

Contents

Notations and symbols	xi
Introduction	1
1 Almost periodic functions	5
1.1 Definition and some properties	5
1.2 Mean values	10
1.3 Convolutions	13
1.4 Approximation theorem	18
1.5 Parseval equality	23
2 Probability measure \mathbf{P} on $\mathbb{R}^{\mathbb{B}}$	27
2.1 Definition of the probability measure \mathbf{P}	27
2.2 Limit theorem on the probability space $(\mathbb{R}^{\mathbb{B}}, \mathbf{P})$	32
3 Complex random variable $\sum_p -\log(1 - \frac{e(-\log p)}{p^\sigma})$ on $(\mathbb{R}^{\mathbb{B}}, \mathbf{P})$	43
3.1 Complex random variables $e(\lambda)$	43
3.2 Logarithm functions of a complex variable	47
3.3 Complex random variable $\sum_p -\log(1 - \frac{e(-\log p)}{p^\sigma})$	49
3.4 Some properties of the distribution of $\sum_p -\log(1 - \frac{e(-\log p)}{p^\sigma})$	51
4 Riemann zeta function	67
4.1 Euler-Maclaurin summation formula	67
4.2 Analytic continuation to the entire complex plane	75
4.3 Functional equation	81
4.4 No zeros on the line $\operatorname{Re} s = 1$	93
5 Bohr-Jessen limit theorem	97
5.1 Log zeta function	97
5.2 Presentation of the main theorem	101
5.3 Proof of the main theorem	104
6 Some facts from analytic number theory	119
6.1 Square mean value estimate of $\zeta(s)$	119
6.2 Stirling's formula and estimate of $\Gamma^{(l)}(\sigma + \sqrt{-1}t)$	148
6.3 Carlson's mean value theorem	171

6.4	Proof of Claim 5.5	189
Appendix		193
A.1	Several facts from probability theory	193
A.1.1	Convergence of probability measures	193
A.1.2	Characteristic functions	194
A.1.3	Kolmogorov's extension theorem	195
A.1.4	Almost sure convergence theorem for independent random variables	195
A.1.5	Lindeberg's central limit theorem	196
A.2	Gauss's product formula of the gamma function	196
A.3	A proof of $\zeta(2) = \frac{\pi^2}{6}$	202
A.4	The second mean value theorem for integrals	205
Bibliography		209
Index of theorem		213
Index		215