Notations and symbols

Here are notations and symbols frequently used in this monograph.

- $\mathbb{N}$ = the set of all natural numbers. In this monograph, natural number means positive integer. Thus $0 \notin \mathbb{N}$.

  $\mathbb{Z}$ = the set of all integers.

  $\mathbb{Q}$ = the set of all rational numbers.

  $\mathbb{R}$ = the set of all real numbers.

  $\mathbb{C}$ = the set of all complex numbers.
- $\mathcal{B}(\mathbb{R}^n)$ = the Borel σ-algebra of $\mathbb{R}^n$, $\mathcal{B}(\mathbb{C})$ = the Borel σ-algebra of $\mathbb{C}$.
- We denote the *imaginary unit* by $\sqrt{-1}$. The letter 'i' is not used for the imaginary unit, because we wish to use this as an index. For $z \in \mathbb{C}$, let

$$\begin{aligned} \operatorname{Re} z &= \text{the } \textit{real part} \text{ of } z, \\ \operatorname{Im} z &= \text{the } \textit{imaginary part} \text{ of } z, \\ \overline{z} &= \text{the } \textit{conjugate} \text{ of } z. \end{aligned}$$

- μ = the 1-*dimensional Lebesgue measure*.
- For $a, b \in \mathbb{R}$, let

$$\begin{aligned} &a \vee b = \max\{a, b\}, \quad a \wedge b = \min\{a, b\}, \\ &a^+ = a \vee 0, \quad a^- = (-a)^+ = (-a) \vee 0. \end{aligned}$$

- For $A \subset X$ where X is a universal set,

$$\mathbf{1}_A(x) = \begin{cases} 1, & x \in A, \\ 0, & x \in X \setminus A, \end{cases}$$

$$\begin{aligned} A^{\complement} &= X \setminus A \\ &= \{x \in X; x \notin A\}. \end{aligned}$$

  $\mathbf{1}_A$ is called the *defining function* (or *indicator function*) of A, and $A^{\complement}$ the *complement* of A.
- For a set A, $\operatorname{card} A$ = the *cardinality* of A. Let $\aleph_0 = \operatorname{card} \mathbb{N}$ and $\aleph = \operatorname{card} \mathbb{R}$. $\aleph_0$ is called the *countable infinite cardinality* and $\aleph$ the *cardinality of the continuum*. When $\operatorname{card} A \leq \aleph_0$, i.e., A is at most countable, this is written as $\# A$. $\# A$ is the number of elements of A.

- For $a \in \mathbb{R}$, let

$$\lfloor a \rfloor = \max\{n \in \mathbb{Z}; n \leq a\},$$
$$\lceil a \rceil = \min\{n \in \mathbb{Z}; a \leq n\}.$$

$\lfloor \cdot \rfloor : \mathbb{R} \to \mathbb{Z}$ and $\lceil \cdot \rceil : \mathbb{R} \to \mathbb{Z}$ are called the *floor function* and *ceiling function*, respectively. Also let

$$\{a\} = a - \lfloor a \rfloor \in [0, 1).$$

$\{a\}$ and $\lfloor a \rfloor$ are called the *fractional part* and *integral part* of a, respectively.

- The letter 'p' is mainly used for prime numbers. Sometimes so is the letter 'q'. Arranging prime numbers in ascending order, we denote the ith prime number by p_i.
- We denote the inverse functions of $\cos : [0, \pi] \to [-1, 1]$, $\tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to (-\infty, \infty)$, $\cot : (0, \pi) \to (-\infty, \infty)$ by $\cos^{-1}$, $\tan^{-1}$, $\cot^{-1}$, respectively.
- We use the following abbreviations:

a.e. = almost every, almost everywhere,
a.s. = almost sure, almost surely,
def = definition,
i.e. = id est (in Latin) = that is (in English),
iff = if and only if,
i.p. = in probability,
L.H.S. = (the) left-hand side,
R.H.S. = (the) right-hand side,
s.t. = such that.