Hao Wang as Philosopher

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In this paper I attempt to convey an idea of Hao Wang's style as a philosopher and to identify some of his contributions to philosophy. Wang was a prolific writer, and the body of text that should be considered in such a task is rather large, even if one separates off his work in mathematical logic, much of which had a philosophical motivation and some of which, such as his work on predicativity, contributed to the philosophy of mathematics. In a short paper one has to be selective. I will concentrate on his book *From Mathematics to Philosophy* [1974][1], since he considered it his principal statement, at least in the philosophy of mathematics. It also has the advantage of reflecting his remarkable relationship with Kurt Gödel (which began with correspondence in 1967[2]) while belonging to a project that was well under way when his extended conversations with Gödel took place. Although Gödel's influence is visible, and in some places he is documenting Gödel's views by arrangement with their author, the main purpose of the book is to expound Wang's philosophy. Although I will comment on the Wang-Gödel relation, a discussion of Wang's work as source for and interpretation of Gödel, work which included two books written after Gödel's death ([1987] and [199?]) will have to be deferred until another occasion.

1. Style, convictions, and method

Wang's writings pose difficulties for someone who wishes to sort out his philosophical views and contributions, because there is something in his style that makes them elusive. FMP, like other writings of Wang, devotes a lot of space to exposition of relevant logic and sometimes mathematics, and of the work and views of others. Sometimes the purpose of the latter is to set the views in question against some of his own (as with Carnap, 381-384); in other cases the view presented seems to be just an exhibit of a view on problems of the general sort considered (as with Aristotle on logic, 131-142). The presence of expository sections might just make Wang's own philosophizing a little harder to find, but it is not the most serious difficulty his reader faces. That

[1] This work is referred to as FMP and cited merely by page number.
[2] In fact Wang first wrote to Gödel in 1949, and they had a few isolated meetings before 1967. But the closer relationship originated with an inquiry of Wang with Gödel in September 1967 about the relation of his completeness theorem to Skolem's work. Gödel's reply is the first of the two letters published in FMP, pp. 8-11.

comes from a typical way Wang adopts of discussing a philosophical issue: to raise questions and to mention a number of considerations and views but with a certain distance from all of them. This makes some of his discussions very frustrating. An example is his discussion (FMP, ch. viii) of necessity, apriority, and the analytic-synthetic distinction. Wang is sensitive to the various considerations on both sides of the controversy about the latter distinction and considers a greater variety of examples than most writers on the subject. But something is lacking, perhaps a theoretical commitment of Wang's own, that would make this collection of expositions and considerations into an *argument*, even to make a definite critical point in a controversy structured by the views of others.[3]

This manner of treating an issue seems to reflect a difference in philosophical aspiration both from older systematic philosophy and from most analytic philosophy. In the Preface to FMP Wang writes:

> This book certainly makes no claim to a philosophical theory or a system of philosophy. In fact, for those who are convinced that philosophy should yield a theory, they may find here merely data for philosophy. However, I believe, in spite of my reservations about the possibility of philosophy as a rigorous science, that philosophy can be relevant, serious, and stable. Philosophy should try to achieve some reasonable overview. There is more philosophical value in placing things in their right perspective than in solving specific problems (x).

Nonetheless one can identify certain convictions with which Wang undertook the discussions in FMP and other works. He is very explicit about one aspect of his general point of view, which he calls "substantial factualism." This is that philosophy should respect existing knowledge, which has "overwhelming importance" for philosophy. "We know more about what we know than how we know what we know" (1). Wang has primarily in mind mathematical and scientific knowledge. Thus he will have no patience with a proposed "first philosophy" that implies that what is accepted as knowledge in the scientific fields themselves does not pass muster on epistemological or metaphysical grounds, so that the sciences have to be revised or reinterpreted in some fundamental way. He would argue that no philosophical argument for modifying some principle that is well established in mathematical and scientific practice could possibly be as well-grounded as the practice itself.[4]

Factualism as thus stated should remind us of views often called naturalism. In rejecting first philosophy, Wang is in agreement with W. V. Quine, as he seems to recognize (3), and yet his discussions of Quine's philosophy

[3] But see the remarks below on Wang's discussion of "analytic empiricism".

[4] All views of this kind have to recognize the fact that scientific practice itself undergoes changes, sometimes involving rejection of previously held principles. There is a fine line between altering a principle for reasons internal to science and doing so because of a prior philosophy.

emphasize their disagreements.[5] Wang's factualism differs from a version of naturalism like Quine's in two respects. First, natural science has no especially privileged role in the knowledge that is to be respected. "We are also interested in less exact knowledge and less clearly separated out gross facts" (2). Even in the exact sphere, Wang's method gives to mathematics an autonomy that Quine's empiricism tends to undermine. Second, in keeping with the remark from the Preface quoted above, Wang has in mind an essentially descriptive method. Quine's project of a naturalistic epistemology that would construct a comprehensive *theory* to explain how the human species constructs science given the stimulations individuals are subjected to is quite alien to Wang. In one place where Wang criticizes epistemology, his target is not only "foundationalism"; he says his point of view "implies a dissatisfaction with epistemology as it is commonly pursued on the ground that it is too abstract and too detached from actual knowledge" (19). That comment could as well have been aimed at Quine's project of naturalistic epistemology as at "traditional" epistemology.[6] Wang proposes to replace epistemology with "epistemography which, roughly speaking, is supposed to treat of actual knowledge as phenomenology proposes to deal with actual phenomena" (ibid.). But he does not make that a formal program; I'm not sure that the term "epistemography" even appears again in his writings. One way of realizing such an aspiration is by concrete, historical studies, and there is some of this in Wang's writing.[7]

There were, I think, convictions related to his "factualism" that are at work in Wang's work from early on. One I find difficult to describe in the form of a thesis; one might call it a "continental" approach to the foundations of mathematics, where both logicism after Frege and Russell and the Vienna Circle's view of mathematics and logic have less prominence, and problems arising from the rise of set theory and infinitary methods in mathematics, and their working out in intuitionism and the Hilbert school, have more. In an early short critical essay on Nelson Goodman's nominalism he wrote:

> ... there is ... ground to suppose that Quine's general criterion of using the values of variables to decide the "ontological commitment" of a theory is not as fruitful as, for instance, the more traditional ways of distinguishing systems according to whether they admit of infinitely many things, or whether impredicative definitions are allowed, and so on.[8]

Wang's sense of what is important in foundations probably reflects the influence of Paul Bernays, under whose auspices he spent much of the academic

[5] See especially [Wang 1985] and [1986].

[6] In a brief comment on Quine's project ([1985], p. 170), Wang expresses a similar philosophical reservation but also asks whether the time is ripe for such a program to achieve *scientific* results.

[7] Good examples are [1957] and the chapter on Russell's logic in FMP.

[8] [1953], pp.416-417 of the reprint in FMP.

year 1950-51 in Zürich. Wang's survey paper [1958] is revealing. There he classifies positions in the foundations of mathematics according to a scheme he attributes to Bernays (and which indeed can be extracted from [Bernays 1935]), so that the article ascends through strict finitism (which he prefers to call "anthropologism"), finitism, intuitionism, predicativism, and platonism. He shared the view already expressed by Bernays that one does not need to make a choice between these viewpoints and that a major task of foundational research is to formulate them precisely and analyze their relations. His own work on predicativity, most of which was done before 1958, was in that spirit. Although he was familiar with the Hilbert school's work in proof theory and already in the mid-1950's collaborated with Georg Kreisel, his own work on problems about the relative logical strength of axiom systems made more use of notions of translation and relative interpretation than of proof-theretic reductions.[9]

Another conviction one can attribute to the early Wang, more tenuously connected with factualism, is of the theoretical importance of computers. Interest in computability would have come naturally to any young logician of the time; recursion theory and decision problems were at the center of interest. Wang concerned himself with actual computers and spent some time working for computer firms. His work in automatic theorem proving is well known. A whole section of the collection [1962] consists of papers that would now be classified as belonging to computer science. Issues about computational feasibility had some concrete reality for him.

One can see Wang's familiarity with computers at work in the essay [1961], partly incorporated into chapter vii of FMP. This essay is one of the most attractive examples of Wang's style. Many of the issues taken up seem to arise from Wittgenstein's *Remarks on the Foundations of Mathematics* [1956], but Wittgenstein's name is not mentioned; in particular he does not venture to argue for or against Wittgenstein's general point of view as he might interpret it. But the notion of perspicuous proof, the question whether a mathematical statement changes its meaning when a proof of it is found, the question

[9] Here I might make a comment about my own brief experience as Wang's student. In the fall of 1955, as a first-year graduate student at Harvard, I took a seminar with him on the foundations of mathematics. I had begun the previous spring to study intuitionism, but without much context in foundational research. Wang supplied some of the context. In particular he lectured on the consistency proof of [Ackermann 1940]. I knew of the existence of some of Kreisel's work (at least [Kreisel 1951]) but was, before Wang's instruction, unequipped to understand it. In the spring semester, in a reading course, he guided me through [Kleene 1952]. Unfortunately for me he left Harvard at the end of that semester, but his instruction was decisive in guiding me toward proof theory and giving me a sense of its importance.

Wang's seminar was memorable for another reason. Its students included two undergraduates, David Mumford and Richard Friedberg. Friedberg gave a presentation on problems about degrees growing out of Post's work; I recall his mentioning Post's problem and perhaps indicating something of an approach to it. It was just a few weeks after the seminar ended that he obtained his solution.

whether contradictions in a formalization are a serious matter for mathematical practice and applications, and a Wittgensteinian line of criticism of logicist reductions of statements about numbers are all to be found in Wang's essay. But it could only have been written by a logician with experience with computers; for example it is by a comparison with what "mechanical mathematics" might produce that Wang discusses the theme of perspicuity. Computers and Wittgenstein combine to enable Wang to present problems about logic in a more concrete way than is typical in logical literature, then or now. But except perhaps for the criticism of Frege and Russell on '$7+5=12$', one always wishes for the argument to be pressed further.

In the remainder of the paper I shall discuss three themes in Wang's philosophical writing where it seems to me he makes undoubted contributions that go beyond the limits of his descriptive method. The first is the concept of set, the subject of chapter vi of FMP. The second is the range of questions concerning minds and machines, the subject of chapter x and returned to in writings on Gödel and the late article [1993]. The third is the discussion of "analytic empiricism", a term which he uses to describe and criticize the positions of Rudolf Carnap and W. V. Quine. This only becomes explicit in [1985] and [1985a]. Each of these discussions reflects the influence of Gödel, but it is only of the second that one could plausibly say that Wang's contribution consists mostly in the exposition and analysis of Gödel's ideas.

2. The concept of set

Chapter vi of FMP is one of the finest examples of Wang's descriptive method. It combines discussion of the question how an intuitive concept of set motivates the accepted axioms of set theory with a wide-ranging exploration of issues about set theory and its history, such as the question of the status of the continuum hypothesis after Cohen's independence proof. As an overview and as a criticsm of some initially plausible ideas,[10] it deserves to be the first piece of writing that anyone turns to once he is ready to seek a sophisticated philosophical understanding of the subject. Not of course the last; in particular, even given the state of research when it was published, one might wish for more discussion of large cardinal axioms,[11] and some of the historical picture would be altered by the later work of Gregory Moore and Michael Hallett.

[10] For example, that independence from ZF itself establishes that a set-theoretic proposition lacks a truth-value (194-196). This discussion has a curious omission, of the obvious point that if ZF is consistent, then on the view in question the statement of its consistency lacks a truth-value. Possibly Wang thought the holder of this view might bite this particular bullet; more likely the view is meant to apply only to properly set-theretic propositions, so that the theorems of a progression of theories generated by adding consistency statements would be conceded to have a truth-value.

[11] Wang himself seems to have come to this conclusion; see [1977].

The inconclusiveness that a reader often complains of in Wang's philosophical writing seems in the case of some of the issues discussed here to be quite appropriate to the actual state of knowledge and to respond to the fact that philosophical reflection by itself is not likely to make a more conclusive position possible, not only on a specific issue such as whether CH has a definite truth-value but even on a more philosophically formulated question such as whether "set-theoretical concepts and theorems describe some well-determined reality" (199). (Incidentally, Wang is here suspending judgment about a claim of Gödel.)

The most distinctive aspect of the chapter, in my view, is his presentation of the iterative conception of set and "intuitive" justifications of axioms of set theory. It is very natural to think of the iterative conception of set in a genetic way: Sets are formed from their elements in successive stages; since sets consist of "already" given objects, the elements of a set must be available (if sets, already formed) at the stage at which the set is formed. Wang's treatment is the most philosophically developed presentation of this idea. A notion of "multitude" (Cantor's *Vielheit* or *Vieles*) is treated as primitive; in practice we could cash this in by plural or second-order quantification. Exactly how this is to be understood, and what it commits us to, is a problem for Wang's account, but not more so than for others, and for most purposes we can regard the term "multitude" as a place-holder for any one of a number of conceptions.

Wang says, "We can form a set from a multitude only in case the range of variability of this multitude is in some sense intuitive" (182). One way in which this condition is satisfied is if we have what Wang calls an intuitive concept that "enables us to overview (or look through or run through or collect together), in an *idealized* sense, all the objects in the multitude which make up the extension of the concept" (ibid.). Thus he entertains an idealized concept of an infinite intuition, apparently intuition of objects. An application of this idea is his justification of the axiom of separation, stated "If a multitude A is included in a set x, then A is a set" (184):

> Since x is a given set, we can run through all members of x, and, therefore, we can do so with arbitrary omissions. In particular, we can in an idealized sense check against A and delete only those members of x which are not in A. In this way, we obtain an overview of all the objects in A and recognize A as a set.

Some years ago I argued that the attempt to use intuitiveness or intuitability as a criterion for a multitude to be a set is not successful. The idealization that he admits is involved in his concept of intuition cuts it too much loose from intuition as a human cognitive faculty.[12] For example, the set x can be very large, so that "running through" its elements would require

[12] [Parsons 1977], pp. 275-279, a paper first presented in a symposium with Wang. (His paper is [1977].)

something more even than immortality: a structure to play the role of time that can be of as large a cardinality as we like. Moreover, it seems we need to be omniscient with regard to A, in order to omit just those elements of x that are not in A. It is not obvious that intuitiveness is doing any work that is not already done by the basic idea that sets are formed from given objects.

I am, however, not sure that my earlier critical discussion captured Wang's underlying intention. Two things raise doubts. First, Wang lists five principles suggested by Gödel "which have actually been used for setting up axioms." The first is "Existence of sets representing intuitive ranges of variability, i. e. multitudes which, in some sense, can be 'overviewed'" (189). This suggests that Gödel gave some level of endorsement to Wang's conception.[13] Second, Wang evidently saw some justice in my criticisms, at least of some of his formulations (see [1977], p. 327) but still holds that the just quoted principle "is sufficient to yield enough of set theory as a foundation of classical mathematics and has in fact been applied ... to justify all the axioms of ZF ([1977], p. 313).

Wang's use of the term "intuition" in chapter vi of FMP is confusing. I don't think it is entirely consistent or fits well either the Kantian paradigm or the common conception of intuition as a more or less reliable inclination to believe. The way the term "intuitive range of variability" is used also departs from Gödel's use of "intuition" in his own writings, although it could be an extension of it rather than inconsistent with it.

One possibly promising line of attack is to think of what Wang calls "overview" as conceptual. In going over some of the same ground in note 4 of [1977], Wang encourages this reading; for example he seems to identify being capable of being overviewed with possessing unity. A reason for being confident that the natural numbers are a set is that the concept of them as what is obtained by iterating the successor operation beginning with 0 gives us, not only a clear concept of natural number, but some sort of clarity about the *extension* of natural number, what will count as a natural number. It is this that makes the natural numbers a "many that can be thought of as one."[14] For well-known reasons we cannot obtain that kind of clarity about all sets or all ordinals. The difficult cases are the situations envisaged in the power set axiom and the axiom of replacement. In the footnote mentioned, Wang discusses the set of natural numbers in a way consistent with this

[13] However, it appears from remarks quoted in [Wang 199?] that Gödel's understanding of "overview" included some of what I was critizing. Gödel is reported to say, "The idealized time concept in the concept of overview has something to do with Kantian intuition" (in remark 71.17 (in ch. 7)). In remark 71.18 he speaks of infinite intuition, and of "the process of selecting integers as given in intution." Speaking of idealizations, he says, "What this idealization ... means is that we conceive and realize the possibility of a mind which can do it" (71.19). It should be clear that when Gödel talks of idealized or infinite intution in these remarks, he is not attributing such a faculty to humans; cf. also 71.15.

[14] Cantor in 1883 famously characterizes a set as "jedes Viele, welches sich als Eines denken läßt" ([1932], p. 204).

approach and then reformulates his treatment of the power set axiom as follows:

> Given a set b, it seems possible to think of all possible ways of deleting certain elements from b; certainly each result of deletion remains a set. The assumption of the formation of the power set of b then says that all these results taken together again make up a set ([1977], p. 327).

The difference between thinking of all possible ways of deleting elements from a given set and thinking of all possible ways of generating sets in the iterative conception is real, but not so tangible as one would like, as Wang concedes by talking of the *assumption* of the formation of the power set. There is a way of looking at the matter that Wang does not use, although other writers on the subject do.[15] In keeping with Wang's own idea that it is the "maximum" iterative conception that is being developed, any multitude of given objects can constitute a set. Suppose now that a set x *is* formed at stage α. Then, since its elements must have been given or available at that stage, any subset of x could have been formed at stage α. But we would like to say that *all* the subsets of x are formed at stage α, so that $\mathcal{P}(x)$ can be formed at stage $\alpha + 1$. This amounts to assuming that if a set could have been formed at α, then it is formed at α. This is a sort of principle of plenitude; it could be regarded as part of the maximality of the maximum iterative conception. But it is certainly not self-evident.[16]

Wang's discussion of the axioms has the signal merit that he works out what one might be committed to by taking seriously the idea that sets are *formed* in successive stages. It thus stimulates one to attempt a formulation of the iterative conception that does not require that metaphor to be taken literally. The task is not easy. An attempt of my own (in [Parsons 1977]) relied on modality in a way that others might not accept and might be objected to on other grounds. Moreover, much of his discussion, for example of the axiom of replacement, can be reformulated so as not to rely on highly idealized intuitability.

I have left out of this discussion the a posteriori aspect of the justification of the axioms of set theory. Following Gödel, Wang does not neglect it, although in the case of the axioms of ZF itself, I think he gives it less weight than I would.

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[15] For example [Boolos 1971], p. 494 of reprint; cf. [Parsons 1977], p. 274.

[16] Cf. [Parsons 1995], pp. 86-87. But there I asked why we should accept the appeal to plenitude in the case of subsets of a set but not in the case of sets in general or ordinals. This question was thoroughly confused. If by "sets in general" is meant all sets, then there is no stage at which they *could* have been formed, and hence no application for plenitude; similarly for ordinals. If one means *any* set, then in the context of the iterative conception, the version of plenitude leading to the power set axiom already implies that when a set could have been formed (i. e. when its elements are available) it is formed.

3. Minds and machines

Clearly Wang was prepared by experience to engage himself seriously with Gödel's thought about the concept of computability and about the question whether the human power of mathematical thought surpasses that of machines. Chapter x of FMP contains the first informed presentation of Gödel's views on the subject, and Wang returned to the question several times later.

Most of the chapter is a judicious survey of issues about physicalism, mechanism, computer simulation as a tool of psychological research, and artificial intelligence. The general tone is rather skeptical of the claims both of the mechanist and the anti-mechanist sides of the debates on these subjects. Section 6 turns to arguments in this area based on the existence of recursively unsolvable problems or on the incompleteness theorems; the difficulties of establishing conclusions about human powers in mathematics by means of these theorems are brought out, but as they had already been in the debate prompted by J. R. Lucas's claim that the second incompleteness theorem shows that no Turing machine can model human mathematical competence ([Lucas 1961]).

Only in the last section of the chapter to Gödel's views enter, and Wang confines himself to reporting. On the basis of the then unpublished 1951 Gibbs Lecture, the "two most interesting rigorously proved results about minds and machines" are said to be (1) that the human mind is incapable of formulating (or mechanizing) all its mathematical intuitions; if it has formulated some of them, this fact yields new ones, e.g. the consistency of the formalism. (2) "Either the human mind surpasses all machines (to be more precise: it can decide more number theoretical questions than any machine) or else there exist number theoretical questions undecidable for the human mind" (324).[17] The disjunction (2) is now well known. A statement follows of Gödel's reasons for rejecting the second disjunct, expressing a point of view that Wang calls "rationalistic optimism".[18] As an expression of the view that "attempted proofs of the equivalence of mind and machines are fallacious" there follows a one-paragraph essay by Gödel criticizing Turing (325-326).[19] Wang reports Gödel's view that the argument attributed to Turing would be valid if one added the premisses (3) "there is no mind separate from matter" and (4) "the brain functions basically like a digital computer" (326).[20] He also reports

[17] Cf. [Gödel *1951], pp. 307-308, 310.

[18] Wang subsequently stated ([1993], p. 119) that the paragraph consisting of this statement (324-325) was written by Gödel. He also described the formulations of (1) and (2) as "published with Gödel's approval" ([1993], p. 118 n. 12).

[19] This essay is an alternate version of Remark 3 of [Gödel 1972a]. Wang states ([1993], p. 123), presumably on Gödel's authority, that the version he published is a revision of what subsequently appeared in the *Collected Works*.

[20] These theses are numbered (1) and (2) in FMP; I have renumbered them to avoid confusion with the theses of Gödel already numbered (1) and (2).

Gödel's conviction that (1) will be disproved, as will mechanism in biology generally.

Wang dropped a kind of bombshell by this reporting of Gödel's views, with very little explanation and only the background of his own discussion of the issues. But he did not leave the matter there. It is taken up in [1987] (pp. 196-198), but only to give a little more explanation of Gödel's theses. The questions are pursued in much greater depth in [1993]. Much of this essay consists of commentary on Gödel, either on the views just mentioned reported in FMP or on other remarks made in their conversations or in other documents such as the 1956 letter to John von Neumann containing a conjecture implying $P = NP$.[21]

By "algorithmism" about a range of processes Wang means that the thesis that they can be captured or adequately modeled by an algorithm (and so simulated by a Turing machine). Physicalism and algorithmism about mental processes have to be distinguished, because one can't take for granted that *physical* processes can be so modeled.

General physical grounds will support algorithmism about the *brain* (some version of Gödel's (4)) only if physical processes are in some appropriate sense computable. Wang thought that it was such grounds that one should look for in order to decide whether to accept (4).[22] Gödel thought it practically certain that physical laws, in their observable consequences, have a finite limit of precision. Wang concludes that numbers obtained by observations can be approximated as well as makes observational sense by computable numbers, and therefore the best approach to the question of algorithmism about the physical is to ask whether physical theories yield computable predictions on the basis of computable initial data. Wang was skeptical about the physical relevance of the well-known negative results of Pour-El and Richards and mentions a conjecture of Wayne Myrvold that "noncomputable consequences cannot be generated from computable initial data within quantum mechanics" ([1993], p. 111).[23] Gödel is reported to have said that physicalism amounts to algorithmism, but Wang expresses doubts because the conclusion is based on arguments that tend to show that computable theories will agree with observation, but that is not the only requirement on a physical theory. Although he is not able to formulate the point in a way he finds clear, he is also given pause by the nonlocality of quantum mechanics.

Thus it is not so clear one way or the other whether physical processes are algorithmic. Wang does not go far into the question whether mental processes, considered apart from any physicalist or anti-physicalist thesis, are

[21] The letter is now published, with an English translation, in [Clote and Krajíček 1993], p. vii-ix.

[22] Wang evidently regards it as most prudent to consider the brain as a physical system, leaving open the question whether characteristic properties of the mind can be attributed to the brain.

[23] Cf. [Myrvold 1995], which contains results and discussion relevant to this question, not limited to quantum mechanics.

algrorithmic, although he gives a useful commentary on Gödel's remarks on Turing's alleged argument for mechanism about the mind (FMP p. 326 and [Gödel 1972a]).[24] The question about the mental has been much discussed by philosophers of mind, often in a highly polemical way. Wang may have thought it not fruitful to engage himself directly in that debate. He follows Gödel in confining himself to what can be said on the basis of rather abstract considerations and the structure of mathematics. There the upshot of his remarks is that is very difficult to make Gödel's considerations about the inexhaustibility of mathematics into a convincing case for anti-mechanism.

Considered as a whole, Wang's discussions of these issues consist of a mixture of his characteristic descriptive method with commentary on Gödel's rather cryptically expressed views. The latter does lead him into an analytical investigation of his own, particularly into whether processes in nature are in some sense computable, and if so what that sense is. Philosophers of mind, when they discuss the question of mechanism, tend to take the "machine" side of the equation for granted. The value of Wang's exploration of Gödel's thoughts is in bringing out that one cannot do that.

4. Wang on "analytic empiricism"

Underlying FMP is undoubtedly a rejection of empiricism as unable to give an adequate account of mathematical knowledge. This had long been recognized as a problem for empiricism. The Vienna Circle thought it saw the way to a solution, based on the reduction of mathematics to logic and Wittgenstein's conception of the propositions of logic as tautologies, as "necessarily true" because they say nothing. The most sophisticated philosophy of logic and mathematics of the Vienna Circle is that of Carnap. Quine rejected Carnap's views on these matters but maintained his own version of empiricism. In [1985a], Wang undertakes to describe a position common to both and to criticize it as not giving an adequate account of mathematics. A more general treatment of Carnap and Quine, in the context of a development beginning with Russell, is given in [1985].

Wang states the "two commandments of analytic empiricism" as follows ([1985a], p. 451):

(a) Empricism is the whole of philosophy, and there can be nothing (fundamental) that can be properly called conceptual experience or conceptual intuition.
(b) Logic is all-important for philosophy, but analyticity (even necessity) can only mean truth by convention.

[24] Wang also raises the corresponding question about the evolutionary process in biology but does not pursue it far, although his quotations from [Edelman 1992] are very provocative.

(b) implies that the Vienna Circle's solution to the problem posed by mathematics for empiricism will be to view it as true by convention. Wang attributes this view to Carnap and criticizes it on lines close to those of Gödel's remarks on the subject ([*1951] and [*1953/9]). Apart from sorting out some different elements in the position being criticized and considering options as to changing one or another of them, Wang's main addition to Gödel's discussion is to include the views of Quine, on whose philosophical writings Gödel nowhere comments (even in the remarks from conversations in [Wang 199?]). Wang brought out the interest of Gödel's views for the Carnap-Quine debate, by pointing to the fact that the stronger sense of analyticity that Gödel appeals to allows the thesis that mathematics is analytic to be separated from the claim that mathematics is true by convention or by virtue of linguistic usage, so that Gödel's view is a third option. I argue elsewhere that Gödel, in his reliance on his notion of concept, does not really have an answer to the deeper Quinian criticism of the ideas about meaning that underlie the analytic-synthetic distinction.[25] But that is not the end of the matter.

Concerning Carnap, Wang assumes, with Gödel, that Carnap aspires to answer a somewhat traditionally posed question about the nature of mathematical truth. His discussion is vulnerable to the objection to Gödel posed by Warren Goldfarb,[26] who points out reasons for questioning that assumption. It is hard to be comparably clear about how Wang intends to criticize Quine on these issues. In [1985a] he makes rather generalized complaints, for example against Quine's tendency to obliterate distinctions. The extended discussion of Quine's philosophy in [1985] is not very helpful; it is rather rambling and does not really engage Quine's arguments.[27]

I think one has to reconstruct Wang's reasons for the claim that Quine's version of empiricism does not give a more satisfactory account of mathematics than that given by logical positivism. An indication of the nature of the disagreement is the difference already noted between Wang's factualism and Quine's naturalism. Wang grants an autonomy to mathematics that Quine seems not to; mathematical practice is answerable largely to internal considerations and at most secondarily to its application in science. By contrast, for Quine mathematics forms a whole of knowledge with science, which is then answerable to observation. At times he views mathematics instrumentally, as serving the purposes of empirical science. This leads Quine to some reserve toward higher set theory, since one can have a more economical scientific theory without it.

[25] [Parsons 1995a], written in 1990. This paper was influenced and in some respects inspired by [Wang 1985a].

[26] See Goldfarb's introductory note to [*1953/9], [Gödel 1995], pp. 329-330.

[27] The reader gets the impression that some rather basic features of Quine's outlook repel Wang, much as he admires Quine's intellectual virtuosity and persistence. The rather limited focus of the present paper means that it does not attempt to do justice to [Wang 1985].

Wang's basic objection, it seems to me, is that Quine does not have a descriptively adequate account of mathematics, because he simply does not deal with such matters as the more direct considerations motivating the axioms of set theory, the phenomena described by Gödel as the inexhaustibility of mathematics, and the essential uniqueness of certain results of analysis of mathematical concepts such as that of natural number (Dedekind) or mechanical computation procedure (Turing). Thus he writes:

> In giving up the first commandment of analytic empiricism, one is in a position to view the wealth of the less concrete mathematical facts and intuitions as a welcome source of material to enrich philosophy, instead of an irritating mystery to be explained away ([1985a], p. 459).

His primary argument against Quine, then, would consist of descriptions and analyses of various kinds, such as those discussed above concerning the concept of set and the axioms of set theory.

Wang goes further in describing mathematics as "conceptual knowledge", where he evidently means more than to give a label to descriptive differences between the mathematician's means of acquiring and justifying claims to knowledge and the empirical scientist's. He is evidently sympathetic to Gödel's way of regarding mathematics as analytic and thus true *by virtue* of the relations of the concepts expressed in mathematical statements. If that is actually Wang's view, it is then troubling that one does not find in his writings any real response to the very powerful objections made by Quine against that view or the availability of a notion of concept that would underwrite it.

Closer examination shows, however, that Wang does not commit himself to Gödel's view. In a later paper, Wang returns to the theme of conceptual knowledge. He states what he calls the Thesis of Conceptualism:

> Given our mathematical experience, the hypothesis stating that concepts give shape to the subject-matter (or universe of discourse) of mathematics is the most natural and philosophically, the most economical.[28]

Wang's discussion of this thesis is too brief to give us a very clear idea of its meaning. One theme that emerges is the importance of a general overview or understanding of mathematical situations and also of organizing mathematical knowledge in terms of certain central concepts; he cites Bourbaki as a way of arguing for some version of the thesis, presumably because of the idea underlying their treatise of organizing mathematics around certain fundamental structures. Wang recognizes that he has not given an explanation

[28] [1991], p. 263. Wang is at pains to distinguish this thesis from realism, which is stated as a separate thesis about which he is more reserved though not unsympathetic.

of the notion of concept.[29] But even though this discussion is undeveloped, it should be clear that he is less committed to the philosophical notion of conceptual truth, of which analyticity as conceived by Gödel is an instance, than appears at first sight. In this passage, what Wang contrasts conceptual knowledge with is not empirical knowledge but technical knowledge or skill.

5. Conclusion

In his discussions of Carnap and Quine (especially in [1985]), Wang expresses general dissatisfaction with the philosophy of his own time. That is not only context in which he does so. Neither time nor space would permit a full discussion of his reasons. By way of conclusion I want to mention a respect in which Wang's philosophical aspirations differed importantly from those of most of his philosophical contemporaries, of different philosophical tendencies and differing levels of excellence. The major philosophers of the past have left us systematic constructions that are or at least aspire to be rather tightly organized logically, and many of the problems of interpreting them derive from the difficulty of maintaining such a consistent structure while holding a comprehensive range of views. Systematic constructions have been attempted by some more conservative figures of recent times as well as some of the important "continental" philosophers, in particular Husserl. But it has also been the aim of some figures in the analytic tradition whom Wang knew well, such as Quine and Dummett. Those who have not sought such systematicity have generally approached problems in a piecemeal manner or relied heavily on a technical apparatus.

As I see it, Wang's aspiration was to be synoptic, in a way that the more specialized analytic philosophers do not rise to, but without being systematic, which I think he saw as incompatible with descriptive accuracy and a kind of concreteness that he sought. Moreover, part of what he sought to be faithful to was philosophical thought itself as it had developed, so that he was led into an eclectic tendency. All but the last of these aspirations might be thought to be shared with a figure who greatly interested Wang but whom I have mentioned only in passing so far: Ludwig Wittgenstein. Yet Wang was no Wittgensteinian. I am probably not the person to explain why. But clearly there were deep differences in their approaches to mathematics and logic, and Wang could, I think, never accept the idea that the philosophical problems should *disappear* if one does one's descriptive work right.

In Gödel, Wang encountered someone who shared many of his views and aspirations but who did have the aspiration to system that he himself re-

[29] [1991], p. 264. In 1959 Gödel made a similar admission in a letter to Paul Arthur Schilpp explaining why he had not finished [*1953/9], quoted in [Parsons 1995a], p. 307. Sidney Morgenbesser informs me that in discussions he and Wang had in the last years of Wang's life the notion of concept figured prominently, but it appears that Wang did not arrive at a settled position.

garded skeptically. I think this was the most significant philosophical difference between them.[30] Because of its rather general nature, it is most often manifested by Wang's reporting and working out rather definite claims of Gödel about which he himself suspends judgment. On some matters, such as the diagnosis of his problems with analytic empiricism, Gödel's influence made it possible for Wang to express his own views in a more definite way; one might say that the ideal of systematicity imposed a discipline on Wang's own philosophizing even if he did not in general share it. I would conjecture that the same was true of Wang' s treatment of the concept of set, although in that case it is a difficult to pin down Gödel's influence very exactly.

Wang has written that Gödel did not believe he had fulfilled his own aspirations in systematic philosophy.[31] Very often he also gives the impression that he did not find what he himself was seeking in philosophical work. Both Gödel and Wang struggled with the tension inherent to the enterprise of general philosophizing taking off from a background of much more specialized research. I would not claim for Wang that he resolved this tension satisfactorily, and often, even when he is most instructive, what one learns from him is less tangible than what one derives from the arguments of more typical philosophers, even when one disagrees. I hope I have made clear, however, that Wang's "logical journey" gave rise to philosophical work that is of general interest and to contributions that are important from standpoints quite different from his own.

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[30] On some issues outside the scope of the present discussion, there are disagreements related to cultural differences. Where Gödel turns to general questions of metaphysics and *Weltanscauung*, he regards the rational theism of pre-Kantian rationalism seriously and manifests a Leibnizian optimism. In his response to these views (which he says Gödel did not discuss much with him), Wang seems to me to show his Chinese cultural background.

[31] [1987], pp. 192, 221.

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