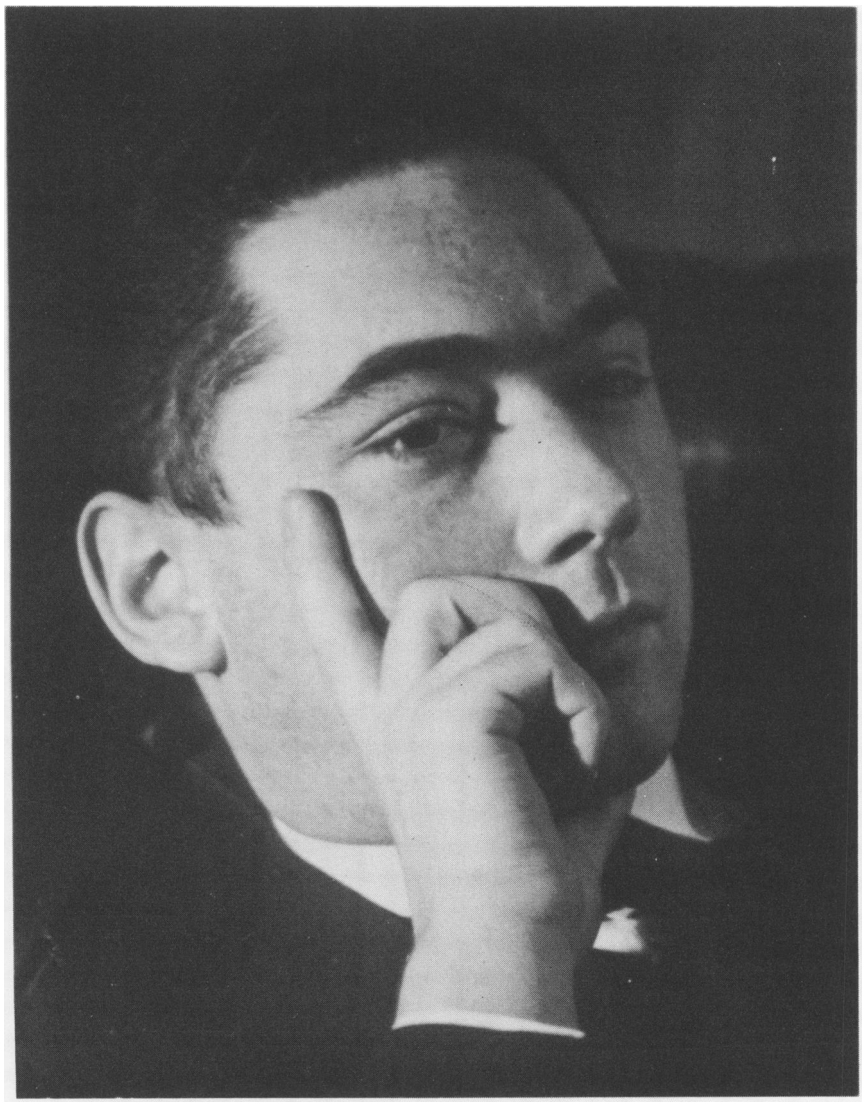
PROCEEDINGS OF THE SIXTH BERKELEY SYMPOSIUM

VOLUME II



William Feller

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PREFACE

BERKELEY SYMPOSIA ON MATHEMATICAL STATISTICS AND PROBABILITY have been held at five year intervals since 1945. The Sixth Berkeley Symposium was divided into four sessions. The first took place from June 21 to July 18, 1970. It covered mostly topics in statistical theory and in theoretical and applied probability. The second session was held from April 9 to April 12, 1971 on the special subject of evolution with emphasis on studies of evolution conducted at the molecular level. The third session held in June 1971 was devoted to problems of biology and health. A fourth session on pollution was held in July 1971.

The first three volumes of the Proceedings cover papers presented in June and July, 1970, as well as papers which were sent to us at that time, but could not be presented in person by their authors. The first volume is entirely devoted to statistics. The second and third are devoted to contributions in probability. Allocation of the papers to the three volumes was made in a manner which we hope is fairly rational, but with an unavoidable amount of arbitrariness and randomness. In the event of doubt, a general index should help the prospective reader locate the desired contribution.

The Berkeley Symposia differ substantially from most other scientific meetings in that they are intended to provide an extended period of contact between participants from all countries in the world. In addition, an effort is made to promote cross contacts between scholars whose fields of specialization cover a broad spectrum from pure probability to applied statistics. However, these fields have expanded so rapidly in the past decades that it is no longer possible to touch upon every domain in a few weeks only. Since time limits the number of invited lectures, the selection of speakers is becoming rapidly an impossible task. We could only sample the abundance of available talent. For this selection, as well as for several other important matters, we were privileged to have the assistance of an advisory committee consisting of Professors Z. W. Birnbaum and L. Schmetterer, representatives of the Institute of Mathematical Statistics, and of Professor Steven Orey, delegate of the American Mathematical Society. The visible success of our gathering is in no small measure attributable to the help we have received from this committee and other scientific friends.

A conference which extends over six weeks with participants from various parts of the world entails expenses. In this respect we feel fortunate that in spite of the general shortage of funds, the University of California and the Federal Agencies found it possible to support our enterprise. We are grateful for the allocation of funds from the Russell S. Springer Memorial Foundation, the National Science Foundation, the Office of Naval Research, the Army Research Office, the Air Force Office of Scientific Research, and the National Institutes of Health, which contributed particularly to the sessions on evolution and on problems of biology and health. In addition the pollution session received support from the Atomic Energy Commission.

The organization of the meetings fell under the responsibility of the under-

signed with the able help of the staff of the Statistical Laboratory and of the Department of Statistics. For assistance with travel arrangements and various organizational matters, special thanks are due to Mrs. Barbara Gaugl.

The end of the actual meeting signals the end of a very exciting period, but not the end of our task, since the editing and publishing of over 3,000 pages of typewritten material still requires an expenditure of time and effort.

In this respect we are indebted to Dr. Morris Friedman for translations of Russian manuscripts. We are particularly grateful to Dr. Amiel Feinstein and Mrs. Margaret Stein who not only translated such manuscripts but acted as editors, checking the references and even verifying the accuracy of mathematical results.

The actual editing and marking of manuscripts was not easy since we attempted to follow a uniform style. We benefitted from the talent and skill of Mrs. Virginia Thompson who also assumed responsibility for organizing and supervising the assistant editors, Miss Carol Conti, Mrs. Margaret Darland, and Miss Jean Kettler. We are extremely grateful to all the editors for the knowledge and patience they have devoted to these manuscripts.

In the actual publication of the material the University of California Press maintained their tradition of excellence. The typesetting was performed by the staff of Oliver Burridge Filmsetting Ltd., in Sussex, England.

The meetings of the Sixth Symposium were saddened by the absence of two of our long time friends and regular participants, William Feller and Alfréd Rényi. Professors J. L. Doob and Mark Kac were kind enough to write a short appreciation of Feller. For a similar appreciation of Rényi, we are indebted to Professor L. Schmetterer. The texts appear at the beginning of the second volume.

L.LC. J.N. E.L.S.

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