ADAPTIVE PROCESSES AND INTELLIGENT MACHINES

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1. Introduction

The extraordinary development of the digital computer has induced a growing concern with the difficult domain of neurophysiology and the very much more inchoate study of thinking and thought processes. Many mathematicians, and others, have been attracted by the challenge of constructing computer programs which will carry out activities ordinarily requiring human intelligence. Here we have in mind such processes as pattern recognition, musical composition, chess playing, and theorem proving.

It is of interest and importance then to see if it is possible to define in precise terms what we mean, or even the many different things we could mean, by the term intelligent machine. Perhaps even more important is to examine in fine detail what is involved in an attempt to introduce such concepts as levels of intelligence, learning, instinct, in such a way as to facilitate reasoned scientific discourse in this area. As we shall see, there are considerable difficulties, and as the reader will soon note, we raise more questions than we answer. These questions do not appear to be insurmountable, but it would appear that their answers require a level of mathematical sophistication and analysis equivalent to that required for the theory of sets, the mathematical theory of logic, and perhaps most closely related to that used in the Liouville theory of the integration of elementary functions in terms of elementary functions [1].

Our basic approach is to imbed the concept of intelligence within the concept of decision making. We then consider various classes of multistage decision processes to which we attach certain familiar names. Admittedly, this is a narrow approach, but perhaps precisely for this reason we may be able to obtain some precision.

2. Multistage decision processes

Let p be a point in a space S, with q a point in a space D, and $T(p, q)$ a transformation with the property that $T(p, q) \in S$ whenever $p \in S$, $q \in D$. Call p the state vector, and q the decision vector (see [2]). Consider a sequence of points in S generated in the following fashion

$$(2.1) \qquad p_1 = T(p, q_1), \qquad p_2 = T(p_1, q_2), \cdots, \qquad p_n = T(p_{n-1}, q_n), \cdots.$$

The decision q_n that is made at the nth stage will in general depend on the past history of the process,

$$q_n = f(p, p_1, \cdots, p_{n-1}; q_1, q_2, \cdots, q_{n-1}). \tag{2.2}$$

Our aim is to delineate various classes of decision processes by recognizing distinct ways in which this dependence can hold. A natural way to begin is with the case where the decision at time n is a function only of the state at time n,

$$q_n = h(p_n). \tag{2.3}$$

This can be regarded as a mathematical model of a class of phenomena lumped under the heading "instinct." A particular stimulus produces a certain effect. If, however, we allow the full dependence of (2.2), the case where the decision depends upon a knowledge and use of past events, we can regard it as a model of the class of phenomena labeled "learning and adaptation."

There are several caveats that must be uttered immediately. Suppose that we redefine our process by introducing the new state vector

$$\pi_n = (p, p_1, \cdots, p_n, q_1, q_2, \cdots, q_n). \tag{2.4}$$

Then,

$$\pi_n = T_1(\pi_{n-1}, q_n), \qquad q_n = \phi(\pi_n). \tag{2.5}$$

This has the same form as (2.3). How now do we distinguish between an instinctive and a learning process?

This is, of course, a familiar question in the theory of stochastic processes where the terms Markovian and non-Markovian are used loosely. The point we wish to raise is that it is no easy matter to make this type of labeling precise. We would prefer to stay away from the questions of detailed analytic structure and dimensionality, but it may be that this is not possible if we wish a useful taxonomy.

Furthermore, is it possible that the foregoing is not merely a mathematical device, but actually represents the possibility that learning itself may be an instinctive phenomenon? This type of question arises again below.

A second point, equally as important as the first, is that it is most certainly unwise at this point to attach ordinary words such as instinctive and learning with a fuzzy cloud of intuitive associations to specific mathematical models of rather simple structure. We know full well that these models are not representative of the entire set of responses connected with the phenomena of instinct and learning. Examples of this narrow use of important terms are information theory, decision theory, learning theory, and a number of others can be given. It is for this reason that we prefer the term adaptive which is not as much a part of the usual vocabulary.

3. Adaptive processes

In order to construct useful models of processes involving learning and adaptation for purposes of simulation and analytic study, it is necessary to consider specific forms of the dependence upon the past. A very useful mathematical model is the following. We begin by replacing (2.3) by

$$q_n = h(p_n, a), \tag{3.1}$$

where a is a vector parameter. We are now talking about a family of responses with the individual member of the family specified by a particular value of a.

To introduce the adaptive aspects, let a_n be made a function of the history of the process,

$$a_n = f(p, p_1, \cdots, p_n; q_1, q_2, \cdots, q_{n-1}; a_1, a_2, \cdots, a_{n-1}). \tag{3.2}$$

Then,

$$q_n = h(p_{n-1}, a_n). \tag{3.3}$$

Processes of this type can be taken to represent certain simple types of learning processes. The question of how a_n should be determined on the basis of the past history of the process is a difficult one which can be discussed by means of a number of mathematical theories ranging from decision theory and dynamic programming to nonlinear prediction theory and quasilinearization [2], [3].

4. Policies and policies for determining policies

We began with a rule for making decisions, (3.1). Let us call this rule a policy. Then we formulated a rule for modifying policies on the basis of experience. This is a policy for producing policies. Can we go one step further? How do we modify these metapolicies? One approach is to mimic what we did before. Write

$$\begin{aligned} p_n &= T(p_{n-1}, q_n), \\ q_n &= h(p_n, a_n), \\ a_n &= f(p, p_1, \cdots, p_n; q_1, q_2, \cdots, q_{n-1}; a_1, a_2, \cdots, a_{n-1}; b), \end{aligned} \tag{4.1}$$

where b is again a vector parameter which we can make dependent upon the history of the process.

A policy for modifying policies is now a prescription for the determination of b at each stage as a function of the past history of the process

$$b_n = \psi(p, p_1, \cdots, p_n; q_1, q_2, \cdots, q_{n-1}; a_1, a_2, \cdots, a_{n-1}; b_1, b_2, \cdots, b_{n-1}). \tag{4.2}$$

But now we encounter a curious difficulty. How do we differentiate between these last two types of learning processes? In both cases, we end up with a policy of the form

$$q_n = f(p, p_1, \cdots, p_{n-1}; q_1, q_2, \cdots, q_{n-1}). \tag{4.3}$$

We know that we have iterated policies, and we feel intuitively that we are operating on a higher level of intelligence. In terms of our initial definition, however, we appear never to be able to rise above a learning process of the simplest type.

The difficulty may lie in the generality we have attempted. If we deal with general functions, there is no way of distinguishing between an arbitrary function and the iterate of an arbitrary function. If, on the other hand, we consider special classes of functions, then we can construct a meaningful hierarchy. This is the basic idea of the Liouville theory of the integration of elementary functions [1].

5. An alternative approach

In place of pursuing the foregoing approach, we can assume that a policy for modifying policies is dependent not only upon the result of a single history, but upon the observation of a number N of histories. Thus, we suppose that we begin with the processes

$$
\begin{aligned}
p_n^{(k)} &= T(p_{n-1}^{(k)}, q_n^{(k)}),\\
q_n^{(k)} &= h(p_n^{(k)}, a_n^{(k)}),\\
a_n^{(k)} &= f(p^{(k)}, \cdots, p_n^{(k)}; q_1^{(k)}, \cdots, q_{n-1}^{(k)}; a_1^{(k)}, \cdots, a_n^{(k)}; b_n),
\end{aligned}
\tag{5.1}
$$

where $k = 1, 2, \cdots, N_1$.

Now let b_n be chosen as a function of the histories of all the processes

$$b_n = g(p_i^{(k)}, q_i^{(k)}, a_i^{(k)}; i = 1, 2, \cdots, n, b_1, b_2, \cdots, b_{n-1}). \tag{5.2}$$

This yields an adaptive process which is on a higher level than the simple learning process described above. We can now enlarge this process in a similar fashion and thus obtain the desired hierarchy of adaptive processes.

6. Discussion

A number of questions arise immediately. Are there other types of hierarchies? One would imagine so. For example, do we have a genuine model of creativity in the foregoing?

A second problem of importance in a number of fields is the inverse problem. Given a description of a type of behavior which is instinctive or intelligent, can we explain it in terms of a mathematical model of the foregoing type? This is the kind of problem studied in [4].

With the construction of various theories of the type proposed above, we can begin to discuss the question, "Can machines think?" in a rational fashion. We would first modify it to read, "Can machines perform level k thinking?" This is now a definite question which can be answered affirmatively by a computer program, or an existence proof for a computer program.

This is the traditional approach used in mathematics to remove mysticism from the unknown, the approach followed in the study of the infinite, in the study of logical statements, in the study of divergent series, the Liouville theory mentioned above, and so on.

REFERENCES

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