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ON HYPER GENERALIZED WEAKLY SYMMETRIC MANIFOLDS

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Abstract. This paper aims to introduce the notion of hyper generalized weakly symmetric manifolds with a non-trivial example.

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1. Introduction

The notion of weakly symmetric Riemannian manifold has been introduced by Tamássy and Binh [23]. Thereafter, it becomes focus of interest for many geometers. For details, we refer to [6], [9], [10], [12], [17], [19–21], [2] and the references there in.

In the spirit of [23], a non flat Riemannian manifold $(M^n, g)(n > 2)$, is said to be weakly symmetric manifold, if its curvature tensor $\bar{R}$ of type $(0, 4)$ is not identically zero and satisfies the identity

$$\begin{aligned}(\nabla_X \bar{R})(Y, U, V, W) = {} & A(X)\bar{R}(Y, U, V, W) \\ & + B(Y)\bar{R}(X, U, V, W) + B(U)\bar{R}(Y, X, V, W) \\ & + D(V)\bar{R}(Y, U, X, W) + D(W)\bar{R}(Y, U, V, X)\end{aligned} \tag{1}$$

where A, B & D are non-zero one-forms defined by $A(X) = g(X, \sigma_1)$, $B(X) = g(X, \pi_1)$ and $D(X) = g(X, \partial_1)$, for all X and $\bar{R}(Y, U, V, W) = g(R(Y, U)V, W)$,

∇ being the operator of the covariant differentiation with respect to the metric tensor g. Such an n-dimensional Riemannian manifold is abbreviated hereafter by $(WS)_n$.

Keeping in tune with Dubey [7], recently the first author[1] introduced a new type of manifold called generalized weakly symmetric manifold which is abbreviated by $(GWS)_n$ and defined as follows.

A non-flat n-dimensional Riemannian manifold (M^n, g) $(n > 2)$, is termed as generalized weakly symmetric manifold, if its Riemannian curvature tensor $\bar{R}$ of type $(0;4)$ is not identically zero and admits the identity

$$\begin{aligned}(\nabla_X \bar{R})(Y,U,V,W) = {} & A(X)\bar{R}(Y,U,V,W) + B(Y)\bar{R}(X,U,V,W) \\ & +B(U)X\bar{R}(Y,X,V,W) + D(V)\bar{R}(Y,U,X,W) \\ & +D(W)\bar{R}(Y,U,V,X) + \alpha(X)G(Y,U,V,W) \\ & +\beta(Y)G(X,U,V,W) + \beta(U)G(Y,X,V,W) \\ & +\gamma(V)G(Y,U,X,W) + \gamma(W)G(Y,U,V,X)\end{aligned} \tag{2}$$

where

$$G(Y,U,V,W) = [g(U,V)g(Y,W) - g(Y,V)g(U,W)] \tag{3}$$

and $A,\ B,\ D,\ \alpha,\ \beta\ \&\ \ \gamma$ are non-zero one-forms which are defined as $A(X) = g(X,\theta_1), B(X) = g(X,\phi_1),\ D(X) = g(X,\pi_1),\ \alpha(X) = g(X,\theta_2), \beta(X) = g(X,\phi_2)$ and $\gamma(X) = g(X,\pi_2)$.

Keeping in tune with Shaikh and Patra [22], we shall call a Riemannian manifold of dimension n, hyper generalized weakly symmetric (which will be abbreviated hereafter as $H(GWS)_n$) if it admits the equation

$$\begin{aligned}(\nabla_X \bar{R})(Y,U,V,W) = {} & A(X)\bar{R}(Y,U,V,W) + B(Y)\bar{R}(X,U,V,W) \\ & +B(U)\bar{R}(Y,X,V,W) + D(V)\bar{R}(Y,U,X,W) \\ & +D(W)\bar{R}(Y,U,V,X) + \alpha(X)(g\wedge S)(Y,U,V,W) \\ & +\beta(Y)(g\wedge S)(X,U,V,W) + \beta(U)(g\wedge S)(Y,X,V,W) \\ & +\gamma(V)(g\wedge S)(Y,U,X,W) + \gamma(W)(g\wedge S)(Y,U,V,X)\end{aligned} \tag{4}$$

where

$$\begin{aligned}(g\wedge S)(Y,U,V,W) = {} & g(Y,W)S(U,V) + g(U,V)S(Y,W) \\ & -g(Y,V)S(U,W) - g(U,W)S(Y,V)\end{aligned} \tag{5}$$

and $A,\ B,\ D,\ \alpha,\ \beta\ \&\ \ \gamma$ are non-zero one-forms which are defined as $A(X) = g(X,\theta_1), B(X) = g(X,\phi_1),\ D(X) = g(X,\pi_1),\ \alpha(X) = g(X,\theta_2), \beta(X) = g(X,\phi_2)$ and $\gamma(X) = g(X,\pi_2)$. The beauty of such $H(GWS)_n$-manifold is that it has the flavour of

i) locally symmetric space [3] (for $A = B = D = \alpha = \beta = \gamma = 0$)

ii) recurrent space [26] (for $A \neq 0$, $B = D = \alpha = \beta = \gamma = 0$)
iii) hyper recurrent space [22] ($A \neq 0$, $\alpha \neq 0$ and $B = D = \beta = \gamma = 0$)
iv) pseudo symmetric space [4] (for $A = B = D = \delta \neq 0$ and $\alpha = \beta = \gamma = 0$)
v) semi-pseudo symmetric space [25] (for $B = D$ and $A = \alpha = \beta = \gamma = 0$)
vi) hyper semi-pseudo symmetric space (for $A = 0 = \alpha, B = D \neq 0$ and $\beta = \gamma \neq 0$)
vii) hyper pseudo symmetric space (for $A = B = D = \alpha = \beta = \gamma \neq 0$)
viii) almost pseudo symmetric space [5] (for $A = B + H$, $H = B = D \neq 0$ and $\alpha = \beta = \gamma = 0$)
ix) almost hyper pseudo symmetric space (for $A = B + H$, $H = B = D \neq 0$, $\alpha = \lambda +, \beta = \gamma = \mu \neq 0$)) and
x) weakly symmetric space[23] (for $\alpha = \beta = \gamma = 0$).

Our work is structured as follows. Section 2 is concerned with some results on $H(GWS)_n$. Among others it is proved that every weakly conharmonically symmetric space which is Ricci symmetric is necessarily a $H(GWS)_n$. In Section 3, we have investigated conformally flat $H(GWS)_n$ and obtained some interesting results. Finally, the existence of $H(GWS)_4$ is ensured by a non-trivial example.

More general types of recurrency can be found in [8, 13–16].

2. Some Results on $(HGWS)_n$

In this section, we consider a Riemann manifold (M^n, g) $n > 2$ which is hyper generalized weakly symmetric. Now, making use of (5) in (4) we find

$$
\begin{aligned}
(\nabla_X \bar{R})(Y, U, V, W)
&= A(X)\bar{R}(Y, U, V, W) + B(Y)\bar{R}(X, U, V, W) \\
&\quad + B(U)\bar{R}(Y, X, V, W) + D(V)\bar{R}(Y, U, X, W) \\
&\quad + D(W)\bar{R}(Y, U, V, X) + \alpha(X)[g(Y, W)S(U, V) + g(U, V)S(Y, W) \\
&\quad - g(Y, V)S(U, W) - g(U, W)S(Y, V)] + \beta(Y)[g(X, W)S(U, V) \\
&\quad + g(U, V)S(X, W) - g(X, V)S(U, W) - g(U, W)S(X, V)] \\
&\quad + \beta(U)\,[g(Y, W)S(X, V) + g(X, V)S(Y, W) - g(Y, V)S(X, W) \\
&\quad - g(X, W)S(Y, V)] + \gamma(V)\,[g(Y, W)S(U, X) + g(U, X)S(Y, W) \\
&\quad - g(Y, X)S(U, W) - g(U, W)S(Y, X)] + \gamma(W)[g(Y, X)S(U, V) \\
&\quad + g(U, V)S(Y, X) - g(Y, V)S(U, X) - g(U, X)S(Y, V)].
\end{aligned}
\tag{6}
$$

Note that for an Einstein space, $H(GWS)_n$ reduces to $(GWS)_n$. This leads to the following:

Theorem 1. *Every $H(GWS)_n$ is $(GWS)_n$ provided that the space is an Einstein.*

Next, contracting (6) we have

$$(\nabla_X S)(U,V)$$
$$\begin{aligned} &= A(X)S(U,V) + B(U)S(X,V) + D(V)S(U,X) + B(R(X,U)V) \\ &\quad +D(R(X,V)U) + \alpha(X)[(n-2)S(U,V) + rg(U,V)] \\ &\quad +\beta(U)\,[(n-2)S(X,V) + rg(X,V)] + \gamma(V)\,[(n-2)S(U,X) \\ &\quad +rg(U,X)] + \beta(X)S(U,V) + \tilde{\beta}(X)g(U,V) - \tilde{\beta}(U)g(X,V) \\ &\quad -\beta(U)S(X,V) + \gamma(X)S(U,V) + \tilde{\gamma}(X)g(U,V)S(Y,) \\ &\quad -\gamma(V)S(U,X) - \tilde{\gamma}(V)g(U,X) \end{aligned} \tag{7}$$

which yields after further contraction

$$\begin{aligned} \mathrm{d}r(X) &= A(X)r + 2\bar{B}(X) + 2\bar{D}(X) + 2(n-1)r\alpha(X) \\ &\quad + 2r[\beta(X) + \gamma(X)] + 2(n-2)[\tilde{\gamma}(X) \ + \tilde{\beta}(X)] \end{aligned} \tag{8}$$

where $\bar{B}(X) = S(X,\phi_1)$, $\bar{D}(X) = S(X,\pi_1)$ $\tilde{\beta}(X) = S(X,\phi_2)$ and $\tilde{\gamma}(X) = S(X,\pi_2)$ for all X.
Next, if we suppose that the scalar curvature of a $H(GWS)_n$ is non-zero constant, then (8) becomes

$$\begin{aligned} &r[A(X) + 2(n-1)\alpha(X) + 2\beta(X) + 2\gamma(X)] \\ &\quad = -2[\bar{B}(X) + \bar{D}(X)] - 2(n-2)[\tilde{\gamma}(X) \ + \tilde{\beta}(X)]. \end{aligned} \tag{9}$$

This leads to

Theorem 2. *Let $(M^n,g)(n>2)$ be a Riemannian manifold with non-zero constant scalar curvature. Then the one-forms are related by the relation* (9).

Claim 3. There does not exist a hyper recurrent Riemannian manifold $(M^n,g)(n > 2)$ whose clear curvature is non-zero constant and the one-forms are co-linear.

In analogous to the definition of $(WS)_n$, we can define the following

Definition 4. *A non flat Riemannian manifold $(M^n,g)(n>2)$, is said to be weakly conharmonically symmetric manifold, if its nonharmonic curvature tensor*

$$\bar{K} = \bar{R} - \frac{1}{n-2}(g \wedge S)(Y,U,V,W) \tag{10}$$

of type $(0,4)$ is not identically zero and satisfies the identity

$$\begin{aligned} (\nabla_X \bar{K})(Y,U,V,\ W) = &\ A(X)\bar{K}(Y,U,V,\ W) \\ &+B(Y)\bar{K}(X,U,V,\ W) + B(U)\bar{K}(Y,X,V,\ W) \\ &+D(V)\bar{K}(Y,U,X,\ W) + D(W)\bar{K}(Y,U,V,\ X) \end{aligned} \tag{11}$$

where A, B and D are non-zero one-forms defined by the formulas $A(X) = g(X, \sigma_1)$, $B(X) = g(X, \pi_1)$ and $D(X) = g(X, \partial_1)$, for all X and $\bar{K}(Y, U, V, W) = g(K(Y, U)V, W)$.

From the above definition, it is follows that

Theorem 5. *A weakly conharmonically symmetric space which is Ricci symmetry is necessarily a $H(GWS)_n$.*

However, the converse of the above Theorem may not be true.

3. Conformally Flat $H(GWS)_n$

In this section, we shall study conformally flat $H(GWS)_n$. Next, in a $H(GWS)_n$ the relation (7) holds which is equivalent to

$$\begin{aligned} S_{ij,l} = {} & A_l S_{ij} + B^h \bar{R}_{lijh} + B_i S_{lj} + D_j S_{il} + D^h \bar{R}_{hijl} + \alpha_l\{(n-2)S_{ij} + g_{ij} r\} \\ & + \beta_l S_{ij} + \beta_i\{(n-3)S_{lj} + g_{lj} r\} + \beta^h (g_{ij} S_{lh} - g_{lj} S_{ih}) \\ & + \gamma_j\{(n-3)S_{il} + r g_{il}\} + \gamma_l S_{ij} + \gamma^h (g_{ij} S_{hl} - g_{il} S_{hj}). \end{aligned} \tag{12}$$

Let as assume that our manifold is conformally flat. Thus we have

$$S_{ij,l} - S_{il,j} = \frac{1}{2(n-1)}(g_{ij} r_{,l} - g_{il} r_{,j}). \tag{13}$$

Multiplying (12) by g^{ij}, we find

$$r_{,l} = \{A_l + 2(n-1)\alpha_l + 2\beta_l + 2\gamma_l\} r + 2S_{lh}\{B^h + D^h + (n-2)\beta^h + (n-2)\gamma^h\}. \tag{14}$$

Comparing (12) with the equations (13) and (14), we obtain

$$\begin{aligned} & A_l S_{ij} + B^h \bar{R}_{lijh} + D_j S_{il} + D^h \bar{R}_{hijl} + \alpha_l\{(n-2)S_{ij} + g_{ij} r\} + \beta_l S_{ij} \\ & \quad + g_{ij}\beta^h S_{lh} + \gamma_l S_{ij} + \gamma_j\{(n-3)S_{il} + g_{ij} r\} + \gamma^h (g_{ij} S_{hl} - g_{il} S_{hj}) \\ & \quad - A_j S_{il} - B^h \bar{R}_{jilh} - D_l S_{ij} - D^h \bar{R}_{hilj} - \alpha_j\{(n-2)S_{il} + g_{il} r\} \\ & \quad - \beta_j S_{il} - g_{il}\beta^h S_{jh} - \gamma_j S_{il} - \gamma_l\{(n-3)S_{ij} + g_{ij} r\} \\ & \quad - \gamma^h (g_{il} S_{hj} - g_{ij} S_{hl}) \\ & = \frac{1}{2(n-1)}[g_{ij}\{A_l + 2(n-1)\alpha_l + 2\beta_l + 2\gamma_l\} r + 2g_{ij} S_{lh}\{B^h + D^h \\ & \quad + (n-2)\beta^h + (n-2)\gamma^h\} - g_{il}\{A_j + 2(n-1)\alpha_j + 2\beta_j + 2\gamma_j\} r \\ & \quad - 2g_{il} S_{jh}\{B^h + D^h + (n-2)\beta^h + (n-2)\gamma^h\}]. \end{aligned} \tag{15}$$

Multiplying (15) by g^{ij}, it can found that

$$\{\frac{1}{2}A_l - D_l + (n-2)\alpha_l - 2(n-2)\gamma_l\}r \\ = \{A^h - 2D^h + (n-2)\alpha^h - 2(n-2)\gamma^h\}S_{lh}. \quad (16)$$

From (16), we get

$$-\frac{1}{2}A_l r + D_l r + \{A_l - 2D_l + (n-2)\alpha_l - 2(n-2)\gamma_l\}r \\ = \{A^h - 2D^h + (n-2)\alpha^h - 2(n-2)\gamma^h\}S_{lh}. \quad (17)$$

Assuming that $\lambda_l = A_l - 2D_l + (n-2)\alpha_l - 2(n-2)\gamma_l$

$$\left[rD_l - \frac{1}{2}A_l r\right] + \lambda_l r = \lambda_h S_{lh}. \quad (18)$$

If r is an eigenvalue of the Ricci tensor S corresponding to the eigenvector ρ we have($r \neq 0$)

$$g(X, \rho) = A(X) - 2D(X) + (n-2)\alpha(X) - 2(n-2)\gamma(X)$$

then we get

$$D_l = 2A_l. \quad (19)$$

In this case, by putting(19) in (17), it can be easily seen that (since $n > 3$)

$$(\alpha_l - 2\gamma_l)r = (\alpha_h - 2\gamma_h)S_{lh} \quad (20)$$

and

$$\lambda_l = (n-2)(\alpha_l - 2\gamma_l). \quad (21)$$

Hence, we have the following theorem

Theorem 6. *If a hyper generalized weakly symmetric manifold is also conformally flat then r is an eigenvalue of the Ricci tensor S corresponding to the eigenvector ρ, where*

$$g(X, \rho) = \alpha(X) - 2\gamma(X) = \lambda(X).$$

Now, rearranging the equation (16)

$$\{A_l - 2D_l + (n-2)\alpha_l - 2(n-2)\gamma_l\}\frac{r}{2} + [(n-2)\alpha_l - 2(n-2)\gamma_l]\frac{r}{2} \\ = \{A^h - 2D^h + (n-2)\alpha^h - 2(n-2)\gamma^h\}S_{lh}. \quad (22)$$

By taking $\lambda_l = A_l - 2D_l + (n-2)\alpha_l - 2(n-2)\gamma_l$

$$\lambda_l \frac{r}{2} + (n-2)[\alpha_l - 2\gamma_l]\frac{r}{2} = \lambda^h S_{lh}. \quad (23)$$

If $\frac{r}{2}$ is an eigenvalue of the Ricci tensor S corresponding to the eigenvector μ we have($r \neq 0$)

$$g(X,\mu) = A(X) - 2D(X) + (n-2)\alpha(X) - 2(n-2)\gamma(X) = \lambda(X)$$

then we get

$$\alpha_l = 2\gamma_l. \tag{24}$$

In this case, by putting (24) in (23), we find

$$(A_l - 2D_l)\frac{r}{2} = (A^h - 2D^h)S_{lh}, \qquad \lambda_l = A_l - 2D_l. \tag{25}$$

Theorem 7. *If a hyper generalized weakly symmetric manifold is also conformally flat then $\frac{r}{2}$ is an eigenvalue of the Ricci tensor S corresponding to the eigenvector μ, where*

$$g(X,\mu) = A(X) - 2D(X) = \lambda(X).$$

4. Existence of Hyper Generalized Weakly Symmetric Space

Example 8. *Consider a four-dimensional space (M^4, g) with the metric g defined by*

$$ds^2 = (dx^2)^2 + 2e^{x^2}\left[dx^1dx^2 + dx^3dx^4\right] \tag{26}$$

where $x^2 > 0$. From the above one can calculate and list the non-vanishing components of Christoffel symbols, as well as of $\bar{R}_{hijk}$, S_{ij} , C_{hijk} and $\nabla_m \bar{R}_{hijk}$ as follows

$$\begin{array}{ll}
\Gamma^1_{22} = -\,e^{-x^2}, & \frac{1}{2}\Gamma^2_{22} = \Gamma^3_{23} = \Gamma^4_{24} = -\Gamma^1_{34} = \frac{1}{2} \\
R_{2324} = \frac{1}{4}e^{x^2}, & S_{22} = -\,\frac{1}{2} \\
R_{2324,2} = -\,\frac{1}{2}e^{x^2}, & S_{22,2} = 1.
\end{array}$$

Making use of the relation, we can easily bring out

$$\begin{aligned}
(g \wedge S)(Y,U,V,W) &= g(Y,W)S(U,V) + g(U,V)S(Y,W) \\
&\quad -g(Y,V)S(U,W) - g(U,W)S(Y,V) \\
(g \wedge S)_{2324} &= S_{23}g_{24} + g_{23}S_{24} - S_{22}g_{34} - S_{34}g_{22} = -\frac{1}{2}e^{x^2} \\
(g \wedge S)_{2224} &= S_{22}g_{24} + g_{22}S_{24} - S_{22}g_{24} - S_{24}g_{22} = 0 \\
(g \wedge S)_{2424} &= S_{24}g_{24} + g_{24}S_{24} - S_{22}g_{44} - S_{44}g_{22} = 0 \\
(g \wedge S)_{2322} &= S_{23}g_{22} + g_{23}S_{22} - S_{22}g_{32} - S_{32}g_{22} = 0
\end{aligned} \tag{27}$$

For the following choice of the one-forms

$$
\begin{aligned}
A_i &= -2,\ \text{for } i = 2 & \alpha_i &= \frac{1}{4}\mathrm{e}^{x^2},\ \text{for } i = 2\\
&= 0,\ \text{otherwise} & &= 0,\ \text{otherwise}\\
B_i &= -\frac{1}{3}\mathrm{e}^{x^2},\ \text{for } i = 2 & \beta_i &= \frac{1}{4}\mathrm{e}^{x^2},\ \text{for } i = 2\\
&= 0,\ \text{otherwise} & &= 0,\ \text{otherwise}\\
D_i &= \frac{1}{3}\mathrm{e}^{x^2},\ \text{for } i = 2 & \gamma_i &= -\frac{1}{2}\mathrm{e}^{x^2},\ \text{for } i = 2\\
&= 0,\ \text{otherwise} & &= 0,\ \text{otherwise}
\end{aligned}
$$

one can easily conclude that

$$
\begin{aligned}
R_{2324,k} = {} & A_k R_{2324} + B_2 R_{k324} + B_3 R_{2k24} + D_2 R_{23k4} + D_4 R_{232k}\\
& + \alpha_k\ (g \wedge S)_{2324} + \beta_2 (g \wedge S)_{k324} + \beta_3 (g \wedge S)_{2k24}\\
& + \gamma_2 (g \wedge S)_{23k4} + \gamma_4 (g \wedge S)_{232k}
\end{aligned}
$$

where, $k = 1, 2, 3, 4$. As a consequence of the above one can state

Theorem 9. *There exists a* $(\mathbb{R}^4, g)$ *which is a hyper generalized weakly symmetric space with non-zero and non-constant scalar curvature for the above mentioned choice of the i-forms.*

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