Preface

Tracking back through the history, the vague shade of Gröbner bases already appeared in the famous paper[1] by F. S. Macaulay in 1927. The problem which Macaulay studied was to find a combinatorial characterization of Hilbert functions of residue class rings of the polynomial ring modulo homogeneous ideals. Macaulay succeeded in discovering the fundamental fact that the Hilbert function of the residue class ring of an arbitrary homogeneous ideal coincides with that of a certain monomial ideal. Macaulay's work had stimulated the algebraic study on enumerative combinatorics and promoted the birth of the historic area called "Commutative Algebra and Combinatorics," which originated in the work by Richard Stanley in 1975 on the upper bound conjecture for spheres by using the algebraic theory of Cohen–Macaulay rings.

The modern definition of Gröbner bases was independently introduced by Heisuke Hironaka in 1964 and Bruno Buchberger in 1965. Hironaka caught the idea of standard bases in the process of solving the outstanding problem, the resolution of singularities of algebraic varieties. On the other hand, Buchberger created the notion of Gröbner bases in his dissertation whose research topic had been given by his advisor Wolfgang Gröbner. Hironaka's standard bases work in the local ring, while Buchberger's Gröbner bases work in the polynomial ring. Apart from discussions of difference between the idea of standard bases and that of Gröbner bases, it turned out that Buchberger algorithm had opened the fascinating research area called "Computer Algebra."

After the pioneering work of Hironaka and Buchberger, however, for about twenty years, Gröbner bases had been out of the limelight. A turning point arose in the middle of 1980s, when David Bayer and Michael Stillman developed the computer software **Macaulay**, which has a great influence on computational aspects of commutative algebra and algebraic geometry. Since the theory of Gröbner bases was indispensable for developing **Macaulay**, Gröbner bases became common knowledge for researchers on commutative algebra and algebraic geometry.

The entry of Gröbner bases into the world of applied mathematics was achieved by Conti and Traverso in 1991, who proposed an algorithm to solve problems on integer programming by means of Gröbner bases of toric ideals. Toric ideals spread rapidly in the middle of 1990s. As one of the effective techniques to compute the dimension of the solution

[1] F. S. Macaulay, Some properties of enumeration in the theory of modular systems, *Proc. London Math. Soc.* **26** (1927), 531–555.

space of a hypergeometric equation, in 1994, Gel'fand, Kapranov and Zelevinsky introduced the notion of regular triangulations. It was shown by Sturmfels that the Stanley–Reisner ideal of a regular triangulation is just the radical of an initial ideal of a toric ideal. As a result, Gröbner bases of toric ideals turned out to be a brilliant bridge between the theory of monomial ideals and that of regular triangulations of convex polytopes. Since then, the algebraic theory of regular triangulations of convex polytopes has developed dramatically.

The study of Gröbner bases in the ring of differential operators started gradually in 1980s. A breakthrough was done by Oaku, who created new and effective algorithms on D-modules based on Buchberger algorithm. Since regular triangulations originated in the study of hypergeometric equations, the algebraic development of toric ideals naturally had a great influence on the study of hypergeometric equations.

An epoch-making application of Gröbner bases to statistics originated in the work by Diaconis and Sturmfels in 1998. When Markov chain Monte Carlo method is achieved in the examination of a statistical model, to find a Markov basis is required. It was shown by Diaconis and Sturmfels that a Markov basis corresponds to a system of generators of the toric ideal arising from a statistical model. Later, the new and exciting research area called "Algebraic Statistics" was born and has been developing rapidly. Algebraic statistics supplies commutative algebra with new problems and, conversely, toric ideals studied in commutative algebra supply algebraic statistics with new statistical models.

Taking the above background into consideration, in order to promote the international exchange of the study on Gröbner bases, the 8th Mathematical Society of Japan Seasonal Institute (MSJ-SI) entitled "Current Trends on Gröbner Bases" with the subtitle "The 50th Anniversary of Gröbner Bases" was held at Hotel Nikko Osaka, Osaka, Japan, during 1 – 10 July 2015. The organizing committee consisted of Satoshi Aoki (Kagoshima University), Takayuki Hibi (Osaka University, Chair), Masayuki Noro (Rikkyo University), Hidefumi Ohsugi (Kwansei Gakuin University), Nobuki Takayama (Kobe University) and Akimichi Takemura (The University of Tokyo). The present volume plays a role as the Proceedings of the conference.

The overall schedule of the 8th MSJ-SI 2015 is worth keeping a record. The first half, 1 – 4 July 2015, was a school for graduate students and the latter half, 6 – 10 July 2017, was a conference for researchers. There were 93 participants and among them 47 participants came from abroad. In the break of Sunday 5 July, a day trip to Mount Koya (Kōyasan), which is the center of Shingon Buddhism, an important Buddhist sect which was introduced to Japan in 805 by Kobo Daishi,

also known as Kukai, was organized. There was a welcome party in the evening of Monday 6 July as well as an official banquet in the evening of Thursday 9 July. A half day trip to each of Kyoto, Nara and Kobe was arranged in the afternoon of Wednesday 8 July.

In the school, the following four series of expository lectures were presented: "Algebraic Problems in Structural Equation Modeling" by Mathias Drton; "Algorithms for D-modules, Integration, and Generalized Functions" by Toshinori Oaku; "Computing in Algebraic Geometry — Sheaf Cohomology and Its Applications in Geometry and Physics" by Michael Stillman; "Identifiability Problems in Statistics and Biology" by Seth Sullivant. Each of the series of expository lectures consisted of three 60-minute lectures together with one 60-minute exercise class.

In the conference, the attempt was made to organically disperse the invited talks into the three different aspects of Gröbner bases; theory (commutative algebra and algebraic geometry), application (algebraic statistics) and computation (D-modules and computational mathematics). The invited speakers were as follows: Bernd Sturmfels, Michael Stillman, Hyungju Park, Thomas Kahle, Toshinori Oaku, Mathias Drton, Seth Sullivant, Ezra Miller, Bruno Buchberger, Jürgen Herzog, Anna M. Bigatti, Anton Leykin, Dongming Wang, Viktor Levandovskyy, Mateusz Michałek, Anders N. Jensen, Caroline Uhler, Sonja Petrović, Raimundas Vidunas, Eva Riccomagno, Constantin Siriteanu and Henry Wynn. In addition, a session of contributed talks was organized from the afternoon through the night of Tuesday 7 July, in which twenty one 20-minute talks were given. At the end of Buchberger's talk in the morning of Wednesday 8 July, he made a highly amusing remark about the birth of the modern theory of Gröbner bases. The atmosphere of the conference had a cooperative spirit between the different areas.

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Takayuki Hibi

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.