STATISTICS OF CLUSTERS OF GALAXIES

DISTRIBUTION OF CENTERS, ANGULAR DIMENSIONS, STRUCTURE, LUMINOSITY FUNCTION OF MEMBER GALAXIES

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1. Preamble

We shall in this study be concerned with the large scale distribution of galaxies and of clusters of galaxies. Particular emphasis will be laid on the morphology of clusters. Physical properties of galaxies and of clusters of galaxies will be discussed qualitatively, but only insofar as knowledge of these properties is necessary to make the objects in question clearly identifiable. In addition to this, a few preliminary suggestions will be made concerning the importance of various physical properties of large scale aggregations of matter for the construction of reasonable cosmological theories. Many of the theories in the past have indeed been built on foundations which are unconvincing to the author and the establishment of reliable pegs of factual knowledge, rather than mathematical theses on the origin and on the evolution of the universe and of its component parts, would appear to constitute the most urgent task in astronomy today.

We shall lay our principal emphasis on those simplest characteristics of the large scale distribution of matter which can be analyzed by the three elementary operations constituting the methodology of *dimensionless morphology*, namely, *identification, counting*, and *recognition of coincidences*. We shall attempt to demonstrate the great power of this approach by means of a few specific applications. Once the groundwork is laid through the observation of a large number of facts and the analysis of these facts by the most elementary statistical methods, the way will be open for fruitful applications of the more detailed and mathematically elegant methods developed by mathematicians who have made statistics their profession.

2. Definitions

2.1. *Galaxies*. It is extraordinarily difficult to give an unambiguous definition of a galaxy which is operationally workable in practice. The lack of a clear understanding of what objects must be treated as individual stellar systems has in the past severely confused all issues related to the luminosity function of galaxies. Likewise, in counting galaxies in general, or galaxies of particular types within well defined regions of the sky, we must first know which of these objects are to be considered as individual units.

There are in principle two avenues open to us. We may characterize stellar systems either by their *appearance* or by their *intrinsic physical properties*, such as their dynamic parameters. An analysis of these latter parameters would yield a definition of galaxies describing them as certain types of *statistically stationary* and *dynamically stable units*. Although such an approach would be theoretically most satisfactory, it is quite unworkable in the statistical investigation of large numbers of extragalactic objects, since no adequate observations have ever been carried out to analyse the dynamic properties of even a few of the nearest galaxies, not to speak about the millions of distant systems with which we are here concerned. There thus remains only the use of the structural appearances of the luminous extragalactic objects. It goes without saying that systems consisting of very cool stars or more highly dispersed dark matter will be disregarded from the start and will have to be dealt with later on by quite different methods.

Concerning the luminous aspects of galaxies, two simple methods suggest themselves for the integral evaluation of the distribution of the surface brightness within the telescopically recorded images. If we restrict ourselves for the present to the use of light which can be easily recorded by ordinary photography, the two methods mentioned are based on the following features:

(a) The average surface brightness of a nebular image within rings between r and $r + dr$ generally declines as a function of the distance r from the nucleus or from some point of highest surface brightness until a critical value r_{crit} is reached where it again begins to increase. Eliminating the effects of foreground stars and of sharply outlined lanes of interstellar absorption, we may assume that the increase of the average surface brightness beyond r_{crit} is due to the inclusion of the light from a neighboring galaxy which thus announces its presence and is consequently identified. The operation "identification" which constitutes one of the pegs of dimensionless morphology is thus essentially based on the visual qualitative appraisal of the surface brightness of various objects recorded photographically.

Paradoxically, the criterion just described works well in the range from medium distances, at which not too many details are seen in the images of galaxies, and out to those remotest distances where images of galaxies are not any more distinguishable from individual stars.

For nearby galaxies, with their complicated knotty and resolved structures clearly distinguishable, our criterion occasionally fails. If automatically applied as stated, it may make two or more individuals out of a single object. Fortunately there are other aspects which allow us to check on the individual identity of nearby galaxies. In any event, in large scale statistics of the type contemplated here, a few errors about nearby galaxies will not affect our results appreciably. On the other hand, if it were our problem to establish the absolute luminosity function of galaxies through a study of the neighboring stellar systems alone, a much more thorough scrutiny of these systems would be necessary in order to decide which objects should be introduced as individuals of the statistical weight unity.

(b) Outstanding structural features may help in some cases to identify individual galaxies. For instance, spirals are often so characteristic that even for tangled objects it is relatively easy to separate the individual units. Photographs taken in different colours will further facilitate this task.

Unfortunately, the structural features do not provide a universal remedy to our difficulties. In the first place these features become hopelessly smeared for distant objects or for systems on edge. In the second place it seems that contrary to earlier statistics, the irregular systems appear to be in the vast majority. Obviously, in groups containing objects devoid of any *a priori* known shape, the structural features cannot be used to separate the individuals.

All conventional methods of counting have been based essentially on the procedure described under (a), although the various operations and definitions necessary were never before clearly formulated and the limitations involved were not evaluated. Among these limitations we mention the difficulty of properly classifying certain open and compact star clusters and dwarf stellar systems as well as the component parts of many of the newly discovered luminous intergalactic formations [1], [2], [3]. For instance, according to the adopted method of identification, all of the star clusters on the outskirts of Messier 31 or of NGC 4486 which appear clearly separated from the mother galaxy will be counted as individual units while the clusters projected on or immersed in the brighter parts of the main body of the principal galaxy would escape a similar identification. There thus exists the problem of the analysis of the fields within and around the images of large galaxies. This analysis refers to objects which are actually close to the galaxy in question as well as to distant nebulae which lie in the line of sight and which are partly or totally covered by the large foreground galaxy. The errors introduced into any statistical analysis because of all the complicating circumstances just described must in every case be investigated. It will be seen that for the statistics of clusters of galaxies the uncertainties in the identification of individual galaxies necessitate only insignificant corrections. This is one among other great advantages of dealing with whole clusters rather than with individual galaxies.

For purposes of illustration two tangled systems of galaxies are shown in the figures 1 and 2, which, according to the adopted mode of identification in our statistics, would be counted as two and five individual galaxies respectively. As a first approximation, luminous intergalactic formations, such as the bridges and filaments seen in figures 1 and 2, are not being considered as separate individuals, but are associated with the galaxies from which they emerge.

2.2. *Clusters of galaxies.* In contradistinction to the difficulties encountered in adequately defining what a galaxy is, our analogous task for clusters of galaxies is relatively simple. This circumstance stems from the fact that we shall include in our analysis only clusters whose brightest member galaxies can be individually identified, while on the other hand in most galaxies not even the brightest constituent stars can be individually resolved.

For a first survey we shall divide clusters of galaxies into three classes:

(a) Compact clusters with one single major central condensation containing a dozen or more galaxies contacting each other. Many of the compact clusters such as those in Perseus, Coma, and Corona Borealis show remarkable spherical symmetry.

(b) Medium compact clusters have either two or more major concentrations of galaxies, or, if there is only one dominant major concentration, the galaxies in it are not visibly contacting each other but are separated by one or several diameters.

(c) Open clusters with the appearance of dispersed clouds do not show any prominent concentrations of the member galaxies. The "nearby" and very extended Ursa Major cloud of galaxies is an example of this class.

Among the three classes mentioned there are rich and poor clusters. In the following analyses we shall deal only with rich clusters, which for purposes of practical operation are defined by the following characteristics. If m_B is the apparent magnitude of the brightest member galaxy we call a cluster rich, if in the magnitude range m_B to $m_B + 3$ it contains more than fifty galaxies. The well known clusters in Hydra, Perseus, Coma and Corona Borealis are richly populated containing several hundred member galaxies within the first three magnitude ranges.

3. Individual clusters of galaxies

Without attempting to be exhaustive we review here a few of the characteristics of individual clusters ([4] through [8]). There is a marked difference of the types of galaxies which constitute compact clusters on the one hand, and medium compact or open clusters on the other hand. Spirals are in fact very rare in compact agglomerations which are mostly made up of elliptical or S_0 types. In addition to complete stellar systems, clusters contain dispersed stars throughout and, as we shall show in this study, dark obscuring intergalactic matter, that is, dust.

Many of the compact clusters are spherically symmetrical and show a radial distribution of the first few hundred of the brightest member galaxies which can be mathematically represented by the radial density distribution function of an Emden isothermal gravitational gas sphere. Furthermore, the structural indices or structural lengths [9], [10]

$$\alpha = (\overline{v^2}/12\pi\Gamma\rho_0)^{\frac{1}{2}}, \tag{1}$$

where $\overline{v^2}$ is the square of the velocity dispersion, Γ is the universal gravitational constant and ρ_0 is the central density of the cluster, are of the order of 20,000 to 40,000 light years for all rich clusters investigated so far. All cosmic lengths given in this study are based on Hubble's original distance scale [11]. The recent revisions are not being applied since they are as yet quantitatively uncertain.

The diameters of the largest clusters are of the order of ten million light years. Clusters fill nearly all of space. In contradistinction to the results reported by the early investigators cluster nebulae are in the majority, rather than field nebulae.

There is a marked segregation of absolutely bright and of faint galaxies, the latter showing less tendency toward clustering than the absolutely bright galaxies.

The internal velocity dispersion in clusters is directly related to the size of the population and the degree of compactness of a cluster. In the richest and most compact agglomerations such as those in Coma and Corona Borealis (see figures 3, 4, 5) the velocity dispersion may be as high as 2,000 to 3,000 km/sec. In small groups the dispersion is of the order of 200 km/sec.

Values of the total masses of large clusters of galaxies were first derived from the observed velocity dispersions by the author in 1933 [12]. Using the virial theorem and assuming for the conspicuous central part of the Coma Cluster a diameter of two million light years, a total mass for this part of the cluster of about $3 \times 10^{14} M_{\odot}$ was obtained. Assuming further that this mass is essentially concentrated in the first thousand brightest member galaxies, their average mass becomes equal to $3 \times 10^{11} M_{\odot}$, that is, at least a hundred times larger than the values which were originally deduced from the absolute luminosities and the rotational characteristics of the brighter stellar systems. Dark and subluminous matter spread throughout a cluster may, of course, contribute much to its total mass. It seems unlikely, however, that in this manner the average mass of the brighter member galaxies of large clusters can be reduced sufficiently to achieve agreement with other methods for the determination of the masses of stellar systems.

From the velocity dispersion of large clusters and their distances D it follows that their smoothed out central densities are of the order of 10^{-23} grams/cm^3. This leads to average densities of matter throughout the visible universe which are $\bar{\rho} = 10^{-26}$ grams/cm^3 or greater [13]. This expectation may be compared with the density derived from the characteristics of the simplest model of a flat expanding universe as it was described by Einstein and de Sitter [14]. According to their theory the expansion constant E is related to $\bar{\rho}$,

$$E = \frac{dD}{dt}/D = (8\pi\Gamma\bar{\rho}/3)^{\frac{1}{2}} . \tag{2}$$

With $E = 550$ km/sec megaparsecs we obtain $\bar{\rho} = 6 \times 10^{-28}$ grams/cm^3 which is at least one order of magnitude too small. This discrepancy which is independent of the adopted distance scale is one among many difficulties which stand in the way of the theories of an expanding universe. For purposes of illustration we here present a few data on the Corona Borealis cluster of galaxies. The three photographs reproduced in the figures 3, 4 and 5 show the results obtainable on this cluster with the 18 and 48-inch Schmidts as well as with the 200-inch telescope.

4. Intercomparison of some characteristics of galaxies and of clusters of galaxies

In order to understand the advantages and disadvantages of using either the galaxies or the clusters of galaxies as the individual objects which are being subjected to large scale counts and a subsequent statistical analysis, it is important that one appreciate certain basic differences in the photographic appearance and in the intrinsic character of the two types of objects.

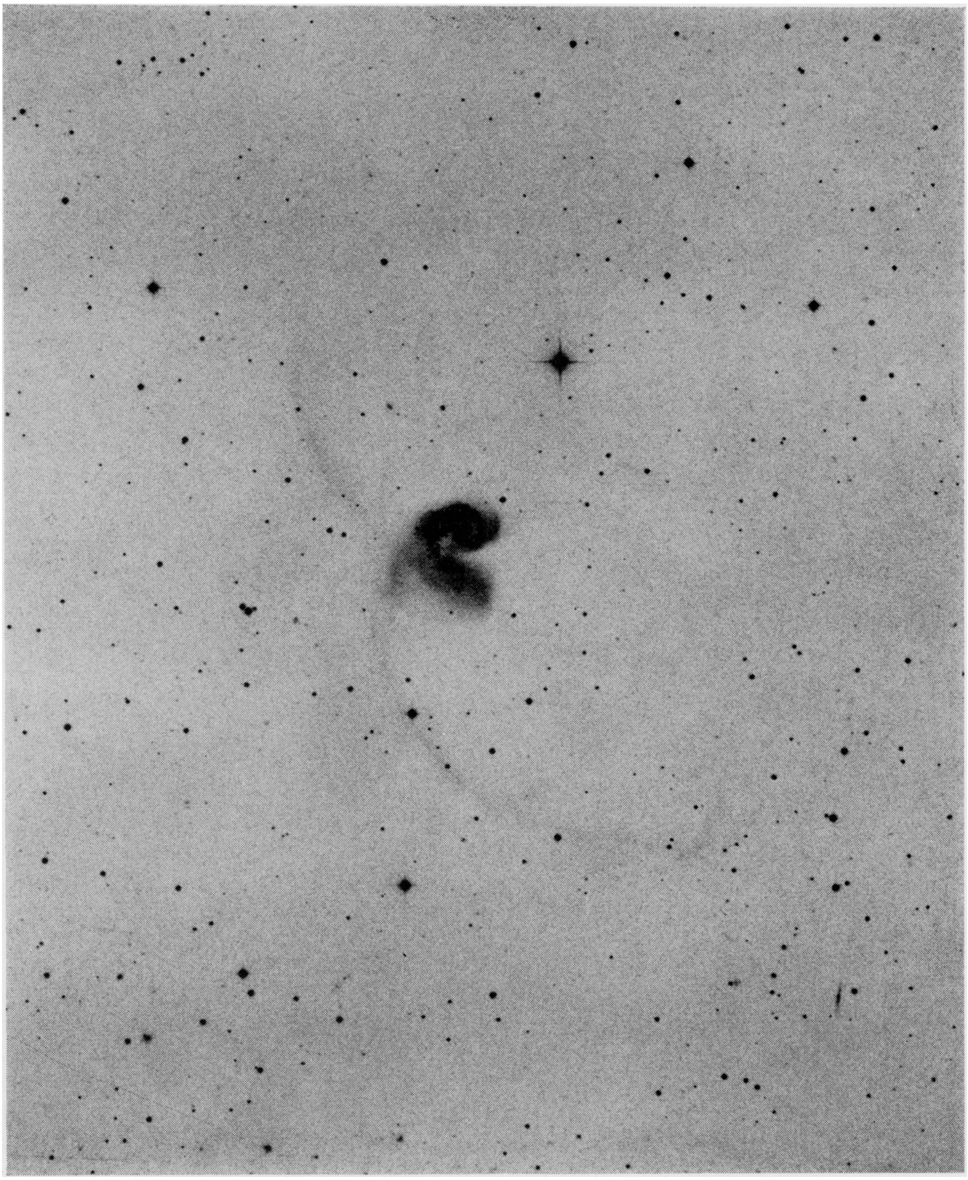

FIGURE 1

Photograph obtained with the 48-inch Schmidt telescope on Palomar Mountain of the double galaxy NGC 4038–4039 at R.A. $11^h59^m18^s$ and Decl. $-18°$ 35′ (Epoch 1950.0). The average apparent symbolic velocity of recession of this system is $v_s = c\Delta\lambda/\lambda = 1636$ km/sec. The spectrum in addition to the continuum with absorption lines shows emission lines also, with λ3727Å particularly strong.

Scale: 1 mm = 12.2 seconds of arc.

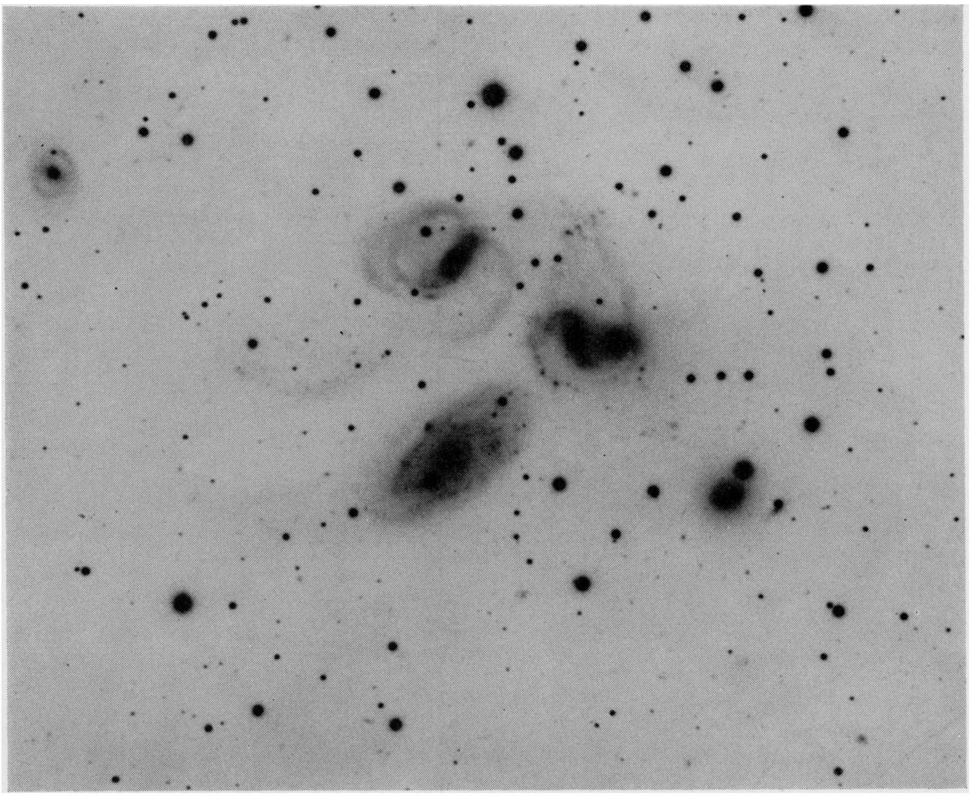

FIGURE 2

Photograph with the 200-inch telescope on Eastman 103 a-D plate behind GG11 schott glass filter of Stephan's Quintet, NGC 7317, 7318a, 7318b, 7319, 7320 at R.A. $22^h33^m43^s$ and Decl. $+33° 42'$ (Epoch 1950.0). The symbolic velocities of recession for the first four galaxies mentioned above are v_s = 6736, 6638, 5638 and 6657 km/sec, respectively. This, according to the law of red-shifts, puts the quintet at a distance of about 12.5 million parsecs, on the old scale.

Scale: 1 mm = 5.20 seconds of arc.

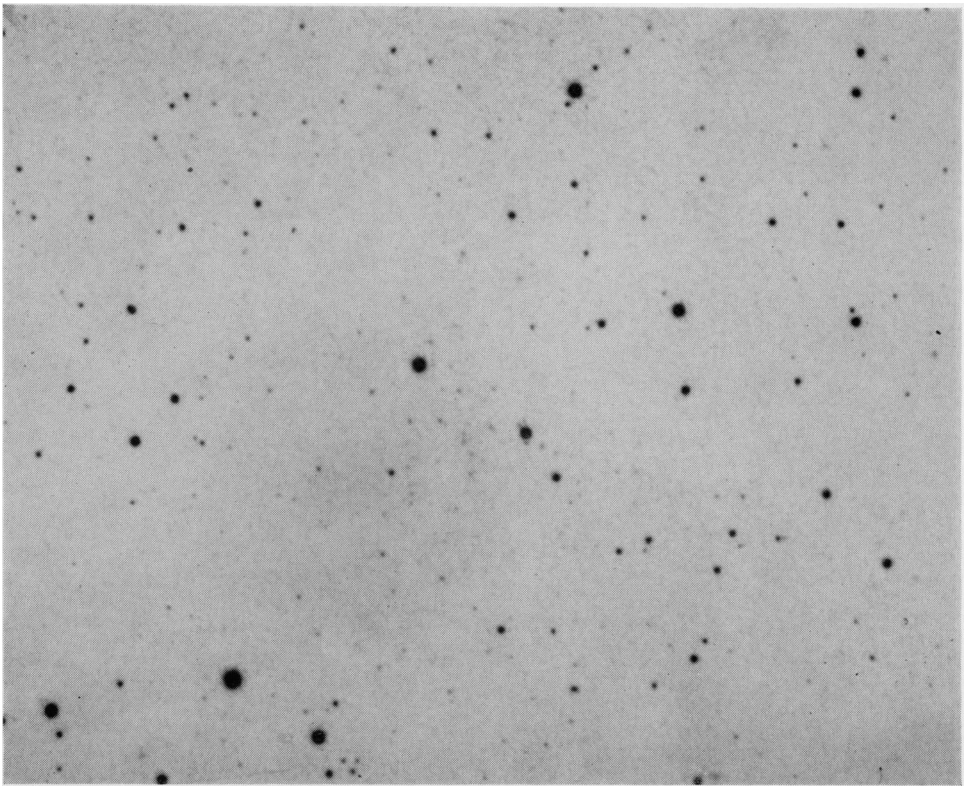

FIGURE 3

Photograph with the 18-inch Schmidt telescope of the Corona Borealis cluster of galaxies at R.A. $15^h20^m23^s$ and Decl. $+27° 53' 12''$ (Epoch 1950.0). The cluster lies at a distance of about 120 million light years, old scale. Its symbolic velocity of recession is about $v_s = 21000$ km/sec and its brightest member galaxy has the apparent photographic and photovisual magnitudes $m_p = +16.52$ and $m_{pv} = +15.38$, respectively (from photoelectric measures of E. Pettit and W. A. Baum).

Scale: 1 cm = 2.92 minutes of arc.

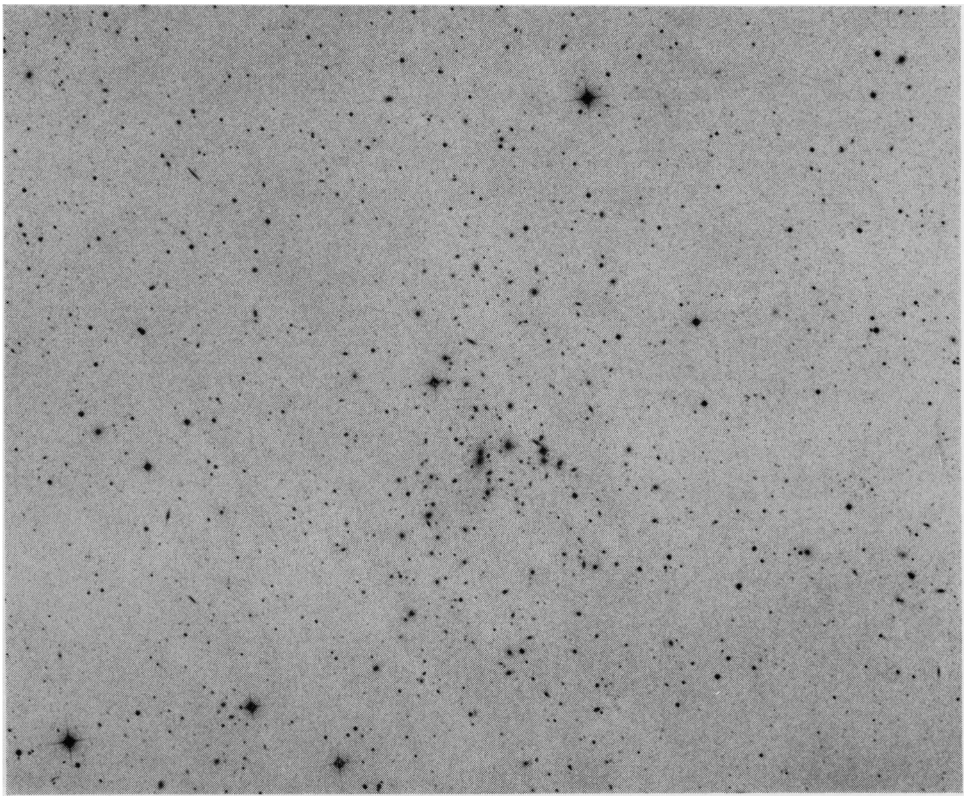

FIGURE 4

Photograph with the 48-inch Schmidt telescope of the Corona Borealis cluster on Eastman 103 a-E plate, behind red plexiglass filter.

Scale: 1 cm = 2.93 minutes of arc.

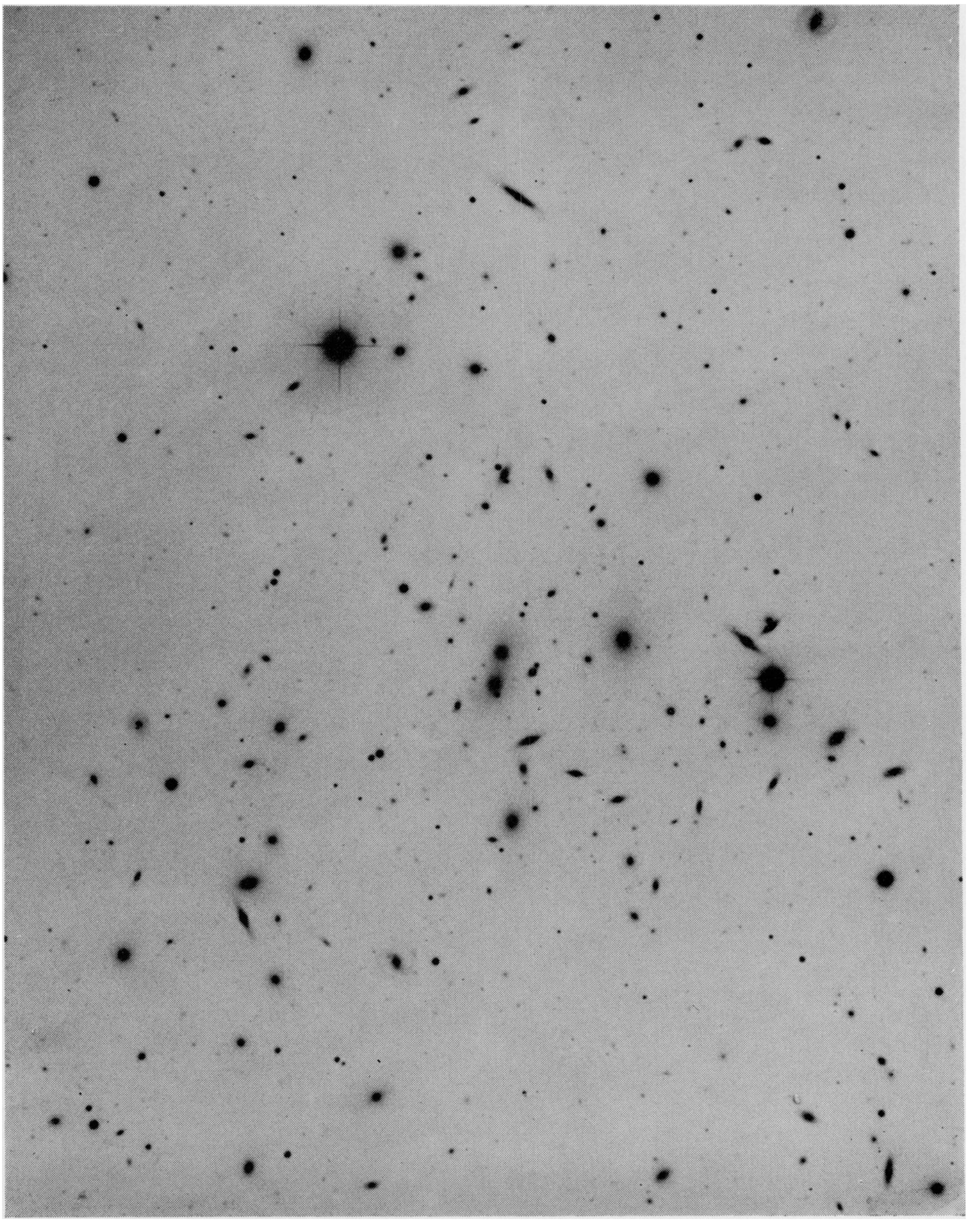

FIGURE 5

Photograph of the Corona Borealis cluster with the 200-inch telescope on Eastman 103 a-D plate behind GG11 schott glass filter. The partial segregation of bright and faint galaxies is particularly conspicuous on this photograph, the bright objects being concentrated in a central area of peculiarly triangular shape. Notice the two bright globular galaxies which are connected and surrounded by intergalactic matter. The symbolic velocity of recession of the northern component of this pair is $v_s = 21962$ km/sec, very close to the average value for the cluster.

Scale: 1 mm = 4.11 seconds of arc.

4.1. *Appearance of distant galaxies and of clusters of galaxies*

(a) *Individual* galaxies, at the limit of the plates obtained with the 48-inch Schmidt, often cannot be distinguished from images of stars, since both of them appear as limiting spots of the order of 40 microns, as determined by the graininess of the emulsion, the resolving power of the telescope, and the effects of astronomical seeing. On plates taken with the 48-inch Schmidt it is therefore exceedingly difficult to go to the limit in the statistics of galaxies, since the smallest images of galaxies can easily be mistaken for stars or for plate defects.

The limitation just mentioned does not apply to clusters of galaxies which on 48-inch Schmidt plates are most easily recognizable.

In contradistinction to the Schmidt telescopes with large focal ratios, the task of distinguishing images of faint stars from those of distant galaxies is much easier when telescopes of relatively large focal length, such as the 20-inch Ross astrograph of the Lick Observatory, are used. On the other hand, clusters of galaxies are much less conspicuous on photographs obtained with telescopes of small focal ratio.

The circumstances just stated largely influenced the division of labour observed at this symposium, the Lick observers occupying themselves mostly with the distribution of galaxies while I concentrated on the statistics of clusters of galaxies.

(b) Most of the distant galaxies with which we are concerned in our large scale statistics cannot be resolved into individual stars. The images of these galaxies are in fact so small that their structural types can only rarely be identified. In general one therefore does not know even approximately the absolute brightness or the absolute distance of any individual very distant galaxy. On the other hand even in the most distant cluster recorded many of the brighter member galaxies appear clearly resolved as individuals and much can be learned about the internal structure of a cluster by simply studying the distribution of its members on photographs taken with different exposure times in various spectral ranges. For instance, diameters of clusters of galaxies can be defined and determined by simply using the operation of counting. Diameters of galaxies cannot be so defined and the observation of degrees of surface brightness as a substitute for counting is practically only applicable for near and large galaxies while images of distant galaxies are too small to be analyzed in this manner.

4.2. *Relative physical characteristics of galaxies and of clusters of galaxies*

(a) Large stellar systems contain *millions of stars* in the range of absolute magnitudes $+10 > M > -5$. In a similar range, namely $-5 > M > -20$, a large cluster of galaxies probably has fewer than 100,000 member galaxies.

(b) While diameters of stars are of the order $d_S = 10^{11}$ cm, they are separated by interstellar distances of the order $D_S = 10^{18}$ cm, so that

$$D_S/d_S = 10^7. \tag{3}$$

Diameters of galaxies are of the order $d_G = 10^{22}$ cm, and the intergalactic distances between them $D_G = 10^{23}$ cm, so that

$$D_G/d_G = 10. \tag{4}$$

Intergalactic spaces are thus relatively much smaller than interstellar spaces from which fact one might *a priori* suspect that they are less likely to be empty. This suspicion has now been verified both theoretically and by direct observation ([1] through [3]).

(c) The smoothed out central densities in compact galaxies and clusters of galaxies are, respectively, of the order of 10^{-19} grams/cm^3 and 10^{-23} grams/cm^3, as derived from the velocity dispersion of stars and galaxies with the use of the virial theorem. Observational reasons for the applicability of the virial theorem were given in the references ([4] through [10] and [12]). An additional *a priori* justification for the determination of the total masses of clusters of galaxies with the aid of the virial theorem is mentioned in the next paragraph (d).

(d) With respect to the kinetic energies residing in the various integral degrees of freedom of galaxies and of clusters of galaxies our knowledge is still very fragmentary.

Translational velocities of the centers of mass: Galaxies, as mentioned before, have peculiar velocities v_G relative to their immediate neighbors which are of the order of 100 km/sec to 5,000 km/sec, depending on whether they belong to small or large groups. Stars within the galaxies have velocities v_S relative to their neighbors of the order of 10 to 100 km/sec. It is thus

$$(5) \qquad v_G \gg v_S .$$

This has the important consequence that on close encounters galaxies can disrupt each other and populate intergalactic space with material fragments of all description.

In contradistinction to the motions of galaxies the translational velocities of clusters of galaxies v_{Cl} are relatively small in the sense that

$$(6) \qquad v_{Cl} < v_G$$

if, for v_G, we use the high values of cluster galaxies. As a matter of fact no translational velocity has as yet been definitely observed for any cluster. This circumstance has the important consequence that clusters of galaxies in their mutual interactions are far more stable than galaxies. In contradistinction to galaxies they can hardly change their character by disrupting each other through violent interactions during close encounters; they do so only through the slow processes of diffusion of galaxies from one group to the other or through growth of stars and galaxies from dispersed matter.

Clusters of galaxies for the reasons stated are quite stationary and durable. They are therefore the most reliable indicators of the large scale characteristics of the universe.

The fact that clusters of galaxies do not show any considerable translational velocities with respect to their immediate neighbors means that on a large scale the average internal total kinetic energy $\bar{E}_k$ is opposite and equal to the average internal gravitational energy $\bar{E}_G$ per unit of matter,

$$(7) \qquad \bar{E}_k = -\bar{E}_G .$$

E_k embraces the translational, rotational and oscillational energies of the member galaxies of a cluster, as well as of the intergalactic matter. The virial theorem

$$(8) \qquad \bar{E}_k\,(\mathrm{Tr}) = -\bar{E}_G/2 ,$$

on the other hand, relates the average translational energy $\bar{E}_k(\mathrm{Tr})$ of a unit of matter to the average gravitational potential energy. The combination of (7) and (8) leads to the fundamental conclusion that the luminosity function of galaxies cannot have a maximum but increases monotonely with decreasing mass of the galaxies [15].

Rotational velocities: While the majority of the galaxies probably possess moments of momenta of various magnitudes, no definite rotation has so far been observed in any cluster of galaxies.

Oscillation: So far we do not possess any very definite evidence that either galaxies or clusters of galaxies are subject to large scale internal oscillations. Such oscillations in mechanical systems are, however, in general much easier to excite than rotation, and we should therefore expect them to exist. There are some peculiar morphological aspects to the structure of certain galaxies and clusters of galaxies which gives us a hint of the action of large scale oscillations. I refer to triangular, quadrangular and other polygonal arrangements of the bright stars in galaxies and of the bright galaxies in some of the clusters of galaxies. Our photographs of the Corona Borealis cluster reproduced in the figures 4 and 5 give a striking illustration of this aspect. Clusters of galaxies and galaxies in that state simulate the analogous aspects of a violently oscillating water droplet. Whether the analogy is a true one or not remains, of course, to be investigated.

(e) The question naturally arises whether clusters of galaxies are the largest organized aggregations of matter in the universe. One might imagine that clusters of galaxies agglomerate to form ever larger systems. Provided that our notions about signal velocities are correct and that gravitational interactions cannot be transmitted with infinitely large velocities there is a limit to the size of organized material units. Indeed, intergalactic distances D_G between the galaxies are traversed by them in times of the order D_G/v_G. This time, for a cluster of the diameter D_{Cl} to be organized, must be larger than the time it takes the gravitational interactions to be announced from one end of the cluster to the other. If the speed of this transmission is equal to c, for instance the velocity of light, it follows that, with $D_G \sim$ 100,000 light years in a cluster,

$$D_{Cl} < cD_G/v_G = 200\,D_G = 20 \text{ million light years} \tag{9}$$

which is a cosmic length only a little larger than the diameter of the largest cluster of galaxies thus far investigated.

5. Methodology of the statistics of galaxies and of clusters of galaxies

There are at our disposal two fundamental approaches to the characteristics of the large scale distribution of matter in the universe. We shall designate these approaches as *dimensionless morphology* and *dimensional morphology*, respectively.

As we have mentioned previously, dimensionless morphology is based on the three operations of identification of objects, counting and the recognition of certain coincidence conditions.

Dimensional morphology in astronomy deals with the physical properties of matter. It occupies itself on the one hand with phenomenological correlations among the apparent qualitative characteristics of celestial bodies, as they are observed from the earth. Beyond that, however, dimensional morphology deals with the absolute physical characteristics of stars, galaxies, clusters of galaxies and other bodies.

In the sense of the methodology just described there are thus three phases to the analysis of any problem concerning distant objects in the universe. As a simple illustration of these three phases we may mention those related to the "geographical" distribution of galaxies.

Within the bounds of dimensionless morphology we may first inquire about the apparent distribution of galaxies over the sky as we make use of more and more powerful telescopes. What can be learned from this distribution will be related in section 6. Entering the realm of dimensional morphology we may then ask what the distribution of galaxies is as a function of the photographic apparent magnitude and, ultimately, what is the actual distribution of the galaxies in space. Dimensional morphology thus involves first, the measurements of apparent light intensities, colours, spectra and second, the determination of absolute distances, absolute luminosities, masses and so on, in terms of terrestrial standards.

6. Dimensionless morphology

We briefly review the results obtained by the author from counts of galaxies in breadth and depth over the sky, as well as within individual large clusters of galaxies. Close to four million galaxies were counted in these surveys.

6.1. *Counts of galaxies in breadth.* Extensive counts were made both with the 18-inch and the 48-inch Schmidts. The first and most conspicuous result is that the distribution of galaxies over the sky, as recorded by either one of the two telescopes, is very far from random. Two phenomena immediately suggest themselves to account for this nonrandomness. In the first place systematic clustering of galaxies appears more conspicuous on photographs taken with Schmidt telescopes of large focal ratios than on those obtained with other instruments. An extensive statistical analysis confirms that clustering is a much more general phenomenon than was previously thought [11] and that, as a first approximation, it may be assumed that most of the galaxies are members of clusters ([4] through [8]). A second possible phenomenon which may contribute to the observed nonuniformity in the distribution of distant stellar systems is the interference of intergalactic obscuration. The existence of intergalactic dust was suspected for some time from investigations with the 18-inch Schmidt, but it could not be proved decisively that the observed effects cannot be accounted for by clustering alone. It was found for instance that near the northern galactic pole on the outskirts of the Coma cluster the average number of galaxies on limiting 18-inch Schmidt films is of the order of ten per square degree. On the other hand, over one hundred galaxies per square degree were counted in a field covering over a hundred square degrees around the Corona Borealis cluster which, however, is surrounded by half a dozen or more equally rich clusters, the presence of which might be made responsible for the excess in the average number of galaxies observed. A final decision became only possible after the 48-inch Schmidt had been put into operation. Still, even with this telescope, counts in width alone were not decisive. Both counts of galaxies to different depths and a survey of the distribution of clusters were necessary to obtain strong evidence for the existence of intergalactic dust. In all of these investigations the tests related to the so-called dispersion-subdivision curves were applied. Briefly, one subdivides the field to be analyzed into z equal squares and calculates the observed dispersion $\sigma_{\text{obs.}}$ as

$$\sigma_{\text{obs.}} = \left[\sum_{i=1}^{z} \delta_i^2/z\right]^{\frac{1}{2}}. \tag{10}$$

where n_t is the total number of galaxies in the whole field, $\bar{n} = n_t/z$ is the average number per square and $n_i = \bar{n} + \delta_i$ is the number of galaxies in the ith square. If the distribution of the n_t objects were random, the most probable dispersion $\sigma_{\text{calc.}}$ would be calculated as

$$\sigma_{\text{calc.}} = [n_t(1 - 1/z)/z]^{\frac{1}{2}}. \tag{11}$$

The *relative dispersion* k

$$k(n_t, z) = \sigma_{\text{obs.}}/\sigma_{\text{calc.}} \tag{12}$$

as a function of n_t and z can be shown to reveal most significant features of clustering, intergalactic and interstellar obscuration and so on [16], [17].

The relative dispersion for counts in width over the sky has the following characteristics. For an indefinitely increasing number z of subdivisions, k approaches unity, no matter what the distribution of the objects may be. If the distribution of these objects is random, then k is near unity for *all values* of n_t and z. If the random distribution is disturbed because of the interference of randomly distributed, more or less distinct intergalactic clouds, k may be materially different from unity if fields and subdivisions are considered whose areas are comparable with that of the obscuring clouds. If, however, the fields and subdivisions chosen are large compared with these clouds, then k for large values of n_t again approaches unity. On the other hand, if clustering is a major feature of the distribution of galaxies, the relative dispersion k will remain materially different from unity (except again for $z > n_t$) no matter how much one increases the area of the sky under investigation, provided that in depth of space one does not cover more than a few "layers" of clusters. It is this feature which makes it impossible to clearly distinguish between the effects of clustering and those of intergalactic obscuration, as long as one works with telescopes which do not record galaxies at distances very much greater than the diameters of individual large clusters of galaxies.

Briefly stated, the counts in breadth with the 48-inch Schmidt lead to analogous results as those with the 18-inch Schmidt. Again, very much smaller numbers of galaxies are observed around the north galactic pole than, for instance, in Corona Borealis and many in locations far removed from the pole. The total numbers counted per plate of 40 square degrees may range from 15,000 to 100,000 galaxies, a range which cannot be accounted for by the effects of clustering alone but can be readily explained only by intergalactic obscuration unless one wishes to postulate that the universe is correspondingly unevenly populated. The assumption of the existence of intergalactic dust is vastly strengthened by the observation that the plates are quite deficient in numbers of images of distant galaxies in all of those regions which cover conspicuous nearby clusters of galaxies, such as the rich Coma cluster which lies near the north galactic pole. Likewise counts of galaxies in the regions of the clusters in Virgo, Ursa Major, Cancer, and Pegasus are low, although the obscuring effects of these clusters seem to be less pronounced than that of the very rich and compact Coma cluster. This result is in keeping with the fact that the mentioned clusters are population poorer and less condensed than the one in Coma. Although intergalactic dust may be expected to be dispersed more or less uniformly throughout the whole universe, the preliminary observation that it is locally concentrated within clusters of galaxies is expected from theory and represents a corollary to the fact that large clouds of luminous intergalactic matter, presumably consisting of stars, have been found to be spread throughout the central parts of large clusters.

In line with the phenomena just described the possibility suggests itself that intergalactic dust might be still more concentrated in the spaces between the most compact local groups of galaxies in a cluster. Indications which verify this suspicion were obtained through counts of very distant galaxies in between and around groups of bright galaxies within some of the large clusters. These investigations are at the present time being continued with the aid of the 200-inch telescope in order to derive values for the total and differential absorption by intergalactic material as well as values for possible reddening effects. It is hoped to calibrate the effects of intergalactic dust through a comparison with the actions of interstellar dust on the counts of galaxies, assuming that these actions have been sufficiently well determined through the fading and the reddening of stars of known types.

6.2. *Counts of galaxies in depth.* We emphasize again that when, in dimensionless morphology, we speak of depth in space we do not in any way endeavour to determine absolute distances. We do not even make any assumptions about possible quantitative relations between photographic exposure times and the magnitudes of the faintest galaxies which appear on the photographic plates. It is quite sufficient to know that with different exposure times different numbers n_t of galaxies appear identifiable. In ordering the plates obtained as sequences according to increasing values of n_t, we may apply our analysis based on the construction of *dispersion-subdivision curves.* The mathematical analysis of these curves will give us information as to clustering, intergalactic obscuration, and effects of a possible evolution of the universe as a whole, or that of matter in particular forms.

We here emphasize in particular the results obtained on intergalactic dust. If this dust is not distributed uniformly but shows local cloudy concentrations of the types we have described in the previous section, it may be shown that the relative dispersion $k(n_t, z)$ for a large range of values of z will for many fields of proper solid angle and of proper location increase as the square root of the total number n_t of galaxies counted

$$k(n_t, z = \text{constant}) \sim n_t^{\frac{1}{2}}. \tag{13}$$

The area of the field in question as well as the subdivisions are held constant.

On the other hand if the distribution of galaxies and of clusters of galaxies in space were entirely random, then, regardless of whether the universe is expanding or not, the relative dispersion k, with increasing value of n_t of the galaxies counted in a given field, would tend toward unity, provided there were no patchy intergalactic absorption and provided that the number of member galaxies of clusters with decreasing brightness of the galaxies does not increase faster than $10^{0.6M}$, where M designates the absolute photographic magnitude of these member galaxies. The latter condition is certainly fulfilled, since according to the author's researches the luminosity function of cluster galaxies increases roughly as $10^{0.2M}$ only. The fact,

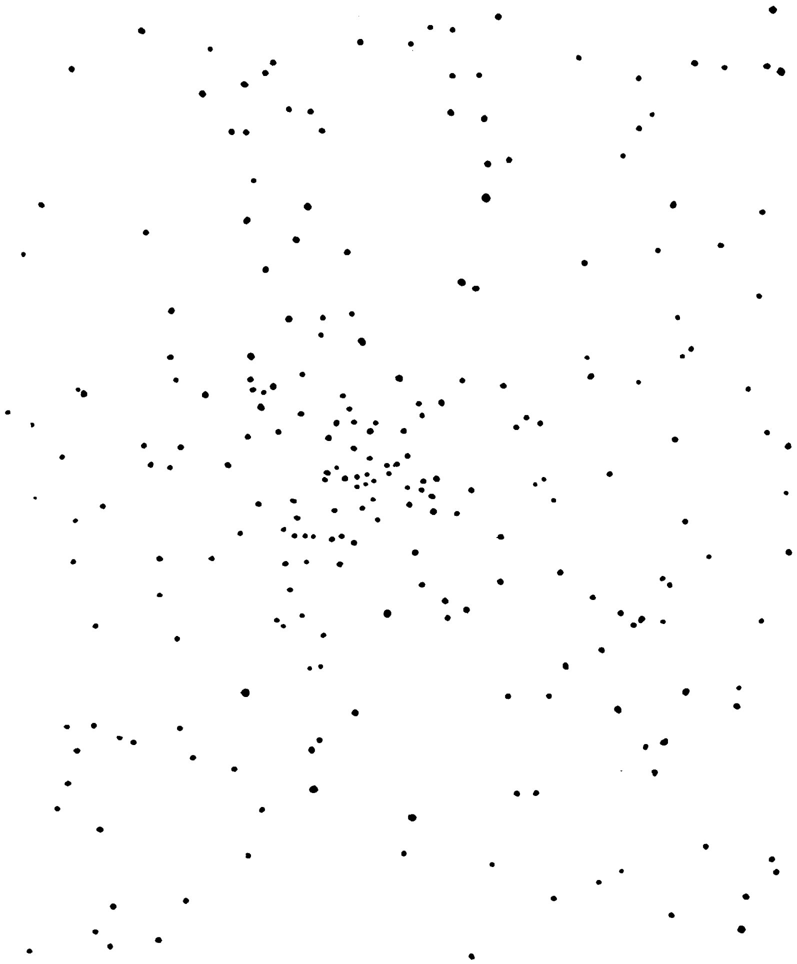

FIGURE 6

Galaxies in the central parts of the Corona Borealis cluster as identified on photographs obtained with the 18-inch Schmidt telescope. Each dot represents one galaxy.

Scale: 1 mm = 2.29 minutes of arc.

therefore, that in many of the fields investigated with the 48-inch Schmidt the relative dispersion k actually increases as the square root of the total number of galaxies counted (on successive longer exposures) represents one of the strongest indications for the existence of observable amounts of obscuring intergalactic dust.

6.3. *Counts of galaxies within the Corona Borealis cluster.* The following data were derived from films and plates obtained with the 18-inch and the 48-inch Schmidt telescopes. They show that the Corona Borealis cluster is rich in member galaxies, spherically symmetrical in appearance and that it possesses the characteristics expected for a statistically stationary system. In table I the results of the counts on two separate films, Eastman 103a–0 films, and executed by two observers (F. Zwicky and P. Wild) are given. See also figure 6.

Within a circle of one degree from the center of the cluster 667 galaxies were counted from which $\pi N_f = 377$ field galaxies must be subtracted, which leaves

$$\mathfrak{N}_{Cl} \text{ (18-inch Schmidt)} = 290 \text{ member galaxies.} \tag{14}$$

TABLE I

COUNTS OF GALAXIES WITH THE 18-INCH SCHMIDT TELESCOPE IN THE CORONA BOREALIS CLUSTER

N_r is the average total number of galaxies per square degree within a ring of the indicated width around the center of the cluster. $N_f = 120$ is the average number of galaxies per square degree in the surrounding field. $N_r - N_f$ represents the number of cluster member galaxies within the rings of indicated width at different distances from the center of the cluster.

Ring Limits in Minutes of Arc	N_r	$N_r - N_f$
0′– 1′	10887	10767
1 – 2	6288	6168
2 – 3	4468	4348
3 – 4	2870	2750
4 – 5	1591	1471
5 – 6	1504	1384
6 – 7	1620	1500
7 – 8	914	794
8 – 9	400	280
9 – 10	636	516
10 – 11	623	503
11 – 12	623	503
12 – 13	504	384
13 – 14	660	540
14 – 15	436	316
15 – 30	216	96
30 – 45	151	31
45 – 60	141	21
60 – 75	127	7
75 – 90	126	6
90 –105	123	3
105 –120	133	13
120 –135	143	23
135 –150	119	. .
150 –165	115	. .
165 –180	128	8
180 –195	128	8
195 –210	108	. .
210 –225	124	4
225 –240	133	13
240 –255	122	2

Beyond one degree from the center the fluctuations of the numbers of galaxies in the general field become so large that the cluster galaxies cannot be counted with any certainty. The diameter of the Corona Borealis cluster, as traced with the 18-inch Schmidt, is therefore approximately two degrees of arc. For the Coma cluster which seems to be an object very similar to the Corona Borealis cluster, and which is located at a distance about one third of the latter, a diameter of about six degrees was obtained from counts of galaxies with the 18-inch Schmidt ([4] through [8]). The diameters of the two clusters are therefore inversely proportional to their distances, as we might expect from the similarity of their structures and the similarity of their populations. We may also represent N_r as a function of r by the standard Emden curve for the radial distribution of mass in an isothermal gravitational gas sphere. In doing so it is found that the so-called structural index or the characteristic structural length of the Emden curve is of the order of 25,000 light years, almost equal to that found previously for the large clusters in Coma, Perseus and Hydra [9], [10].

TABLE II

Counts of Galaxies in the Corona Borealis Cluster with the 48-Inch Schmidt

In the columns 2, 3, 4 and 5 the numbers of galaxies within the rings of indicated width are given in each of the four quadrants. n_r is the total number of galaxies in each ring. N_r is the average number of galaxies per square degree in the ring of average radius r, while $N_r - N_f$ is the corresponding number of cluster galaxies per square degree.

Ring Limits in Minutes of Arc	NW	NE	SE	SW	n_r	N_r	$N_r - N_f$
0′- 1′	7	4	2	3	16	18336	17736
1 - 2	6	6	6	11	29	11082	10482
2 - 3	8	8	13	10	39	8939	8339
3 - 4	10	11	9	7	37	6074	5474
4 - 5	11	6	6	8	31	3953	3353
5 - 6	12	12	14	16	54	5626	5026
6 - 7	7	7	11	10	35	3094	2494
7 - 8	13	10	14	11	48	3667	3067
8 - 9	8	9	10	11	38	2555	1955
9 -10	8	7	11	8	34	2063	1463
10 -11	11	13	9	9	42	2292	1692
11 -12	16	10	9	13	48	2395	1795
12 -13	10	10	14	11	45	2063	1463
13 -14	13	13	8	7	41	1742	1142
14 -15	17	22	16	14	69	2727	2127
Totals	157	148	152	149	606		
15′-20′	54	51	63	69	237	1730	1130
20 -25	101	107	49	69	326	1203	603
25 -30	110	113	81	68	372	1227	627
30 -40	315	285	132	185	917	1043	443
40 -50	307	274	180	198	959	963	363
50 -60	351	335	180	150	986	699	99
60 -80	889	1018	347	586	2840	756	156

The data obtained with the 48-inch Schmidt are given in table II.

We notice first that within a circle of fifteen minutes of arc the Corona Borealis cluster is spherically symmetrical. The total numbers of galaxies within the four quadrants are indeed even closer to the average number of 151 galaxies per quadrant than can generally be expected for a random distribution of noninteracting objects. Beyond fifteen minutes of arc, however, there are about twice as many galaxies north of the cluster than south of it. The reason for this asymmetry lies in the fact that there are half a dozen independent clusters of galaxies which to the north interfere with the Corona Borealis cluster. For this reason the values of N_r beyond 15′ of arc from the center of the cluster were determined from the counts in the SE and the SW quadrants alone. Likewise, the adopted value of $N_f = 600$ galaxies per square degree refers to the southern surroundings of the Corona Borealis cluster. Actually it was not possible to trace the membership of the cluster beyond a distance of about 90′ of arc from its center, since beyond this distance independent rich clusters to the south vitiate the relevant counts of galaxies. The membership could thus be estimated only within a circle of 80′ of arc, for which we obtain

$$(15) \qquad \mathfrak{N}_{Cl}(\text{48-inch Schmidt}) = 1977 \text{ cluster galaxies.}$$

In this connection it is of interest to compare the results (14) and (15) with the approximate luminosity function of cluster galaxies which the writer recently derived from counts of clusters of galaxies in depth of space [16], [17]. This function is

$$(16) \qquad \mathfrak{N}(M) = \text{constant} \times [10^{0.2(M-M_{\max})} - 1]$$

where $M_{\max}$ is the absolute magnitude of the brightest galaxy in a cluster, while $\mathfrak{N}(M)$ is the total number of member galaxies in the range of absolute magnitudes from $M_{\max}$ to M. The multiplying constant may be expected to be the same for clusters of the same total population and the same morphological character. A rough check of the relation (16) can be made with the aid of the data given in (14) and (15). Indeed, in the case of the Corona Borealis cluster for the 18-inch Schmidt we have $M - M_{\max} = 1.5$, while for the 48-inch $M - M_{\max} = 4.5$, where for M we have substituted the absolute magnitude of the faintest galaxies recognizable with the two instruments. Consequently we may expect from (16) that

$$(17) \qquad \mathfrak{N}\,(\text{48-inch Schmidt})/\mathfrak{N}\,(\text{18-inch Schmidt}) = (10^{0.9} - 1)/(10^{0.3} - 1) = 7$$

while the observed ratio, from (14) and (15), is equal to 6.82.

As we have found previously, notably in the case of the clusters in Coma and in Cancer [18], [19], bright and faint galaxies are partly segregated. This is also true for the Corona Borealis cluster as shown in table III from the comparison of our counts with the two Schmidt telescopes.

The fact that the absolutely brighter nebulae show a greater tendency toward clustering than the fainter ones is in agreement with the segregation effects which are to be expected in stationary assemblies on the basis of Boltzmann's principle of statistical mechanics. The data in table III also clearly illustrate why some of the early investigators [11] derived quite erroneous luminosity functions for cluster nebulae since, because of the very small fields of the large reflectors, they sampled only the central parts of the large clusters where the bright galaxies predominate.

6.4. *Distribution of clusters of galaxies in breadth over the sky.* In section 2.2 a definition was given for clusters of galaxies, classifying them into three simple types designated as compact, medium compact and open. The following discussion deals with the distribution of these various types of clusters over the sky and with their populations and diameters as determined with the 48-inch Schmidt telescope. The population is simply taken as the total number of galaxies counted, minus the estimated number of background galaxies, using the procedure outlined in section 6.3 where the membership of the Corona Borealis cluster was discussed. On the other hand angular diameters γ are determined by using only the member galaxies in the first three brightest magnitude ranges. In order to make the diameter γ operationally quite definite the said brightest galaxies are counted only to that contour along which their number per square degree is twice the number of the correspondingly bright galaxies in the field. This means of course that the diameters which we list in table IV are considerably smaller than the diameters obtained when tracing the cluster members as far out into the general field as possible as we have done it in section 6.3 with the counts of galaxies in and around the Corona Borealis cluster as photographed with the 18-inch Schmidt telescope.

TABLE III

PARTIAL SEGREGATION OF BRIGHT AND FAINT GALAXIES IN THE CORONA BOREALIS CLUSTER

N_{bright} are the numbers per square degree of the member galaxies recorded with the 18-inch Schmidt. N_{faint} are the corresponding numbers of galaxies recorded with the 48-inch Schmidt within various rings after subtraction of N_{bright}.

Ring Limits in Minutes of Arc	N_{bright}	N_{faint}	$N_{\text{bright}}/N_{\text{faint}}$
0′- 1	10767	6969	1.54
1 - 2	6168	4314	1.43
2 - 3	4348	3991	1.09
3 - 4	2750	2724	1.01
4 - 5	1471	1882	0.79
5 - 6	1384	3642	0.38
6 - 7	1500	994	1.51
7 - 8	794	2273	0.35
8 - 9	280	1675	0.17
9 -10	561	947	0.54
10 -11	503	1189	0.43
11 -12	503	1292	0.39
12 -13	384	1079	0.36
13 -14	540	602	0.89
14 -15	316	1811	0.17
15 -20	110	1020	0.11
20 -25	90	513	0.17
25 -30	80	547	0.15
30 -40	31	412	0.08
40 -50	24	339	0.07
50 -60	21	78	0.27
60 -80	12	144	0.08

Clusters of galaxies on photographs made with different telescopes may become unrecognizable or all-together invisible for two reasons. In the first place the cluster

TABLE IV

Clusters of Galaxies on 48" Schmidt Plate E 70

Coordinates of center of plate (Epoch 1950) R.A. = $14^h22^m8^s$; Decl. = $+29°24'26''$
Brightest star at x = 27.4 cm. y = 72.6 cm.

Number	Population	Character	Diameter cm.	Distance	East ← x cm.	North ↑ y cm.
1	111	Medium compact	1.6	D.	23.7	84.2
2	61	Medium compact	0.8	E.D.	23.1	83.6
3	43	Medium compact	0.9	V.D.	7.4	84.7
4	97	Medium compact	1.4	V.D.	19.4	83.6
5	158	Medium compact	1.6	V.D.	21.4	82.3
6	84	Compact	0.9	E.D	16.5	80.9
7	51	Compact	0.8	E.D.	11.8	81.2
8	112	Medium compact	1.2	V.D.	32.2	80.0
9	263	Medium compact	2.5	D.	28.3	79.2
10	149	Medium compact	1.6	V.D.	26.2	78.8
11	91	Medium compact	1.3	V.D.	24.1	78.8
12	163	Compact	2.3	V.D.	9.7	74.6
13	127	Medium compact	1.4	V.D.	24.0	79.9
14	77	Compact	1.1	V.D.	31.5	76.7
15	106	Medium compact	1.6	V.D.	19.9	78.6
16	55	Compact	0.7	E.D.	11.8	79.6
17	98	Compact	1.3	D.	13.8	77.1
18	56	Compact	0.7	E.D.	34.1	75.9
19	197	Compact	1.7	V.D.	34.1	73.0
20	353	Medium compact	3.6	D.	32.4	73.5
21	68	Compact	0.8	V.D.	28.5	73.4
22	322	Open	3.5	M.D.	24.6	72.0
23	304	Compact	2.5	E.D.	29.4	67.8
24	205	Compact	1.8	E.D.	27.4	68.2
25	157	Open	2.7	M.D.	13.7	72.7
26	80	Medium compact	1.1	E.D.	3.0	70.5
27	236	Open	4.5	M.D.	18.7	69.1
28	144	Medium compact	1.6	V.D.	22.8	68.1
29	186	Compact	2.0	V.D.	5.1	66.8
30	155	Open	2.1	E.D.	2.5	66.8
31	137	Medium compact	1.3	V.D.	33.3	69.4
32	340	Very compact	2.4	V.D.	34.0	71.8
33	102	Medium compact	1.4	V.D.	35.5	65.4
34	113	Very compact	1.3	E.D.	32.9	65.5
35	99	Medium compact	1.5	V.D.	32.0	64.1
36	53	Compact	0.8	E.D.	18.0	66.6
37	75	Compact	1.3	V.D.	26.6	66.0
38	60	Compact	1.0	E.D.	31.4	65.3
39	56	Medium compact	0.8	V.D.	20.0	64.5
40	105	Medium compact	1.2	V.D.	11.0	64.6
41	51	Compact	0.8	E.D.	7.1	63.1
42	76	Compact	1.0	V.D.	8.2	62.2
43	100	Medium compact	2.3	D.	3.4	61.1
44	280	Medium compact	2.7	D.	10.8	60.3
45	146	Medium compact	2.0	D.	13.0	59.5
46	122	Medium compact	1.4	V.D.	31.8	59.4
47	93	Compact	1.1	E.D.	33.0	63.3

TABLE IV—*Continued*

Number	Population	Character	Diameter cm.	Distance	East ← x cm.	North ↑ y cm.
48	90	Compact	1.4	E.D.	33.1	62.7
49	110	Compact	1.6	V.D.	30.8	63.6
50	106	Medium compact	1.1	V.D.	33.4	59.1
51	75	Medium compact	1.2	E.D.	21.8	56.0
52	282	Medium compact	3.0	V.D.	7.0	55.4
53	158	Medium compact	1.8	D.	16.9	56.5
54	37	Compact	0.6	E.D.	23.6	57.0
55	88	Medium compact	1.3	V.D.	21.9	54.1
56	185	Medium compact	3.5	Near to M.D.	11.4	52.0
57	75	Medium compact	1.4	D.	2.4	51.7
58	112	Medium compact	1.8	D.	20.1	53.2
59	184	Compact	2.0	D.	18.2	51.3
60	61	Compact	0.9	E.D.	17.9	52.2
61	130	Open	3.3	Near	18.5	52.7
62	50	Compact	0.8	E.D.	27.1	52.9
63	65	Medium compact	1.0	V.D.	21.8	50.7
64	81	Medium compact	1.1	V.D.	3.9	69.5

Total number of galaxies on plate = 8206 galaxies
Average number of galaxies per cluster = 128.22 galaxies

galaxies may be too faint to register at all, although if they could be seen, the diameters of the cluster would be quite finite. This means that clusters of a given absolute type vanish from sight if they are at distances at which their angular diameter becomes smaller than some critical γ_0. The population $\mathfrak{N}$ of a cluster is therefore a function of $(\gamma - \gamma_0)$. It was found previously [16], [17] that on the average and as a first approximation it is

$$\mathfrak{N}_{Cl} = \text{constant} \times (\gamma - \gamma_0). \tag{18}$$

This relation, when transformed to the scale of magnitudes, leads to the integrated luminosity function (16).

Clusters of galaxies may in the second place become unrecognizable because of the interference of intergalactic (or interstellar) dust. Although many of the member galaxies of a cluster may appear on the photographic plates others will not show because of local obscuration. As a result of such disrupting effects a large cluster of galaxies may not any more be identifiable as such but take the appearance of a number of small groups and clouds.

In table IV one of our work sheets is reproduced giving the number of clusters of galaxies on one 48-inch Schmidt plate covering about forty square degrees. Because of the considerable apparent redness of distant galaxies, 103 a–E plates were used which were partly taken by the author and which partly belong to the Sky Survey supported by a grant of the National Geographic Society.

Positions are given in centimeters with the coordinates of the brightest star fixing the zero point. Diameters of the clusters are likewise expressed in centimeters. For conversion to angles it is to be remembered that on the 48-inch Schmidt plates 1 mm = 67.1 seconds of arc.

The distances indicated in column 5 of table IV are crude estimates only. On the old distance scale they correspond approximately to: "Near" equals distances smaller than seventy million light years (for instance, the clusters in Virgo, Cancer, Perseus, Coma, etc.), "Medium Distant" = MD and is equivalent to the approximate range from 70 to 150 million light years (for instance, the Corona Borealis cluster); "Distant" = D, "Very Distant" = VD, and "Extremely Distant" = ED describe successively the ranges from 150 to 250, 250 to 350, and greater than 350 million light years, respectively.

TABLE V

NUMBERS ν OF CLUSTERS OF GALAXIES PER FIELD OF 40 SQUARE DEGREES PHOTOGRAPHED WITH THE 48-INCH SCHMIDT TELESCOPE

N_{galaxies} = total number of cluster galaxies on the plate.
$\mathfrak{N}_{\mathrm{Cl}} = N_{\mathrm{galaxies}}/\nu$ = average number of member galaxies per cluster for the individual plate.
$\overline{\mathfrak{N}}_{\mathrm{Cl}}$ = average number of member galaxies per cluster for all 26 plates.

Number	Field centered at				
	R.A	Decl.	ν	N_{galaxies}	$\overline{\mathfrak{N}_{\mathrm{Cl}}}$
1	$10^h\ 27^m\ 05^s$	+11° 30′ 30″	41	5366	131
2	10 29 07	+17 30 41	53	6053	120
3	10 53 01	+11 30 24	54	7296	135
4	10 53 04	+17 29 37	34	3711	109
5	11 05 20	+41 29 17	64	6374	100
6	11 16 55	+11 28 36	36	4301	120
7	11 17 00	+17 28 49	45	6611	147
8	11 21 05	+29 28 45	35	4918	141
9	12 04 52	+5 28 15	30	3770	126
10	12 04 49	+11 27 15	15	1474	98
11	12 12 48	+29 28 18	22	4007	182
12	12 28 51	+5 28 29	9	1237	137
13	12 28 48	+11 28 42	17	2587	152
14	12 28 47	+17 28 30	56	5759	103
15	12 38 43	+23 28 39	11	914	83
16	12 38 40	+29 28 42	24	2768	115
17	13 04 37	+23 29 25	20	2429	121
18	13 04 33	+29 29 25	8	1084	136
19	13 30 31	+23 30 35	34	4476	132
20	13 30 25	+29 30 33	24	2828	118
21	13 56 20	+23 32 06	36	4616	128
22	13 56 17	+29 32 06	54	9224	171
23	14 22 19	+23 34 10	41	5552	135
24	14 22 08	+29 24 26	64	8206	128
25	15 13 59	+29 38 40	55	9952	181
26	15 40 07	+23 41 39	39	2678	69
Total			921	118191	$\overline{\overline{\mathfrak{N}}}_{\mathrm{Cl}} = 128.33$

In table V and figure 7 some of the essential data on the distribution of clusters of galaxies are summarized. From the tables IV and V, as well as from figure 7, the following conclusions can be drawn. A remarkable number of the rich clusters are compact and spherically symmetrical, similar to those in Hydra (I), Cancer, Perseus, Coma, and Corona Borealis. This observational fact greatly strengthens the concept of a stationary universe in which there was ample time available to allow the great aggregates of galaxies to reach statistically stable configurations. There is no evidence whatever, to the limit of the 200-inch telescope, that distant rich and compact clusters have any morphological characteristics which differ from the nearby ones. Richness of population, radial distribution, segregation effects and so on, appear to be independent of the angular diameters of these clusters and of the apparent magnitudes of their brightest members. All phenomena which might suggest evolution of the universe such as the nebular redshift and the enhanced reddening found by Stebbins and Whitford [20] are therefore subject to a thorough reappraisal.

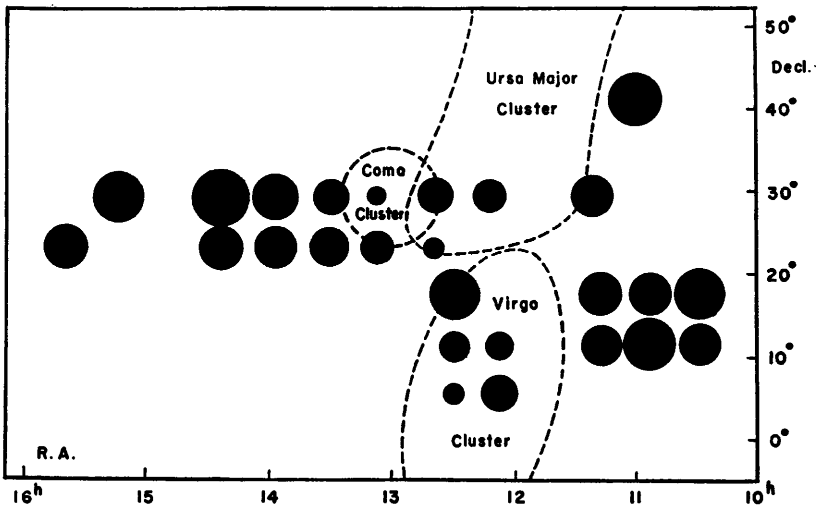

FIGURE 7

Distribution of 921 clusters of galaxies and intergalactic obscuration. The areas of the disks are proportional to the number of clusters found with the 48-inch Schmidt telescope in areas of 40 square degrees in the 26 fields listed in table V. The largest and the smallest disk stand for 64 and 8 clusters per 40 square degrees, respectively. The shaded lines roughly indicate the areas occupied by the nearby clouds and clusters of galaxies in Ursa Major, Virgo and Coma.

We further notice from our data the important fact that the *average population per cluster* as determined from the 48-inch Schmidt plates does not vary greatly from one field to another. In other words the deviations $(\mathfrak{N}_{cl} - \overline{\overline{\mathfrak{N}}}_{cl})$ are hardly greater than would be expected for a random distribution. This again is a strong indication for stationary conditions in the universe. On the other hand, the total number of clusters and of cluster galaxies varies considerably from plate to plate. These facts suggest either superclustering or the existence of intergalactic dust. The latter assumption is to be preferred since, although the total number of clusters and of cluster galaxies is very variable from plate to plate, their ratio is roughly constant. Also, as we have discussed in the sections 6.1 and 6.2 the distribution of galaxies both in breadth and in depth can be most easily explained by intergalactic obscuration. Furthermore, there is a very great deficiency both in the numbers of galaxies and of clusters of galaxies in those fields which contain large nearby aggregations of galaxies. This is quite apparent from the pictorial representation of the results of table V in figure 7. The number of distant clusters of galaxies per 48-inch

Schmidt plate, covering each an area of about forty square degrees, is by far the smallest (8 clusters) in the region of the very compact Coma cluster. Somewhat smaller deficiencies are likewise found in the fields including the Virgo cluster and the Ursa Major cloud. Similar results have been obtained from a preliminary exploration of the areas near the clusters in Cancer and Pegasus. On the other hand, the maximum number of distant clusters is found in the large gaps between the nearby rich clusters. In these gaps we observe that the total number of clusters per unit area varies in a more or less random way from plate to plate and that the cluster centers are randomly distributed. There is so far no observational evidence for superclustering, a fact which is in keeping with our theoretical expectations provided that gravitational interactions are transmitted with a finite velocity equal to, or not too different from, the velocity of light. Off and on several clusters appear to be closely bunched, of course. There are, for instance, the groups of clusters in the Perseus Pisces region (p. 28 in [21], [4]) as well as in Corona Borealis. The frequency and compactness of such aggregates is, however, not any greater than should be expected for that of local accumulations in a random distribution of noninteracting objects.

In continuation of the researches reported here it is intended to analyse all of the plates of the 48-inch Schmidt Sky Survey in order to prove conclusively that intergalactic dust is not only dispersed throughout the whole visible universe but that it is partially concentrated within the large clusters of galaxies. In order to calibrate the degree of obscuration and the effects of possible intergalactic reddening, the effects of intergalactic dust will be compared with those of interstellar dust. This comparison can be achieved through simultaneous counts of stars, galaxies and clusters of galaxies in regions of low galactic latitudes where quantitative data on the specific actions of interstellar dust are available.

It is perhaps not superfluous to mention that in addition to galaxies and clusters of galaxies, stars were also counted in several of the regions investigated and shown in figure 7. In the crucial regions around the northern galactic pole faint stars are nearly randomly distributed within each area of 40 square degrees covered by one 48-inch Schmidt plate. Any deviations from randomness, although they certainly exist, are far too small to account for the irregularities in the distribution of galaxies and of clusters of galaxies which we have discussed.

6.5. *Counts of clusters of galaxies in depth; numbers as a function of angular size.* One of the most powerful tools of dimensionless morphology lies in the analysis of the distribution of angular sizes of various objects. If one knows or suspects for instance that certain objects are distributed randomly and uniformly throughout a stationary Euclidean space and that these objects are all of the same absolute size, the number of such objects within a given solid angle which subtend a linear angular diameter γ lying in the interval γ to $\gamma + \Delta\gamma$ is equal to

$$N_\gamma \Delta\gamma = \text{constant} \times \Delta\gamma/\gamma^4 \tag{19}$$

provided that no optical obscuring effects or fading because of distance effects remove any of the objects from visibility. For a flat *expanding* universe of the type described by Einstein and de Sitter [14] we should expect

$$N'_\gamma = \text{constant}/\gamma^4[1 - \bar{v}(\gamma)/c]^3 \tag{20}$$

where $\bar{v}(\gamma)$ is the average apparent velocity of recession for objects whose average angular diameter is γ.

TABLE VI

Distribution of the Angular Diameters γ of 706 Clusters of Galaxies Located in Areas Totaling 600 Square Degrees in Regions Around the Northern Galactic Pole

Those regions listed in table V were chosen which contain the largest numbers of clusters thus indicating a relative minimum of intergalactic obscuration. The first column gives the ranges $\Delta\gamma$ of the diameters γ in minutes of arc. In the second column are the numbers $\mathcal{N}^*$ of clusters for the whole area of 600 square degrees which lie in the various ranges $\Delta\gamma$, as identified from 48-inch Schmidt plates. The final column lists the function $N_\gamma = \mathcal{N}^*/6\Delta\gamma$ which gives the average number of clusters per one hundred square degrees in a range of angular diameters equal to one minute of arc around the midpoint of the range $\Delta\gamma$.

$\Delta\gamma$	$\mathcal{N}^*$	$\mathcal{N}_\gamma$
1800′ -300′	(see text)	(3.33×10^{-6})
300 - 60	10	6.96×10^{-3}
60 - 30	67	0.372
30 - 15	250	2.78
15 - 7.5	273	6.07
7.5 - 4	80	3.81
4 - 2	27	2.25

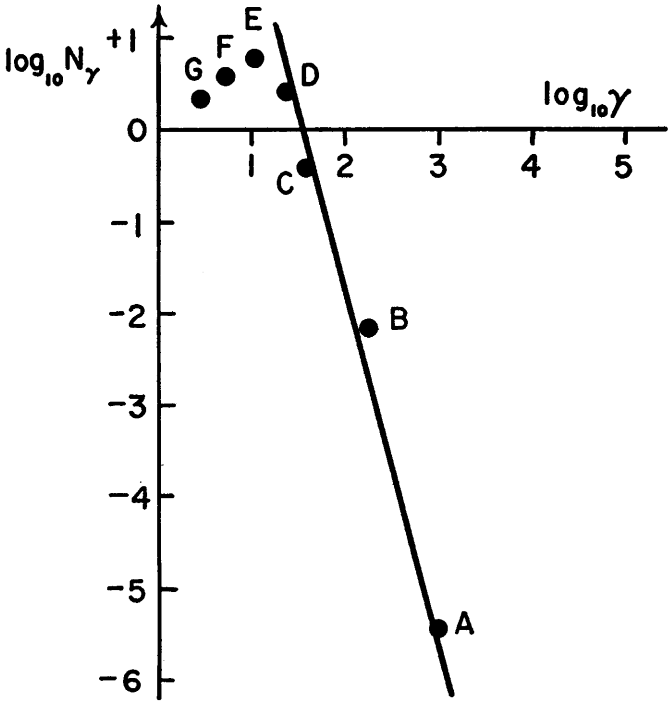

Figure 8

Distribution of 706 clusters of galaxies in dependence of their angular diameters. $N\gamma\Delta\gamma$ is the number of clusters per one hundred square degrees whose angular diameters lie in the range γ to $\gamma + \Delta\gamma$.

The question therefore arises whether there are any objects about whose absolute nature we know enough to make them suitable for a statistical investigation of the features expressed by the relations (19) and (20). The galaxies themselves clearly will not do for our purpose. In the first place, diameters of galaxies can only be determined from criteria referring to surface brightness which become impossible of application for very distant objects because of the small size of the images. In the second place, we must have some *a priori* criterion regarding the probable absolute diameters of the objects whose apparent diameters we wish to compare. No such criterion is known in the case of galaxies. The situation, however, is quite different with the clusters of galaxies. Diameters of clusters are indeed easily measurable to all distances of interest to us by simply applying the operations described in section 6.4. Furthermore it is also possible to determine the morphological structure, such as spherical symmetry, the total population in the first three magnitude classes of the member nebulae, the degree of segregation of the nebulae in these three classes as well as the types of the brightest nebulae for various clusters. Choosing for our statistics of angular diameters only clusters for which the mentioned properties are equal within narrow limits, we achieve the greatest possible certainty that these clusters are of the same absolute dimensions. From Hubble's results [22] on the linear dependence of the logarithm of the universal redshift $\Delta\lambda/\lambda$ on the photovisual magnitudes of the tenth brightest nebula of large clusters, it appears furthermore that not only the geometrical dimensions of morphologically similar clusters are the same but that the equality applies to some of the physical characteristics as well. This consequently means that from counts of nebulae, from their geographical distribution within the clusters, and from the distribution in apparent magnitudes and the relative frequency of types, we can judge the absolute equality of clusters located at very different distances. Once several thousands of clusters have been observationally analysed by the methods described in the present study it will then be quite possible to determine the function N_γ separately for different classes of clusters such that the members of each class may be assumed to be of equal absolute dimensions. For the present we have applied a more primitive test by choosing for our analysis those fifteen fields listed in table V which appear the least affected by any possible intergalactic obscuration caused by the dust in the three nearby large clusters in Virgo, Coma, and Ursa Major. The mentioned fifteen fields (Numbers 1, 2, 3, 4, 5, 6, 7, 8, 14, 21, 22, 23, 24, 25, 26 of table V) contain 706 rich clusters. The statistical distribution of their angular diameters, as determined by the operational procedure which we have defined in section 6.4 is shown in table VI. In figure 8 the results of our investigation on the distribution of angular diameters of clusters of galaxies are represented graphically.

The point A of the figure is naturally quite uncertain since there are only a few clusters of galaxies of large apparent diameters. The possible relative fluctuations to be expected for such small numbers are of course considerable. In agreement with the operational definition of clusters of galaxies and of their diameters we have admitted for our purposes within the unobscured 20,000 square degrees around the two galactic poles only the Virgo cluster into the range of γ from 1800 to 300 minutes of arc. The value of N_γ chosen for point A is consequently equal to 3.33×10^{-6}.

In analysing figure 8 we notice in the first place that the points A, B, C and D lie as closely on the theoretical straight line for a nonexpanding universe (19) as may be reasonably expected. It is also noteworthy that the analysis which was carried out some time ago [16], [17] on the basis of only 156 clusters checks very closely with the fuller analysis of the 706 clusters presented here. In both cases the rise of N_γ' with decreasing values of γ, above the straight line shown, which we should expect from the theory of the expanding universe, does not materialize. This is in agreement with the fact that, except for the spectral redshift itself, none of the effects predicted for the case of an expanding universe has as yet been found to exist. The investigations presented here will be extended in the near future to the analysis of several thousand clusters and a definite decision between the concepts of expansion and nonexpansion should then become possible. The functions N_γ and N_γ' in the equations (19) and (20) show differences which are greater than any other so far pointed out. For an apparent velocity of recession corresponding to $v/c = 1/6$ the ratio N_γ'/N_γ is equal to about two. In relation to the data shown in figure 8 this means that to the left of the point B the points C, D, E should rise above the straight line shown, and somewhere between C and D the observational point should lie by an amount log 2 above this line.

The location of the points E, F, and G is of course due to the failing power of the telescope and may also be caused partly by the intergalactic dust which presumably is spread thinly throughout the whole visible universe.

7. Dimensional morphology

As stated before, we are not in the present study concerned actively with the physical properties and the absolute character of the large scale distribution of matter. In order to round out the perspectives achieved with the methods of dimensionless morphology it is nevertheless desirable to discuss briefly some of the essentials of a dimensional morphology.

7.1. *Phenomenological relations.* Among the many possible relations we mention briefly five on which the author as well as other investigators have spent considerable efforts, and on which work is being continued.

The first of these relations is concerned with the number N_m of galaxies per square degree of the sky in various ranges of apparent magnitude m. If galaxies were uniformly and randomly distributed throughout a nonexpanding flat universe, and if there were neither absorption of light nor reddening because of interfering dust or because of the effects of the universal redshift, we should expect N_m to be given by

$$\log_{10} N_m = 0.6\, m + \text{constant}. \tag{21}$$

Observationally established deviations from this formula would furnish us with pertinent information on the clustering of galaxies, on interstellar and intergalactic absorptions, and on the nature of the universal redshift. Hubble [11] and others undertook large surveys of distant galaxies to check the deviations from (21). In these early researches it was unfortunately not realised that, because of the tremendous variation in absolute size and surface brightness of various classes of galaxies, the relation (21) cannot be satisfactorily checked with counts alone but that it is actually necessary to determine individually the apparent magnitudes of all objects introduced into the statistics leading to the relation between N_m and m. This task, which is of enormous proportions, has been worked upon during the past decade by the author and his collaborators, in particular by Dr. E. Herzog and Mr. P. Wild. The work, which has been partly supported by a grant from the Office of Naval

Research, includes only galaxies brighter than the photographic magnitude $m_p = +15.2$. The results are expected to be of importance for the analysis of nearby clusters, the mapping of the obscuration caused by interstellar and intergalactic dust and, above all, for the determination of the luminosity function of galaxies which at the present is only poorly known. There is, however, little hope that analogous accurate surveys can be carried out as far as the twentieth apparent magnitude and any use of a relation of the type (21) for the purpose of achieving a decision for or against the concept of an expansion of the universe is thus futile.

A second type of survey which we have in mind is that now being carried out by radio astronomers. It should be of the greatest interest to compare their "isophotes" both for the continuous radiation and for the 21 cm. wave with isopleths and isophotes derived from the accurate photographic surveys of galaxies to the fifteenth apparent magnitude.

A third most important relation is that between the redshift $\Delta\lambda/\lambda$ and the angular diameters γ of equal clusters of galaxies lying at different distances. For a nonexpanding flat universe and small values of $\gamma \ll \pi$ we should have

$$\Delta\lambda/\lambda = \text{constant}/\gamma \tag{22}$$

provided that the redshift is strictly proportional to the distance. The few data which are available so far check this relation quite closely. Unfortunately, many more observational values of $\Delta\lambda/\lambda$ will have to be obtained before we can decide whether or not there are any significant deviations from (22) which might indicate the effects either of an expansion of the universe, of a systematic evolution of clusters of galaxies, or of a reddening of light, which in a flat nonexpanding universe is not exactly proportional to the distance.

Fourthly, a most intricate test for various cosmological theories can be made by analysing the width of the lines in the spectra of the member galaxies of distant clusters of galaxies. Morphologically speaking the following four possibilities exist: (1) the spectrum may be shifted toward the red as a whole without any change in the width of any of the lines or bands; (2) both the absorption lines and the emission lines may for instance be shifted and simultaneously widened; (3) only the absorption lines are broadened while the emission lines remain sharp; and (4) absorption lines retain their original width while the emission lines broaden. Peculiarly enough there are indications that the case (3) may be the one corresponding to actual reality, which, if correct, may necessitate a theory of the universal redshift quite different from any as yet proposed.

The fifth phenomenological law which we have in mind is that relating the redshift to the apparent magnitude of the nth brightest nebula in large clusters of nebulae. Hubble [11] and Humason originally chose $n = 5$. More recently Hubble [22] changed over to the tenth brightest nebula in clusters using its apparent photovisual magnitude m_{pv} (corrected for the redshift) rather than the photographic magnitude. The observations available could be represented by

$$\log_{10} v_s = 0.2 m_{pv}\,(\text{tenth nebula}) + 1.16 \tag{23}$$

where $v_s = c\Delta\lambda/\lambda$ is the *symbolic* velocity of recession, with c equal to the velocity of light. Except for the data on two clusters in Pegasus the available observations on a dozen clusters very closely check the relation (23). These observations have been recently continued by Humason, Pettit, Baum and Sandage of the Mount Wilson Observatory. The results which were reported by A. Sandage at the AAAS meeting in Berkeley on December 27, 1954, again very closely approximate the relation (23). As a consequence of these researches some cosmologists are likely to be lured into an overdiscussion of the now available observations. There have even been attempts to deduce the curvature of the observational curve (23). In order to call attention to the danger of drawing important conclusions from statistically insufficient data the following circumstances should be pointed out. The differences to be expected in the form of (23) for the cases of an expanding and a nonexpanding universe amount to a few tenths of magnitude at most, unless the range in v_s is greatly extended. There are, however, a dozen or more sources of systematic and of accidental errors involved which make it quite impossible to achieve the accuracy which would be necessary to decide between different cosmological theories. Some of these sources of errors are as follows. A scale of magnitudes must be used which obviously is not mathematically accurate. The measured magnitudes of the galaxies must be corrected for zenith distance, for interstellar and intergalactic absorption, for the effects of the redshift itself and for the Stebbins-Whitford reddening. The last two corrections can only be applied if assumptions are made as to the spectral characteristics of brightest nebulae in every cluster and as to the laws governing the effects of reddening. A further fundamental difficulty resides in the fact that the total brightness of a galaxy seems to increase continually as one includes larger and larger areas around its center, this increase being affected, however, by conditions of seeing and by background radiation either due to the sky glow or to light from distant sources such as neighboring overlapping nebulae. Also, in any given cluster it is impossible *a priori* to be certain that any specific nebula is a member of the cluster. If therefore, without eliminating any "unsuitable" nebulae, one included all which were initially judged by different observers to be the first ten brightest nebulae in a cluster it is likely that a far greater spread of points would result than that given by the above-mentioned Mount Wilson investigators. Finally we must keep in mind that the clusters used by these observers include a wide variety of types both as far as total population and types of member nebulae are concerned. For instance the Virgo cluster is open and contains many spirals while the Coma cluster, which is compact and richer in population, has hardly any spirals among its member nebulae. Also, as far as we now know, the range of apparent (symbolic) radial velocities of the member galaxies of a large cluster covers several thousand kilometers per second, which again should add to the spread of the observational points. The rather surprisingly small deviations from the relation (23) reported by the mentioned observers must thus be considered as largely fortuitous, and a considerably larger scattering is to be expected if the same investigations are being carried out on more clusters and by several groups of entirely independent observers.

7.2. *Absolute physical characteristics of galaxies and of clusters of galaxies.* Absolute distances and luminosities of galaxies have been derived for several decades from assumptions about the absolute brightness of the brightest stars in stellar systems as well as about the absolute brightness of certain types of variables and of common novae. It might have been realized from the start that these methods of determining absolute characteristics of extragalactic objects are largely based on anthropomorphic reasoning, and it is therefore not surprising that the distance scale had to be changed recently and that it now is in a continuous flux of change. The question thus arises if no scientifically sound methods are available, which not only dispense with any *a priori* assumptions about the absolute nature of distant stars and galaxies, but which allow us to determine the distances of these objects even though an unknown amount of interstellar and intergalactic dust may introduce partial obscuration and reddening. Successful answers can again be found within the realm of dimensionless morphology on the basis of which the writer has recently developed an absolute method of distance determination for some not too remote galaxies in which supernovae have made their appearance in the past few decades [23].

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