This volume is warmly dedicated to

Prof. Ichiro Satake
and
Prof. Friedrich Hirzebruch

on their sixtieth birthdays in 1987
with sincere gratitude
from us all who owe so much to them mathematically
through both their works and their personal guidance

Preface

Since Gauß, automorphic forms have played one of the most important role in number theory together with other but deeply related functicons such as zeta functions.

In this century, the theory of automorphic forms has been extensively enlarged and enriched by relating it to the geometry of symmetric spaces. This generalization allows us to use huge techniques in the Lie group theory, especially the representation theory with harmonic analysis, differential geometry and algebraic geometry. In particular, it has become one of the central problems to study the geometry of the quotients of the Siegel upper half-space or its generalization, the hermitian bounded symmetric domains, by arithmetic subgroups.

The importance of automorphic forms seems to become still greater when we see the very recent development of the theory of strings in quantum field theory where they appear unexpectedly as an essential tool to describe fundamental, physically important quantities. This new tendency would suggest a completely new theory of automorphic forms, namely the one over the moduli space of curves, which would be closely related to non-abelian class field theory.

Here in Japan intensive study on automorphic forms has been made, and it has given, we believe, a not small historical contribution to the theory. It would be our great pleasure if this volume would also contribute something.

This book grew out of a series of symposia held in 1985–86 where the main topic was to study the dimension formulas of various automorphic forms. (For these conferences we obtained financial support from the Research Institute for Mathematical Sciences, Kyoto University, and the Japan Society for the Promotion of Science, to which we together with all participants would express our gratitude.) One of our main concerns was to clarify the relation between two fundamental methods to obtain dimension formulas (for cusp forms), the Selberg trace formula and the index theorem (Riemann-Roch's theorem and the Lefschetz fixed point formula). This relation is well understood when the quotient is compact. But in the case of non-compact quotients, one should analyse the term corresponding to the so-called η-invariants which are related with special values of some zeta functions, and here the relation between the above two methods is not clear. Though this aim is still to be attained (cf. the articles by Satake-Ogata, Sczech,

Tsushima), several important results were obtained which we might consider as fruits of these meetings.

This volume is divided into two parts. Part I consists of survey articles, most of which originated from the lectures given in these meetings but were newly written to be up-to-date. We regret that we could not include here some other interesting survey talks given there, which treat much wider related topics such as those in differential and algebraic geometry.

Part II consists of original papers which have enlarged the subjects treated in the final form of this volume. These subjects cover a large part of those which are now studied in Japan. We also refer the reader to Vol. 13 in this series to understand the whole trend of research in this area in Japan.

This volume is dedicated to Prof. Ichiro Satake and Prof. Friedrich Hirzebruch with our hearty congratulations and thanks on the occasions of their sixtieth birthdays in 1987.

Prof. Satake has long been one of our most important leaders through his work. His theory of compactifications, now known by his name together with Baily and Borel, of quotients of hermitian bounded symmetric domains by arithmetic subgroups, is a fundamental tool to the study of automorphic forms by the geometric method (cf. Tsuyumine's and Tsushima's articles in this volume). Our main aim to clarify the relation between this geometric method and the group-theoretic one (using the trace formula), also originated from his recent work (cf. Satake-Ogata here). Moreover, after his return to Japan, we are enjoying his personal warm encouragements and appropriate advice. The series of meetings mentioned above as the origin of this book was also held under his guidance. Therefore we would like to dedicate this whole volume to Prof. Satake from all the participants of the meetings.

To Prof. Hirzebruch also we owe much, of course, through his Riemann-Roch formula and Lefschetz fixed point formula (which are the main tools to calculate the dimension of cusp forms in geometric method) as well as his deep work on Hilbert modular varieties. But this is not the only reason to dedicate this volume to him. We, the two editors of this volume, ask the reader to allow us to recount our personal experience. Our joint work began with a private seminar at the Max-Planck-Institut für Mathematik in Bonn at the beginning of 1984 by six Japanese members at that time (Prof. Ibukiyama was also one of them) to read Atiyah-Donnelly-Singer's article on the proof of Hirzebruch's conjecture on Hilbert modular varieties. In the course of this seminar we were led to our main motivation for the series of meetings mentioned above. Without this prehistory at MPI, whose

director has been Prof. Hirzebruch since its foundation in 1982, we would never have been coeditors of this volume. Already nearly a hundred Japanese mathematicians, including most of the authors in this book, have stayed at Bonn when we consider SFB "Theoretische Mathematik" at Bonn University as MPI's antecedent. Prof. Hirzebruch has always taken the initiative to invite Japanese mathematicians, especially young ones, and given them an opportunity to study in active and international circumstances. His contribution to mathematics in Japan is hence immeasurable, for which we would like to thank him cordially on this occasion.

Lastly, the editors would ask pardon for the long delay of publishing this volume, which is entirely due to their laziness. They are afraid that this delay might have caused the authors some disadvantage.

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