

ON CHARGE CONSERVATION IN A GRAVITATIONAL FIELD

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Abstract. According to the “gravitationally-modified” Maxwell equations that were proposed for an alternative scalar theory with an “ether”, electric charge would not be conserved in a time-dependent gravitational field. We define an asymptotic expansion scheme for the electromagnetic field in a weak gravitational field. This allows us to assess the amounts of charge production or destruction which are thus predicted. These amounts seem high enough to discard that version of the gravitationally-modified Maxwell equations. We show that this failure is due to the former assumption of additivity of the energy tensors: an “interaction energy tensor” has to be added. Then the standard Maxwell equations in a curved spacetime become compatible with that scalar theory, and they predict charge conservation.

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1. Introduction

An alternative theory of gravitation based on a scalar field alone, and having a preferred reference frame, has been proposed previously [1, 2, 7]. Recently, equations for the electromagnetic field in the presence of a gravitational field have been derived for this theory [9] and it has been found that they lead to a violation of the conservation of electric charge. The aim of this contribution is to give a short account of the work that has been done to assess the magnitude of this effect, and to outline the consequences that followed for the theory from this assessment. (A detailed account has been given elsewhere [10].) Let us first summarize the three main motivations for this “scalar ether theory” of gravitation, in short SET.

1) The first motivation is to extend to the situation with gravitation the *Lorentz-Poincaré version of special relativity*. The latter version is more often called

“Lorentz ether theory”, although undoubtedly Poincaré made essential contributions to it. This is the theory according to which the “ether” is a rigid inertial frame such that: i) any object that is moving with respect to it is subjected to the Lorentz contraction, and ii) in this frame, energy propagation is isotropic, in particular the Maxwell equations are assumed valid in that frame. This theory obtains the Lorentz transformation and then all “relativistic” effects as following from *a*) “absolute” effects of motion through that ether, and *b*) Clock synchronization. It is exposed in particular in Prokhovnik [19], and a summary can be found in [2]. In that theory, the restriction “ $v < c$ ” is not absolute, it concerns mass particles.

2) Newton’s $1/r^2$ force, which acts instantaneously at a distance, cannot be intuitively understood. (This was already Newton himself’s opinion.) Einstein’s general relativity (GR) sees gravity as propagating at the velocity of light c , but says that (“free”) particles follow geodesic curves in the four-dimensional “space-time” manifold V endowed with a curved Lorentzian metric γ . This gives a fundamental physical status to the spacetime — although, to more than one physicist, spacetime looks more like a mere mathematical construction. (The latter certainly is its status in the Lorentz-Poincaré version of special relativity.) In contrast, SET makes gravity thinkable as the pressure force of the ether: Archimedes’ thrust on extended particles seen as organized flows in the ether [1].

3) Despite its successes, GR has problems: i) The unavoidable singularities (e.g. in a gravitational collapse and in the “big bang”). ii) The interpretation of the necessary gauge condition. iii) The problems regarding the coupling with quantum theory. iv) The need for dark matter. v) The need for dark energy. In contrast: i) SET has no singularity either in a gravitational collapse [4] or in the past high-density state [5] and ii) no gauge condition. iii) The assumed preferred reference frame makes it possible to write quantum theory in a gravitational field unambiguously. For instance, this avoids the non-uniqueness problem of the covariant Dirac theory [8]. iv) Preferred-frame effects should be more important at large scales (for they have more time to accumulate) and might possibly contribute to explain motion at a galactical scale. v) That theory necessarily *predicts* accelerated expansion, without assuming any dark energy [5].

2. Equations for the Electromagnetic Field

SET has a preferred reference frame \mathcal{E} . It has also a curved spacetime metric noted γ . The equations below are usually valid only in coordinates adapted to that reference frame and such that the synchronization condition $\gamma_{0i} = 0$ ($i = 1, 2, 3$) is verified. The *spatial* metric in the frame \mathcal{E} [3, 11, 14, 17] will be noted g . The first Maxwell group is unchanged. In terms of the antisymmetric field tensor F

$$F_{\sigma\mu;\nu} + F_{\mu\nu;\sigma} + F_{\nu\sigma;\mu} = F_{\sigma\mu;\nu} + F_{\mu\nu;\sigma} + F_{\nu\sigma;\mu} = 0. \quad (1)$$

Semicolon means a covariant derivative associated with the metric γ . The first equality is an identity for an antisymmetric tensor field and a torsionless connection, as is the metric (Levi-Civita) connection. To get the second group [9], we used the equation for continuum dynamics that is valid in SET and we applied it to the *charged medium* subjected to the *Lorentz force*, assuming that

- i) The total energy(-momentum) tensor is $\mathbf{T}_{\text{total}} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}}$.
- ii) The total energy tensor $\mathbf{T}_{\text{total}}$ obeys the general equation for continuum dynamics, without any non-gravitational force.

After some algebra using the explicit form of the energy tensor $\mathbf{T}_{\text{field}}$ of the electromagnetic (EM) field, this gives (writing the equations “in SI units”)

$$F^\mu{}_\sigma F^{\sigma\nu}{}_{;\nu} = \mu_0 [b^\mu (\mathbf{T}_{\text{field}}) - F^\mu{}_\sigma J^\sigma] \quad (2)$$

where

$$b^0(\mathbf{T}) \equiv \frac{1}{2} \gamma^{00} g_{ij,0} T^{ij}, \quad b^i(\mathbf{T}) \equiv \frac{1}{2} g^{ij} g_{jk,0} T^{0k}. \quad (3)$$

3. Charge Non-Conservation

If $\det \mathbf{F} \neq 0$ (where \mathbf{F} is the matrix $\mathbf{F} \equiv (F^\mu{}_\nu)$), which is equivalent [9] to $\mathbf{E} \cdot \mathbf{B} \neq 0$ with \mathbf{E} and \mathbf{B} the electric and magnetic fields, we get from equation (2) [9]

$$\hat{\rho} \equiv J^\mu{}_{;\mu} = (G^\mu{}_\nu b^\nu (\mathbf{T}_{\text{field}}))_{;\mu}, \quad (G^\mu{}_\nu) \equiv (F^\mu{}_\nu)^{-1}. \quad (4)$$

Thus, charge conservation ($J^\mu{}_{;\mu} = 0$) is not true in general with the second group (2), in contrast with what happens with the second Maxwell group assumed in GR and other metric theories of gravitation [9].

Let Ω be any “substantial” domain of the charged continuum, i.e., it is followed in its motion. We can prove that the evolution rate of the charge contained in Ω is

$$\frac{d}{dt} \left(\int_\Omega \delta q \right) = \int_\Omega \hat{\rho} \sqrt{-\gamma} d^3 x, \quad \gamma \equiv \det(\gamma_{\mu\nu}) \quad (5)$$

in any coordinates x^μ . (Here $t \equiv x^0/c$.) Of course the domain Ω as well as its boundary depend on t in general spatial coordinates x^i .

4. Weak-Field Approximation of the Gravitational Field

The *gravitational field* is assumed weak and slowly varying for the system of interest S (e.g. the Earth with some EM source on it). We use an *asymptotic* post-Newtonian (PN) scheme. This scheme associates with S a *family* (S_λ) of systems, depending on $\lambda \rightarrow 0$, with $\lambda = 1/c^2$ in a specific λ -dependent time unit [7]. It

leads to write Taylor expansions with respect to c^{-2} , using also a λ -dependent mass unit. E.g.

$$\beta \equiv \sqrt{\gamma_{00}} = 1 - U c^{-2} + O(c^{-4}) \quad (6)$$

where $U \geq 0$ is the Newtonian potential, which obeys the Poisson equation [7]. The spatial metric assumed in the theory is [7]

$$\mathbf{g} = \beta^{-2} \mathbf{g}^0 \quad (7)$$

with \mathbf{g}^0 an invariable Euclidean metric. We will take Cartesian coordinates for \mathbf{g}^0 , i.e., $g_{ij}^0 = \delta_{ij}$.

5. Asymptotic Expansions for the EM Field and Current

We assume that \mathbf{F} and the 4-current \mathbf{J} depend smoothly on λ , hence they too admit Taylor expansions w.r.t. λ (with $\lambda = c^{-2}$ in the specific λ -dependent units)

$$\mathbf{F} = c^n (\overset{0}{\mathbf{F}} + c^{-2} \overset{1}{\mathbf{F}} + O(c^{-4})) \quad (8)$$

and

$$\mathbf{J} = c^m (\overset{0}{\mathbf{J}} + c^{-2} \overset{1}{\mathbf{J}} + O(c^{-4})). \quad (9)$$

The integers n and m are not known and can be positive, negative, or zero. Since $\lambda = 1/c^2$ is the gravitational weak-field parameter, this means that the fields \mathbf{F} and \mathbf{J} are not assumed weak. Moreover, \mathbf{F} is not assumed slowly varying: in this respect, the expansions (8)–(9) are similar to post-Minkowskian (PM) expansions.

6. Expansion of the Modified Maxwell Second Group

For the PM-like expansions (8)–(9), the time variable (such that the expansions are true at a fixed value of it) is $x^0 = cT$ [6, 12], not T as it is for PN expansions [7, 12]. (This is not neutral since $c = \sqrt{\lambda^{-1}}$ with λ the weak-field parameter.) Moreover, from dimensional analysis we get that, with the λ -dependent units, we have $\mu_0 = \mu_{00} c^2$ with μ_{00} a constant. From this, we find by examining the orders in equation (2) that we must have

$$2n = n + m + 2, \quad \text{i.e.,} \quad m = n - 2. \quad (10)$$

Using the foregoing, one gets the lowest-order term in the weak-field expansion of (2) as

$$\overset{0}{F}{}^\mu{}_\sigma \overset{0}{F}{}^{\sigma\nu}{}_{,\nu} = -\mu_{00} \overset{0}{F}{}^\mu{}_\sigma \overset{0}{J}{}^\sigma. \quad (11)$$

Thus if $\overset{0}{\mathbf{F}} \equiv (\overset{0}{F}{}^\sigma{}_\nu)$ is invertible, it is an exact solution of the flat-spacetime Maxwell equation

$$\overset{0}{F}{}^{\sigma\nu}{}_{,\nu} = -\mu_{00} \overset{0}{J}{}^\sigma. \quad (12)$$

7. Expansion of the Charge Production Rate

Due to equations (8)-(9), $\overset{0}{\mathbf{F}}$, $\overset{0}{\mathbf{G}}$, etc., do not have the physical dimensions of the corresponding fields \mathbf{F} , \mathbf{G} , etc. However, $\mathbf{F}_1 \equiv c^n \overset{0}{\mathbf{F}}$ and $\mathbf{J}_1 \equiv c^m \overset{0}{\mathbf{J}}$ are solutions of the flat-spacetime Maxwell equation with the correct dimensions in the SI units. Define T_1 as the EM T-tensor associated with \mathbf{F}_1 . Assume matrix F_1 , with components $(F_1)^\sigma{}_\nu$, is invertible. Define $\mathbf{G}_1 \equiv \mathbf{F}_1^{-1}$. Using (8), we can show that

$$\hat{\rho} = c^{-3} [(G_1^{\mu 0} T_1^{jj} - G_1^{\mu i} T_1^{0i}) \partial_T U]_{,\mu} + O(c^{-5}). \quad (13)$$

Expressing \mathbf{F} in terms of the electric and magnetic fields \mathbf{E} and \mathbf{B} , we can rewrite (13) as

$$\hat{\rho} = c^{-3} (e^i \partial_T U)_{,i} + O(c^{-5}) \quad (14)$$

with

$$e^i = \begin{pmatrix} \frac{B_1^3 c^2 + B_1 B_2^2 c^2 + B_1 B_3^2 c^2 + B_1 E_1^2 - B_1 E_2^2 - B_1 E_3^2 + 2 B_2 E_1 E_2 + 2 B_3 E_1 E_3}{2 c \mu_0 (B_1 E_1 + B_2 E_2 + B_3 E_3)} \\ \frac{B_1^2 B_2 c^2 + 2 B_1 E_1 E_2 + B_2^3 c^2 + B_2 B_3^2 c^2 - B_2 E_1^2 + B_2 E_2^2 - B_2 E_3^2 + 2 B_3 E_2 E_3}{2 c \mu_0 (B_1 E_1 + B_2 E_2 + B_3 E_3)} \\ \frac{B_1^2 B_3 c^2 + 2 B_1 E_1 E_3 + B_2^2 B_3 c^2 + 2 B_2 E_2 E_3 + B_3^3 c^2 - B_3 E_1^2 - B_3 E_2^2 + B_3 E_3^2}{2 c \mu_0 (B_1 E_1 + B_2 E_2 + B_3 E_3)} \end{pmatrix}.$$

To evaluate (14), we have to assess $\partial_T U$ and $\partial_T \nabla U$. Both have to be calculated in the preferred reference frame \mathcal{E} , since it is in that frame that the equations are valid. The system of interest producing the EM field should move through \mathcal{E} , with a velocity field \mathbf{v} . One expects that $|\mathbf{v}|$ should be in the range 10 – 1000 km/s. We can show that, for such velocities, the main contribution to $\partial_T U$ comes from the translation motion of a nearly spherically symmetric body (with center $\mathbf{a}(T)$) through \mathcal{E}

$$\partial_T U \simeq -\mathbf{V} \cdot \nabla U \simeq \frac{GM(r)}{r^2} \mathbf{V} \cdot \mathbf{e}_r, \quad r \equiv |\mathbf{x} - \mathbf{a}(T)|, \quad \mathbf{e}_r \equiv (\mathbf{x} - \mathbf{a}(T))/r$$

with $\mathbf{V} \equiv \dot{\mathbf{a}}$ and $M(r) \equiv 4\pi \int_0^r u^2 \rho(u) du$, $\rho(r)$ being the Newtonian mass density. On the Earth's surface, this gives $\partial_T U \simeq gV_r \lesssim 10V \simeq 10^5$ (MKSA) for $V = 10$ km/s. If moreover the rotating spherical body is homogeneous (which can be assumed to get an order-of-magnitude estimate), we have

$$\partial_T \nabla U = \frac{GM(r)}{r^3} \mathbf{V}. \quad (15)$$

On Earth: $\partial_T \nabla U \simeq g\mathbf{V}/R$, hence $|\partial_T \nabla U| \simeq 10^{-2}$ (MKSA) for $V = 10$ km/s.

8. Application to Representative Fields

8.1. Case of a Plane Wave

A monochromatic, “pure”, plane EM wave propagating in the direction $\mathbf{i} \parallel Ox$ has

$$E^1 = 0, \quad E^i = E_0^i \cos(kx - \omega T + \varphi_i), \quad i = 2, 3, \quad c\mathbf{B} = \mathbf{i} \wedge \mathbf{E}. \quad (16)$$

Then of course $\mathbf{E} \cdot \mathbf{B} = 0$, hence the field matrix $\mathbf{F} \equiv (F^\mu{}_\nu)$ is not invertible. But we may add any constant EM field $(\mathbf{E}', \mathbf{B}')$: the field is still a plane wave (though not a pure one, in that the condition $c\mathbf{B} = \mathbf{i} \wedge \mathbf{E}$ is lost). Then generically \mathbf{F} is invertible. Moreover, e^i [equation (14)] has $e^i{}_{,i} = 0$, for $e^1 = 0$ and $e^i = e^i(x^1)$. Neglecting the term $c^{-3}e^i(\partial_T U)_{,i}$ in view of (15) and the extremely small figure $c^{-3} \simeq 3.7 \times 10^{-26}$ (MKSA), we get that

$$\hat{\rho} = 0 \quad (\text{Plane wave, } c^{-3}e^i(\partial_T U)_{,i} \text{ neglected}). \quad (17)$$

However, for very strong values of the constant EM field, the neglected term may give high values of $\hat{\rho}$. (Check the case without the wave part.)

8.2. The Case with Hertzian Dipoles

Hertz’s oscillating dipole is the charge *distribution*

$$\rho = T_{\mathbf{d}, \mathbf{b}, \omega} \equiv -e^{-i\omega t} \mathbf{d} \cdot \nabla \delta_{\mathbf{b}} \quad (18)$$

with \mathbf{b} the dipole position, and \mathbf{d} the dipole vector. The associated three-current is

$$\mathbf{j} = -i\omega \mathbf{d} e^{-i\omega t} \delta_{\mathbf{b}}. \quad (19)$$

With this source distribution, corresponds a solution [13, 16] of the flat Maxwell equations (in the distributional sense)

$$\mathbf{E} = \alpha \left\{ \frac{k^2}{r} (\mathbf{d} - (\mathbf{n} \cdot \mathbf{d})\mathbf{n}) \cos \varphi + [3(\mathbf{n} \cdot \mathbf{d})\mathbf{n} - \mathbf{d}] \left(\frac{\cos \varphi}{r^3} + \frac{k \sin \varphi}{r^2} \right) \right\} \quad (20)$$

$$\mathbf{B} = \beta k^2 (\mathbf{n} \wedge \mathbf{d}) \left(\frac{\cos \varphi}{r} - \frac{\sin \varphi}{kr^2} \right), \quad k = \frac{\omega}{c}, \quad \varphi \equiv kr - \omega t.$$

Here $\alpha \equiv \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$, $\beta \equiv \frac{c}{4\pi} \simeq 2.39 \times 10^7$ (MKSA). This solution can be shown to be exact. It has $\mathbf{E} \cdot \mathbf{B} = 0$. However, adding dipoles with different \mathbf{b} 's and \mathbf{d} 's gives generically $\mathbf{E} \cdot \mathbf{B} \neq 0$.

We thus consider a group of Hertzian dipoles. All of them are at rest in a common frame $\mathcal{E}_{\mathbf{V}}$, moving at a constant velocity \mathbf{V} w.r.t. \mathcal{E} . Their EM field is obtained in the moving frame $\mathcal{E}_{\mathbf{V}}$ by summing the fields (20) of each dipole. It is then Lorentz-transformed to \mathcal{E} . In view of (14), we compute

$$\hat{\rho}(T, \mathbf{x}) = c^{-3} (e^i \partial_T U)_{,i} \approx c^{-3} \int_{\partial C_{\mathcal{E}}(T)} e^i n_i \partial_T U \, dS/v(C_{\mathcal{E}}(T)). \quad (21)$$

Here, C being a small cube bound to the moving frame $\mathcal{E}_{\mathbf{v}}$, centered at the calculation point \mathbf{x} , we denote $C_{\mathcal{E}}(T)$ the domain occupied by this cube at the time T in the ether frame \mathcal{E} . (\mathbf{n} is the external normal to the boundary $\partial C_{\mathcal{E}}(T)$ of the domain $C_{\mathcal{E}}(T)$, dS is the surface element on $\partial C_{\mathcal{E}}(T)$, and $v(C_{\mathcal{E}}(T))$ is the volume of $C_{\mathcal{E}}(T)$.) For three dipoles with $d = 100 \text{ nC}\cdot\text{m}$, $\nu = 100 \text{ MHz}$ ($\lambda = 3 \text{ m}$), situated at $\lesssim \lambda$ from one another, we get fields $E \lesssim$ a few 10^5 V/m , $B \lesssim 15 \text{ T}$. With $V = 10 \text{ km/s}$, $\hat{\rho}(T, \mathbf{x})$ has peaks at $\approx \pm 2 \times 10^8 \text{ e/m}^3/\text{period}$. This seems untenable, even though the peaks are very narrow and their sign alternates in space.

Hence, this version of the gravitationally-modified Maxwell equations seems discarded.

9. The Reason for the Problem and its Solution

9.1. Why Were not These the Right Maxwell Equations of the Theory?

In SET, the dynamical equation for a general continuous medium having a well-defined velocity field \mathbf{v} , subjected to an external force density field \mathbf{f} , is [9]

$$T_{\text{medium};\nu}^{0\nu} = b^0(\mathbf{T}_{\text{medium}}) + \frac{\mathbf{f}\cdot\mathbf{v}}{c\beta}, \quad T_{\text{medium};\nu}^{i\nu} = b^i(\mathbf{T}_{\text{medium}}) + f^i. \quad (22)$$

Assumption i) (Section 2) says: $\mathbf{T}_{\text{total}} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}}$.

Assumption ii) (Section 2) states: $T_{\text{total};\nu}^{0\nu} = b^0(\mathbf{T}_{\text{total}})$, $T_{\text{total};\nu}^{i\nu} = b^i(\mathbf{T}_{\text{total}})$.

Assumptions i) and ii), plus equation (22) with “medium” = “charged medium”, lead easily to

$$T_{\text{field};\nu}^{0\nu} = b^0(\mathbf{T}_{\text{field}}) - \frac{\mathbf{f}\cdot\mathbf{v}}{c\beta}, \quad T_{\text{field};\nu}^{i\nu} = b^i(\mathbf{T}_{\text{field}}) - f^i. \quad (23)$$

This has the form (22) applied to the EM field (“medium” = “field”), with $f_{\text{field}}^i = -f_{\text{charged medium}}^i \equiv -f^i$ and $\mathbf{v}_{\text{field}} = \mathbf{v}_{\text{charged medium}} \equiv \mathbf{v}$. But $\mathbf{v}_{\text{field}} \neq \mathbf{v}_{\text{charged medium}}$! Also, the action-reaction opposition, $f_{\text{field}}^i = -f_{\text{charged medium}}^i$, is not true in a theory with finite velocity of propagation [15, 18].

9.2. What Are the Right Maxwell Equations of the Theory?

Equation (22) is derived from Newton’s second law of the theory, at least if the continuous medium behaves like a dust. Also, Assumption ii) is necessary to the theory of gravitation. However, we may abandon Assumption i), which means to write

$$\mathbf{T}_{\text{total}} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}} + \mathbf{T}_{\text{interact}} \quad (24)$$

where $\mathbf{T}_{\text{interact}} \equiv \mathbf{T}_{\text{total}} - \mathbf{T}_{\text{charged medium}} - \mathbf{T}_{\text{field}}$ is some “interaction energy tensor”. With (24), Assumption ii) and equation (22) do not imply equation (23) any more, and do not determine the second group any more. One may postulate the *standard* gravitationally-modified second group (e.g. [14, 20])

$$F^{\sigma\nu}_{;\nu} = -\mu_0 J^\sigma \quad (25)$$

which, one may show [10], is writing almost the usual (three-vector-form) second group in terms of the local time and the space metric in the frame \mathcal{E} .

10. Conclusion

Maxwell equations for the “scalar ether theory” of gravity (SET) were proposed in a previous work [9]. They predict charge non-conservation in a variable gravitational field. This occurs already for a translation through SET’s “ether”. Using asymptotic PN and asymptotic “PM-like” expansions for the gravitational field and the EM field respectively, an explicit expression for the charge production rate $\hat{\rho}$ was obtained. For a group of Hertzian dipoles producing a strong but realistic EM field (and with a moderate translation velocity $V = 10$ km/s), $|\hat{\rho}|$ seems unrealistically high. In fact, those Maxwell equations are not consistent with the continuum dynamics of SET as applied to the EM field itself. There must be an additional, “interaction”, energy tensor. Then the standard gravitationally-modified Maxwell equations become consistent with SET. The interaction energy might contribute to the “dark matter” [10].

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