Preface

The conference "Algebraic and Arithmetic Structures of Moduli Spaces" was held in September 2007, at Sapporo. Twenty talks were delivered by invited speakers from USA, UK, Germany, Holland, India, Russia and Japan. The subjects of the talks are mainly on algebraic geometry and arithmetic geometry, but also on complex geometry, though many of them were focused more or less on moduli spaces.

The plan to publish this proceedings was made at the very beginning of the preparation of the conference. Although all the speakers of the conference were invited to contribute their articles to this proceedings, many of the speakers were unfortunately unable to do so partly because of lack of time. All the papers submitted by the deadline were sent to the referees and the proceedings here presents eleven papers which have undergone rigorous refereeing. We would like to thank all the referees for their assistance.

The topics that are discussed in the articles are diverse in nature such as class field theory, zeta functions, moduli of arithmetic vector bundles, moduli of complex vector bundles, moduli of abelian varieties and theory of display which relates to arithmetic theory of moduli of abelian varieties, moduli of Fermat varieties and some topics on cubic threefolds. This reflects the atomosphere of the conference pretty well.

The articles of Deninger and Werner, Rajan, Pappas and Rapoport, Weng and Yoshida are more or less around class field theory, Galois representation or Langlands' program. Meanwhile, the Narashimhan and Seshadri correspondence is, for a given compact Riemann surface R, the bijective correspondence between irreducible unitary representations of its fundamental group $\pi_1(R)$ and stable vector bundles of degree zero over R. Thus stable vector bundles of degree zero will give Galois representations once we admit an analogy between $\pi_1(R)$ and the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$.

If one pursues Galois representations in this direction, it would be natural to generalize Narashimhan and Seshadri correspondence into an arithmetic version of it. This would be the major interest of the article by Deninger and Werner. In their paper, Deninger and Werner define a functorial parallel transport, which gives rise to a representation of the étale fundamental group of an algebraic curve over $\overline{\mathbb{Q}}_p$ on the fiber of a certain semistable vector bundle. This gives almost a converse to an earlier result of Faltings, which says that p-adic representations of the fundamental group of a p-adic curve give rise to semistable p-adic Higgs bundles of slope zero.

In his paper, Rajan addresses many ambitious questions, geometric or arithmetic, about L-functions. The paper of Pappas and Rapoport also address many interesting questions on group scheme bundles over curves. The recent access of theoretical physics to algebraic or arithmetic geometry is reflected in the papers of Pappas and Rapoport, and Weng. The conformal blocks in the cohomology sheaves over the certain moduli spaces will give a natural realization of Galois representations. In this context, their papers remind us of two possible ways of understanding the word CFT as class field theory and conformal field theory.

The paper of Yoshida reproves a part of the local Langlands' correspondence for GL_n, which has been proved by Harris–Taylor by a global approach. In contrast with Harris–Taylor, Yoshida's approach is purely local, making a precise study of the local deformation space of a certain formal group with Drinfeld level structure.

Meanwhile, the paper of van der Geer and Kouvidakis gives a new proof of nonrationality of general cubic hypersurfaces in $\mathbb{P}^4$ in arbitrary characteristic. The paper of Looijenga studies primarily the period maps of cubic fourfolds, and cubics of lower dimension.

The other papers by Yoshioka, Nakamura and Zink concern primarily the moduli theory of vector bundles over K3 surfaces, or abelian varieties respectively. The paper of Yoshioka generalizes Nakajima's construction of an sl_2-action on the homology groups of certain varieties to a more general Lie algebra action on the homology group of the moduli space of stable sheaves over any fixed projective K3 surface.

The paper of Nakamura presents the second canonical compactification of the moduli space of abelian varieties, and proves that there is a canonical morphism from the second compactification (2009) to his first compactification (1999), which induces an isomorphism of their normalizations. On the other hand, the paper of Vasiu and Zink discusses p-divisible groups over certain natural classes of rings over the Witt ring by applying the theory of "display" which Zink developed to study the moduli space of abelian varieties, especially around the supersingular loci. Their paper recovers the results due to Breuil and Kisin.

The longest among all the papers is the one by Weng: stability and arithmetic, which presents an ambitious project on general class field theory by combining a traditional p-adic approach using Fontaine's theory of p-adic Galois representations and a somewhat new approach to the problem using Mumford's stability. As a by-product of his approach, he discovered a new class of zeta functions associated with all pairs of a reductive algebraic group and its maximal parabolic subgroup. As was discussed in the other article of Weng, some of them have been proved to

satisfy Riemann Hypothesis, and in general, all of them are conjectured to satisfy it.

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All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.