

Contents

Preface	iii
Chapter I. Overview	1
1. Introduction	1
1.a. Zariski-decomposition	1
1.b. Numerical D -dimension	5
1.c. Canonical divisor	6
1.d. Addition theorem	8
1.e. Invariance of plurigenera	8
1.f. Log-terminal singularities	9
2. History	10
3. Notation	11
Chapter II. Preliminaries	13
1. Complex analytic varieties	13
1.a. General theory	13
1.b. Spec and Proj	15
1.c. Ample line bundles	23
1.d. Bimeromorphic geometry	27
2. Divisors	29
2.a. Weil and Cartier divisors	29
2.b. Reflexive sheaves of rank one	30
2.c. Intersection numbers	33
2.d. \mathbb{Q} -divisors and \mathbb{R} -divisors	35
2.e. Pullback and push-forward	38
3. D -dimension	42
3.a. Linear systems of \mathbb{R} -divisors	42
3.b. D -dimensions of \mathbb{R} -divisors	45
3.c. Relative D -dimension	50
3.d. Big divisors	52
4. Canonical divisor	54
4.a. Kodaira dimension	54
4.b. Logarithmic ramification formula	55
4.c. Terminal, canonical, and log-terminal singularities	59

4.d. Bimeromorphic pairs	60
5. Numerical properties of divisors	62
5.a. Ample and nef cones	62
5.b. Big and pseudo-effective cones	63
5.c. Vanishing theorems	65
5.d. Relative numerical properties	67
6. Algebraic cycles	72
6.a. Chow groups	72
6.b. Chern classes of vector bundles	73
6.c. Semistable vector bundles	77
Chapter III. Zariski-decomposition Problem	79
1. σ -decomposition	79
1.a. Invariants σ_Γ and τ_Γ	79
1.b. Zariski-decomposition problem	85
2. Invariant σ along subvarieties	88
3. ν -decomposition	93
4. Relative version	97
4.a. Relative σ -decomposition	97
4.b. Threefolds	100
5. Pullbacks of divisors	103
5.a. Remarks on exceptional divisors	103
5.b. Mumford pullback	107
5.c. σ -decompositions of pullbacks	110
Chapter IV. Divisors on bundles	113
1. Toric varieties	113
1.a. Fans	113
1.b. Support functions	117
1.c. Relative toric situations	123
2. Toric bundles	127
2.a. Definition of toric bundles	127
2.b. Pseudo-effective divisors on toric bundles	129
2.c. Examples of toric bundles	133
2.d. Explicit toric blowing-up	138
3. Vector bundles over a curve	142
3.a. Filtration of vector bundles	142
3.b. Projective bundles over a curve	146
4. Normalized tautological divisors	153
4.a. Projectively flatness and semi-stability	153
4.b. The case of vector bundles of rank two	157
Chapter V. Numerical Kodaira dimension	165
1. Pseudo-effective \mathbb{R} -divisors	165

1.a. Base-point freeness	165
1.b. Restriction to general subvarieties	170
2. Numerical D -dimensions	173
2.a. Numerical D -dimensions for nef \mathbb{R} -divisors	173
2.b. κ_σ	174
2.c. Numerical domination	177
2.d. κ_ν	181
2.e. Geometrically abundant divisors	184
3. Direct images of canonical sheaves	187
3.a. Variation of Hodge structure	187
3.b. ω -sheaves	192
3.c. Weak positivity and pseudo-effectivity	199
3.d. ω -sheaves and weak positivity	205
3.e. Direct images of relative pluricanonical sheaves	209
4. Abundance and Addition	220
4.a. Addition Theorem	220
4.b. Abundance theorem for $\kappa_\sigma = 0$	225
Chapter VI. Invariance of plurigenera	229
1. Background	229
2. Special ideals	230
2.a. Setting	230
2.b. Inclusions of ideals	235
3. Surjectivity of restriction maps	238
3.a. Big case	238
3.b. Abundant case	244
4. Degeneration of projective varieties	250
5. Deformation of singularities	252
Chapter VII. Around log-terminal singularities	257
1. Admissible and strongly log-canonical singularities	258
1.a. Admissible singularities	258
1.b. Quasi log-terminal and strongly log-canonical singularities	261
2. Minimal model program	263
3. ω -sheaves and log-terminal singularities	267
Bibliography	269
Index	275