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VOLUME II

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PREFACE

THE FOURTH BERKELEY SYMPOSIUM on Mathematical Statistics and Probability was organized by the personnel of the Statistical Laboratory and of the Department of Statistics, University of California, Berkeley, comprised of Professors E. W. Barankin, D. Blackwell, F. Cogburn, E. Fix, J. L. Hodges, Jr., L. Le Cam, E. L. Lehmann, M. Loève, J. Neyman, R. Radner, H. Scheffé, E. L. Scott, and A. J. Thomasian. The plan of the Symposium was drawn and the contributors selected with an active participation of two Advisory Committees. For advice regarding theory of probability and statistics and also regarding some applications, we are indebted to the Advisory Committee composed of delegates of the American Mathematical Society, Professor J. H. Curtiss, J. L. Doob, and William Feller, of delegates of the Institute of Mathematical Statistics, Professors Albert H. Bowker and H. E. Robbins, and of the Editor of the *Annals of Mathematical Statistics*, Professor W. H. Kruskal. The program on biology and on problems of health was arranged with the help of another Advisory Committee composed of representatives of the National Institutes of Health, Drs. M. Berman, R. Bryan, H. F. Dorn, S. Geisser, S. Greenhouse, J. Hearon, M. Shimkin, and J. L. Stephenson, and also Drs. A. Berger, J. Jacquez, H. B. Jones, and C. Tobias. This help is very gratefully acknowledged.

The purpose of the Berkeley Symposia is to stimulate research through the lectures of the carefully selected speakers and by providing opportunity for personal contacts extending over several weeks spent in Berkeley for scholars from different centers, and by publishing the *Proceedings*.

The *Proceedings* are intended to represent a comprehensive cross section of contemporary thinking on problems of probability and mathematical statistics. Although completeness is difficult to achieve, the Statistical Laboratory is gratified by the gradual increase in the number of intellectual centers throughout the world represented at the successive Symposia. In particular, the present *Proceedings* are much richer than those of the earlier Symposia because of the several contributions from members of the great Russian school of probability. These contributions were secured through the kind cooperation of the National Academy of Sciences in Washington, D. C. and of the Academy of Sciences of the U.S.S.R. in Moscow, and hearty thanks are due to both institutions.

Volume I of the *Proceedings* is given to the theory of statistics. Volume II contains papers on probability. As a preliminary section this volume contains the record of a special meeting of the Symposium dedicated to the memory of the recently deceased remarkable probabilist A. I. Khinchin. It is a pleasure to thank Professor B. V. Gnedenko for providing us with a photograph of the late Professor Khinchin. It appears as the frontispiece of Volume II.

Although a comprehensive representation of theoretical developments is difficult to achieve, the field of application of the theory of probability and of statistics is currently so broad and contains so many different domains that any significant approach to completeness of coverage is simply impossible, even in a meeting extending over six weeks. Particular domains of application require separate symposia. The several domains that were actually discussed at the Fourth Symposium were selected partly on the basis of personal preference of certain scholars invited to the Symposium for the excellence of their work in a number of fields and partly because of the related research conducted at the Statistical Laboratory.

Applications to physical sciences are published in Volume III of the *Proceedings*. They include astronomy, meteorology, and physics.

Volume IV of the *Proceedings* is given to biology and problems of medicine. Here, with reference to one particular subject, namely theory of carcinogenesis, substantial effort was expended to collect not only a comprehensive representation of the current theoretical statistical and probabilistic work but also papers characterizing various relevant empirical findings. Thus, the fourth volume of these *Proceedings* includes papers outlining the important biological studies relevant to the theory of carcinogenesis, from experiments on cellular phenomena, through studies of viruses, to large-scale surveys on the possible effects of low level radiation, one of the most dreaded carcinogens.

A realistic and precise theory of carcinogenesis seems to belong to the category of those problems in which the solution depends on a close cooperation and mutual understanding of experimenters on the one hand and of statisticians on the other. It is hoped that Volume IV of these *Proceedings* will contribute to the establishment of such understanding and cooperation.

The sources of financial support of the Fourth Symposium are listed on the title page of each volume of these *Proceedings* and this support is here very gratefully acknowledged. First and foremost, hearty thanks are due to Professor Clark Kerr, President of the University of California, for his generous grant made several years before the Symposium. This initial grant insured the organization of the Symposium. However, without the subsequent generous financial support of the National Science Foundation, followed by those of the Air Force Office of Research and Development, of the Office of Naval Research, and of the Office of Ordnance Research, the meeting could not have been held on the scale that actually was achieved. A special grant and also considerable moral support of the National Institutes of Health, helped to organize a series of sessions on biology and medicine.

In addition to direct grants of funds, the organization of the Symposium obtained a most valuable indirect support kindly provided by the U. S. Air Force Office of Scientific Research and, to an even greater extent, by the Office of Naval Research. This indirect support consisted in the air transportation for a number of foreign guests over the oceans, both the Atlantic and the Pacific.

Without this form of help, foreign participation in the Fourth Symposium would have been greatly reduced.

The growth of the Symposia is naturally accompanied by the corresponding growth of the *Proceedings*, from about 500 pages for the First Symposium of 1945/46 to about 2000 printed pages for the Fourth. Speedy publication of this amount of scientific material naturally presents a number of problems. This is particularly true when a substantial part of the material is originally written in foreign languages and requires translation into English. This is even more particularly true when it is desired to produce books at a relatively low price which will make them accessible to young scholars.

In the above connection I am greatly indebted to a number of colleagues and friends for their work on the translation of manuscripts and for their help to simplify some of the formulas so as to make them less expensive to set in type. In particular my thanks are due to Professor Evelyn Fix, to Professor Lucien Le Cam, to Dr. Emma Lehmer, to Professor and Mrs. J. G. Mauldon, and to Professor Elizabeth L. Scott. Further thanks for help in translations are also due to Drs. I. J. Abrams, A. R. Kraiman, and L. Neustadt of the Space Technology Laboratories.

For work on the preparation of some of the manuscripts for the printers my thanks go to several Research Assistants in the Statistical Laboratory, S. Bhuchongkul, N. L. Cook, M. Darland, J. Denny, J. Fabius, S. S. Jogdeo, J. Karush, W. Klonecki, J. Kraft, W. Lawton, J. L. Lovasich, P. Mikulski, L. Regelson, and G. D. Woodard. Before being sent to the printers, many manuscripts had to be retyped. This was excellently done by Mrs. Sharlee Guise and Mrs. Julia Rubalcava.

Special thanks are due W. H. Newkirk who, as copy editor of the *Proceedings*, took care of the many complexities of publishing with uncommon care and unceasing patience.

Financial difficulties connected with the publication were overcome with the effective help of the Editorial Committee of the University of California, due to the generous subvention of the IBM Company and to additional grants from the RAND Corporation and from the Space Technology Research Laboratories. All this help is gratefully acknowledged.

Funds and adequate personnel are, of course, indispensable in any substantial enterprise. However, in addition to these most necessary elements, for an organization to run smoothly it is necessary that it include someone whose continuous care, initiative, foresight, and good common sense would make the organization "click." For this role in the organization of the Symposium it is a pleasure to thank my colleague and friend, Professor Elizabeth L. Scott.

Mr. August Frugé, the Director of the University of California Press, was most cooperative and helpful in organizing the publication of the *Proceedings*. The speed with which the *Proceedings* are published is very essential, and one year taken to produce about 2000 pages in print is an excellent record. However,

the actual speed in manufacturing the books is even better than it looks, for, to this editor's deep shame and regret, the manuscripts of which he himself is a co-author were the last to be delivered to the Press. In fact, they were delivered in the last days of June 1961, and my thanks to Mr. Frugé are accompanied by sincere apologies.

JERZY NEYMAN
Director, Statistical Laboratory

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