Preface

(1) This volume is a collection of thirteen papers written mainly by the invited speakers of the following:

1. International Symposium, "Algebraic Combinatorics (Fukuoka, 1993)."

2. Research Project: "Algebraic Combinatorics," Research Institute for Mathematical Sciences (RIMS), Kyoto University, April 1994–March 1995.

More detailed information concerning these conferences will be given at the end of this preface, including a list of the invited speakers and members of the organizing committees. Many of the papers in this volume are closely connected with the talks given at one or the other of these conferences. Strictly speaking, however, this collection of works does not constitute the proceedings of the above conferences. Each author was asked to contribute a paper with the understanding that this volume is independent of the conferences.

(2) Before stating the philosophy behind editing this volume and describing each of the papers contained herein, let us briefly mention our personal view on algebraic combinatorics. The term 'algebraic combinatorics' has various meanings and has been used in somewhat different ways by different mathematicians. However, it seems that there is a common understanding of the term when interpreted in a wider sense, as is seen in the 1991 Mathematics Subject Classification number 05E and in the journal name *"Journal of Algebraic Combinatorics"*, the first volume of which was published by Kluwer Academic Publishers in 1992.

One of the editors of this volume (Bannai) originated the use of this term in the late 1970's [B1], and he and Tatsuro Ito wrote a book by that title in 1984 [BI1]. In that book the term algebraic combinatorics is defined to mean "combinatorial representation theory" or "group theory without groups". As described there, we believe that the work of Delsarte [D] symbolizes the start of algebraic combinatorics, or at least algebraic combinatorics in the sense we understand it. Admittedly, there are many other important works, both preceding and contemporary with the work of Delsarte [D], which served as the start of algebraic combinatorics, for instance [Bi]. However, Delsarte's way of looking at many combinatorial problems in the framework of association schemes and combining design theory and coding theory in a single framework was a remarkable new approach which has been extremely successful. The concept of association schemes was defined by Bose and Shimamoto [BS] and was studied algebraically by Bose and Mesner [BM]. A similar concept, with the additional assumption of group action, can be traced back to the earlier works of Schur [S] and Wielandt [W], and more recently, in the 1960's, to the work of D. G. Higman [H1,H2]. Through this permutation group theoretical tradition, one of the editors of this volume (Bannai) became interested in association schemes. Under the strong influence of group theory, he proposed to study association schemes which have both P-polynomial and Q-polynomial properties in the sense of Delsarte [D]. The first step toward the classification of such association schemes was successfully carried out by D. Leonard [L], as a characterization of the associated pair of orthogonal polynomials attached to such association schemes. In particular, it was shown by Leonard [L] that such associated polynomials must in general be Askey-Wilson polynomials. Based on the work of Leonard, the classification problem of the class of association schemes referred to as P- and Q-polynomial schemes has become a driving force for the theory of association schemes. Paul Terwilliger and his group have made tremendous progress in this field (see, e.g., [T1,T2]).

The classification of P- and Q-polynomial association schemes (of large diameters) is a very important problem, which will probably be solvable in the future. At least, the classification of a class of such association schemes referred to as "thin" P- and Q-polynomial association schemes, will very likely be completed by Terwilliger and his colleagues in the not so distant future. The classification problem of distance-regular graphs (i.e., P-polynomial association schemes) of large diameters is considerably more difficult than that of P- and Q-polynomial association schemes, but it may not be impossible. The situation is probably similar for Q-polynomial association schemes. It would be extremely unrealistic to expect that all the primitive and commutative (or symmetric) association schemes could be classified someday while human being still exist, but nevertheless we believe that it is extremely important to set this as one of our ultimate goals and try to approach this goal, epsilon by epsilon. We truly hope, although this may not likely be accomplished imminently, that the classification of finite simple groups could be understood from the viewpoint of association schemes or algebraic combinatorics.

Another direction we would like to proceed in the research of algebraic combinatorics is the exploration of the connections between algebraic combinatorics and various other branches of mathematics. We will return to this point in a later section of this preface, but let us mention here a conference devoted to such a study has already been held, (for example, see [RC]), and we hope to continue this approach.

We are afraid that the description of our view on algebraic combinatorics given here is quite sketchy, but we have included it to help the reader of this volume understand the underlying philosophy of our RIMS research project and associated conferences. We refer the reader to [B2–B4, BI2–3] for further explanation of this view (cf. [BCN, FIK, FIKW, G]).

(3) In this section, we describe our philosophy in editing this volume and organizing the conferences mentioned at the beginning of this preface. This philosophy is closely related to the viewpoint on algebraic combinatorics touched upon in the previous section. In editing this collection of works, we did not attempt to cover all aspects of algebraic combinatorics. We rather placed the emphasis on promoting the connections between algebraic combinatorics and other branches of mathematics. This is reflected in the selection of the invited speakers. Therefore, the scope of topics covered in this volume is quite wide, and not necessarily completely within the boundaries of algebraic combinatorics. Our intention was to consider a spectrum of mathematics through the viewpoint of algebraic combinatorics and intentionally ignore the boundaries between branches of mathematics. We believe that removing the barriers between branches of mathematics is desirable for the progress of mathematics as a whole, and also for the progress of each individual branch of mathematics. Because of the limitation on the space allotted for each volume of this series, we were forced to limit the number of papers included in this volume. However, the thirteen papers in this volume reflect a diversity of subjects greater than normally expected in the proceedings of a conference dedicated to a specific topic.

(4) Now, we wish to comment on each of the papers in this volume. Although the thirteen papers appear in alphabetical order according to the first author's name, we can categorize the papers as follows. Four papers may be regarded as survey papers. Dong and Mason's paper surveys vertex operator algebras, Kantor's paper covers Kerdock codes, spreads and related groups and geometries, Seidel's paper gives a survey on spherical designs and related combinatorial structures, and, Wan's paper summarizes the work on geometries of matrices by himself and other Chinese mathematicians. Although these papers may not contain many new results, they all give interesting and important new viewpoints which will be extremely useful in obtaining insights into these topics. Also, they are quite readable for any mathematician.

The other nine papers may be regarded as original research papers. Among these, Combinatorial Cell Complexes by Aschbacher and Quantum Matroids by Terwilliger are quite extraordinary not only in length but also in content. They are 80 and 119 pages long, respectively, but their contents are nonetheless quite dense.

Aschbacher's paper introduces combinatorial cell complexes as a generalization of topological cell complexes from a geometrical and combinatorial point of view. This is a formal treatment of the groundwork for the study of finite group actions on topological spaces.

Terwilliger's paper describes a new generalization of the concept of matroids. He introduces a regularity condition for quantum matroids and classifies all such quantum matroids. Many known P- and Q-polynomial association schemes arise as the top graph of quantum matroids.

These two papers could be used as textbooks for an advanced topic course for graduate students. In fact, one of the editors of this volume (Munemasa) gave a semester course in 1995 based on Terwilliger's paper. This course was attended by many students and researchers and was quite successful. We believe that Aschbacher's paper is an extremely important and successful attempt to apply the ideas of group theory to the study of mathematical objects in topology, and this paper could also be used as a textbook for an advanced graduate course. This paper also describes a current trend toward the enrichment of finite group theory, which has been receiving mostly retrospective attention after the completion of the classification of finite simple groups around 1980.

Roughly speaking, among the remaining seven original research papers, the papers by Godsil, Ivanov and Saxl, Liebler, and Nomura treat subjects in the field of traditional algebraic combinatorics.

Godsil discusses distance-regular antipodal covers of complete graphs and their relationships with various other combinatorial objects. Ivanov and Saxl compute the character table of a sporadic multiplicity-free permutation representation of ${}^2E_6(2)$ on the cosets of Fi_{22}. Liebler regards incidence matrices of finite projective geometries as elements of the integral Hecke algebra and derives some results on elementary divisors. Nomura explores a connection between certain distance-regular graphs and spin models (see the last paragraph of this section), and classifies almost bipartite distance-regular graphs affording spin models.

The remaining three papers treat subjects usually regarded as topics in mathematical physics or in topology, although there is a strong connection between these fields and algebraic combinatorics, as may be observed from each of these papers.

Deguchi's paper describes a connection between four-weight spin models and exactly solvable models. He introduces gauge transformations for certain spin models and shows that the associated link invariants are invariant under them. Jaeger's paper gives a construction of Bose-Mesner algebras from symmetric spin models using the ingenious idea of a matrix-valued partition function. Kohno and Takata's paper formulates the level-rank duality of Witten's 3-manifold invariants corresponding to $SU(n)$ and gives a rigorous proof.

Let us mention here a wonderful new development on the connection between spin models and association schemes (algebraic combinatorics) which is not fully described in the papers in this volume. As shown in Jaeger's paper in this volume, to each symmetric spin model in the sense of Jones [Jo] there is associated a symmetric self-dual association scheme. Immediately after Jaeger obtained this result using a topological method, Nomura [N] discovered a purely algebraic method to prove the same result. Further developments were obtained and will appear in a paper being prepared jointly by F. Jaeger, M. Matsumoto and K. Nomura [JMN]. These results show that association schemes are inevitably necessary tools in the study of spin models (also generalized spin models [KMW] and other related statistical mechanical models) and that spin models are quite enriching in the study of algebraic combinatorics. This was a major achievement resulting from the organization of the conferences, and we wish to encourage such a viewpoint by the publication of this volume in the *Advanced Studies in Pure Mathematics* series. We believe that to find more mathematical objects which interplay with algebraic combinatorics, but not necessarily inside traditional algebraic combinatorics, is extremely important for the future development of this field. We believe that there are many such objects which are waiting to be discovered.

(5) We would like to mention that we could not include many topics in this volume which we wanted to include because of the space limitation. We would like to emphasize that the conferences included many interesting talks from invited and contributing speakers, in addition to those included in this volume. This can clearly be seen by looking at their programs. Parts of these programs can be found in the appendix to this preface. We are extremely happy to have been able to organize these conferences, and we are happy to acknowledge that this was due to the mathematical power and potential of young mathematicians in Japan who are interested in algebraic combinatorics and many related areas. We do not believe that algebraic combinatorics (or any branch of mathematics) is a fixed or closed object. The nature of algebraic combinatorics will change day by day, and will grow and develop into many new forms. We do hope that our present approach to this field will help its advance in this way. At least, we hope this volume gives a clear and

vivid description of this ever-growing mathematical development.

(6) Finally, but not least, we would like to thank those who helped us in many ways. We thank all the speakers and the participants at the conferences. We also thank the organizers of these conferences, as well as those who helped us in many practical matters. We thank the colleagues, secretaries and students of the Department of Mathematics of Kyushu University, and of the Research Institute for Mathematical Sciences of Kyoto University, for helping us a great deal. We would like to thank the editors of the *Advanced Studies in Pure Mathematics* series, and in particular Professor Tadao Oda for inviting us to edit this volume in this prestigeous series and for giving us valuable advice. We also thank Professor Toshikazu Sunada and Professor Shigeru Mukai for providing financial support for our activities related to the organization of this volume. We would like to thank the secretaries of the editorial office of the *Advanced Studies in Pure Mathematics* series, in particular Kazuko Kozaki and Chiharu Kanasaki for help in practical matters while editing this volume. We also thank the referees who gave valuable comments to contributors to this volume. Finally, we would like to thank the Mathematical Society of Japan for publishing this volume in the *Advanced Studies in Pure Mathematics* series.

January 1996
Eiichi BANNAI and Akihiro MUNEMASA

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.

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Appendix to Preface (Information on the conferences)

1. Algebraic Combinatorics (Fukuoka,1993) November 22–26,1993 at Kyushu University in Fukuoka. Invited Speakers: Michael Aschbacher, Andries E. Brouwer, Tetsuo Deguchi, Chris D. Godsil, Alexander A. Ivanov, François Jaeger, William M. Kantor, Helmut Koch, Toshitake Kohno, Robert A. Liebler, Neil Robertson, J. J. Seidel, Navin M. Singhi, N. J. A. Sloane, Dennis Stanton, Paul Terwilliger, Minoru Wakimoto and Zhe-xian Wan. Organizers: (Executive Committee) Eiichi Bannai, Mieko Yamada, Akihiro Munemasa, (Supporting Committee) Andries E. Brouwer, Hikoe Enomoto, Tatsuro Ito, Alexander A. Ivanov, François Jaeger, Toshitake Kohno, N. J. A. Sloane, Zhe-xian Wan.

Supported by Monbusho (Ministry of Education, Science and Culture of Japan), the Yokatopia Foundation, various individual Grants-in-Aid from the Ministry of Education, Science and Culture of Japan.

2. RIMS Research Project: Algebraic Combinatorics, April 1994–March 1995. Organizing Committee: Eiichi Bannai, Kyoji Saito, Noriaki Kawanaka, Toshitake Kohno, Tatsuro Ito.

(a) Algebraic Combinatorics Summer School; July 20–24 at Ebara Garden, in Matsuyama, Ehime. Local organizer: Hiroshi Kimura. Part 1. Review on Group Theory in Japan. Speakers: Michio Suzuki, Noboru Ito, Hirosi Nagao, Takeshi Kondo, Koichiro Harada, Takeo Yokonuma, Noriaki Kawanaka, Tomoyuki Yoshida, Eiichi Bannai. Part 2. Introduction to Algebraic Combinatorics. Speakers: Eiichi Bannai, Tatsuro Ito, Akihiro Munemasa, Etsuko Bannai. Part 3. Special lectures (3 hours each): Kyoji Saito, Mikio Sato, Toshitake Kohno.

(b) First Meeting; Nov. 21–24, 1994, at RIMS. (1 hour invited speakers) William Kantor, François Jaeger, Eiichi Bannai, Masahiko Miyamoto, Michio Ozeki, Donald Higman, Toshiaki Shoji.

(c) Second Meeting; March 13–15, 1995, at RIMS. (1 hour invited speakers) Hikoe Enomoto, Kazumasa Nomura, Paul Terwilliger, Alexander A. Ivanov, Roland Bacher, Tomoyuki Yoshida.

(d) Workshops at RIMS; Moonshine and Vertex Operator Algebras, (Sept. 5–8, 1994) organized by Masahiko Miyamoto, Algebraic Combinatorics and Low Dimensional Topology, (Nov. 28–30, 1994), organized by Toshitake Kohno, Distance-Regular Graphs and Incidence Geometries, (March 6–10, 1995), organized by Hiroshi Suzuki.

(e) Long term visitors to RIMS for this research project; Geoffrey Mason (June–Sept., 1994), William Kantor (Sept.–Dec., 1994), François Jaeger (Oct.–Dec., 1994), and Alexander A. Ivanov (Jan.–April, 1995).