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GROUP INVARIANCE APPLICATIONS IN STATISTICS

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Preface

These lecture notes are a result of the NSF/CBMS Regional Conference held at the University of Michigan, 15–19 June 1987. Topics in invariance with applications in statistics were discussed in a series of eighteen 45 minute lectures. Because of space and time limitations no material beyond that given in the lectures is presented here. Unfortunately, my original intention of including material on the invariance-sufficiency connection and on amenability had to be scrapped.

Professor Robb Muirhead who organized this conference is to be congratulated on a job well done. The keen interest and helpful suggestions of Professor Roger Farrell are here acknowledged. My thanks also go to Meera Somasundaram for turning my scribblings into a finished manuscript.

Morris L. Eaton

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