer to the introduction given at the beginning of each chapter.

There is some interdependence between the five chapters, but they are not serious, and references are explicitly indicated. Therefore, each chapter can be read independently without the knowledge of the others. The only exception is Chapter 1, §§1 ~ 3, which is presupposed throughout the volume.