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THE ELASTICITY OF QUANTUM SPACETIME FABRIC

VIOREL LAURENTIU CARTAS

Faculty of Engineering and Agronomy of Brail, "Dunarea de Joss" University of Galati, Domneasca Str., Galati 47, Romania

Abstract. The present paper aims to emphasize the geometrical features of the quantum spacetime, considering gravity as an emergent feature similar to the elasticity of the solid state. A small scale structure is needed to explain the emergent gravity and how spacetime atoms are continuously created in the process of the expansion of the universe. A simple geometrical model has been introduced.

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1. Introduction

After 10 years of hard work, Einstein has succeeded to construct a General Theory of Gravity. He has started from Riemann achievements into the differential geometry. He has developed the tensor calculus and eventually put his well-known equations on paper [3]. His stem work created a plethora of very technical scientific papers written by many renown and high class physicists. The central idea is that the notion of gravitational force is replaced by the curvature of the spacetime fabric. The surface of the spacetime fabric is curved in the presence of mass. "The mass tells to spacetime how to curve and the curved spacetime tells to the body bearing this mass how to move". This is not so easy to deal with. Firstly, Einstein made a strong and strange assumption: the space time is not only a mathematical tool, anymore. Actually the spacetime is a real object with structure and properties which can be study in the frame of physics. Confident with this assumption, Einstein went further and has predicted a lot of things, the gravitational waves being one of the most interesting predictions. The quest for the gravitational wave has lasted long time. Actually, it lasted decades. During this time, skeptical voices had risen but when the necessary technology did appear in 2015, the gravitational waves were eventually, successfully observed [1]. Few physicists begin to take into consideration the elastic nature of the spacetime fabric [16–18]. Even now I am not sure if I should use the quotation marks on the word "elastic". Bravely enough, we say that the most revolutionary contribution Einstein had to science was formulating this statement: the fabric of spacetime is elastic. From now on, a wide perspective is open.

2. The Elasticity of Spacetime

We are allowed to consider the spacetime similar to the solid state, even to search for a certain spacetime structure [4], [6], [13], [14]. If we reduce it to only one temporal dimension and only one spatial dimension (1, 1) then we have a surface which could be stress in the time dimension, implying a spatial compression. Because of the peculiar elastic properties of the spacetime fabric such a phenomenon is possible at relativistic velocities or at significant deformation of the spacetime (aka intense gravity). So, we can consider time dilation together with the length contraction: $\varepsilon_{11}{\sim}\frac{1}{\gamma}$, $\varepsilon_{12}{\sim}\gamma$, $\varepsilon_{21}= \varepsilon_{22}$=0. Here ε_{11}, ε_{12}, ε_{21} and ε_{22} are the components of the strain tensor and γ=$\sqrt{(1-\frac{\phi}{c^2})}$, with ϕ , the gravitational potential (Fig.1).

Considering the Liouville theorem, the volume of the space time fabric (actually in this case, the spacetime surface) has to be constant. There is a consequence called the Poisson effect: ε_{11}=$-\nu{\cdot}\varepsilon_{12}$, where ν is the Poisson's ratio and has a value of one for our approach. If we return to the four dimensional case (3,1), the tensor form of the Hooke's theory of elasticity states that

$$\sigma^{ij} = K(g^{mi}g^{nj} - g^{mn}g^{ij})\varepsilon_{mn}. \tag{1}$$

This is true if we consider a global isometry of the spacetime fabric. K is half of the Young's constant and g^{mn} is the metric tensor. The σ^{ij} is the stress tensor. We can also use the rank-four elastic stiffness tensor C^{ijmn}, as Cauchy did

$$\sigma^{ij} = C^{ijmn}\varepsilon_{mn}\,. \tag{2}$$

We have used the Einstein's summation convention, m and n being the dummy indices. This tensor equation (2) characterizes all the material objects from the macroscopic point of view. Actually, the elasticity is an electric feature of the matter and is emergent. It implies a microscopic structure and microscopic (rather nanoscopic) constituents. Hence, the similitude with the behavior of the spacetime fabric, is revolutionary. Nevertheless, Einstein has introduced the celebrated dependence between stress-momentum tensor and the curvature of the spacetime, which is not only similar but (we may say) is even the same with the Cauchy's relation (2)

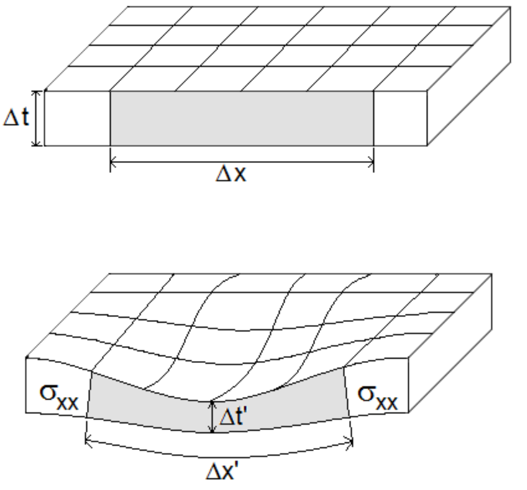

Figure 1. The deformation of the spacetime, due to the presence of a heavy body.

$$8\pi k T_{ij} = R_{ij} - \frac{1}{2} g_{ij} R\,. \tag{3}$$

Again g_{ij} is the metric tensor. Here $8\pi k$ is a constant, meant to fit the Einstein's equation to the Newton's law. R_{ij} is the first contraction and R is the second contraction of the Riemann tensor. R^i_{jlk} is the Riemann curvature tensor, R_{ij} is the Ricci tensor and R is the Ricci scalar curvature

$$R_{jlk} := R^i_{jlk}, \qquad R := g^{ij} R_{ij} = R^j_j. \tag{4}$$

A heavy body bends the membrane which is the fabric of the spacetime, an extreme heavy body may even disrupts the fabric, which corresponds to the existence of a blackhole [7], [10], [15]. The disruption does appear at a certain value of the mass which means that the spacetime fabric features a universal constant. The stress-energy tensor determines the spacetime to compress on transversal direction and to expand longitudinally. So the spacetime object bends. If we consider a membrane which is already stressed and have a supplementary dot of material on it, than the membrane bends. On the contrary, if instead a supplementary material we make a hole into the membrane, than it will present an opposite bending. We can introduce a shallow analogy with the presence of a finite mass (in the first case) and a black hole (in the second). Some physicist are working at this analogy.

Considering the elasticity of spacetime we can refer to a very interesting idea: the elastic dark energy of the universe as the energy of the initially compressed, elastic spacetime of the actual universe (like an elastic strain fluid). The work of Riess [12] and Perlmutter [11] establishes that the universe is accelerated. Because of this acceleration the physicists have to take into consideration the existence of the dark matter featuring, which we use to call nowadays, the dark energy. The dark energy is related to the cosmological constant, Λ, which Einstein was not too sure about. This is the base of a new model, the Λ CDM model (cool dark matter model). It fits well with the experimental data. Considering the fabric of spacetime as an isotropic fabric we use the elastic density of energy in vacuum, which is scalar and global

$$w = \frac{1}{2}\sigma^{ij}\varepsilon_{ij} = \frac{1}{2}C^{ijmn}\varepsilon_{ij}\varepsilon_{mn}\,. \tag{5}$$

Searching for a similar equation in the domain of cosmological gravity, we have to address to the Friedmann theoretical work. Alexander Friedmann has established his cosmological equations [9] that deal with the expansion of spacetime in a homogeneous and isotropic universe, considering the general relativity

$$w = k \cdot (\frac{\dot{a}}{a})^2 = k \cdot H^2. \tag{6}$$

Here, a is a function of time which is called the scalar cosmological factor and it is dimensionless. H is the Hubble parameter and is similar to the strain of an elastic physical object.

The density of energy in vacuum is also proportional to H squared. Regarding the scalar cosmological factor, it relates the proper distance between a pair of physical objects at an arbitrary time t to their distance at some reference time t_0

$$d(t) = a(t) \cdot d_0 \tag{7}$$

where $d(t)$ is the proper distance at time t and d_0 is the distance at the reference time t_0. The similitude between (7) and (8) constitutes another argument for the elasticity of spacetime fabric. The elasticity is an emergent feature of the solid state. Trying to describe the spacetime from the quantum point of view we'll find that it is very different from the classical description (in the same manner the quantum theory of the solid state is different from the macroscopic continuum theory of the solid objects). At the nanoscopic level, we do not use density, stress, strain, etc. All these are macroscopic concepts. Such variables could not be quantized in order to build a proper quantum description of molecules. The continuity concept is only a limit of discontinuous description of the world. Continuity is emergent. So it is the elasticity and so it is the gravity. The conclusion of all these is that the spacetime has to have a small-scale structure, which is granular. The "atoms"of the space time have to be subjects of a peculiar force. From the elastic point of view, there are good arguments for the Casimir force to qualify as the necessary force. This force acts on the spacetime at a certain small level. In this respect, the assertion that the Einstein's theory of General Relativity is incomplete, because at atomic scale the gravity appears to be completely unimportant (even, is well tested from cosmological scale down to a millimeter range) is not valid anymore. Maybe, there is not such a thing like Quantum Gravity. Gravity emerges at large scale, like elasticity does. Quantum description is valid for the small scale (sometimes for the macroscopic scale, when there is a coherence for a large number of constituents), but we do not find any gravity at that small scale. At the small scale we only find a structure of spacetime which is determined by the quantum fluctuation. In 1995 some physicists have introduced another approach to the theory of gravity. They have considered that in every spacetime point exist very small black holes. Their peculiar approach, based on the thermodynamic concepts, sent them to the same general gravity's results Einstein has founded using the idea of bending the spacetime fabric. Ted Jacobson (the University of Maryland in College Park) considers that the statistic which is involved into this approach is a powerful reason to take into consideration that the spacetime fabric has an "atomic" structure [5].

3. The Interaction Between Spacetime "Atoms"

Introducing a mathematical model in order to put the idea of spacetime "atoms"at work, is a difficult task. We have only few points of departure: the spacetime is discrete, the spacetime "atoms"are permanently and everywhere, moving apart, in the supplementary room between the old generation of spacetime "atoms"does appear a new generation of spacetime "atoms". We do not know if the small-scale means quantum scale. Maybe the small scale refers to a new and unknown world which is situated under quantum dimension and has the quantum world as a limit (in the same way the quantum physics describes the classical physics at the proper limit). Anyway, considering that the small-scale is quantum, than we may calculate the quantum force between the spacetime "atoms". We take into consideration a pair of free, identical such "atoms", caring a certain amount of energy W. Our system is one-dimensional and spineless. The wave function corresponding to a quantum spacetime "particle" is real and well localized in the vicinities of the point $+a$ and the point $-a$

$$\psi_{\mp}(x) = (\frac{\beta}{\pi})^{\frac{1}{4}}\ \mathrm{e}^{-\frac{\beta}{2}(x\pm a)^2}. \tag{8}$$

For a parameter $\beta \gg a^{-2}$, the states of our "particles" are very well localized. The wave function for the two particle system is described by

$$\psi(x_1 - x_2) = N[\psi_+(x_1)\psi_-(x_2) \pm \psi_-(x_1)\psi_+(x_2)] \tag{9}$$

with N

$$N = \frac{1}{\sqrt{2}}(1 \pm \mathrm{e}^{-2\beta a^2})^{-1/2}. \tag{10}$$

The bosons correspond to the symmetry (+) and the fermions to the anti-symmetry (-). The Hamiltonian which describes the system is $H = \frac{c^2}{2W}(p_1^2 + p_2^2)$ and so, the energy of the two particle system is

$$E = \frac{\hbar^2 c^2 \beta}{2w} \cdot \frac{1 \pm \mathrm{e}^{-2\beta a^2}(1 - 2\beta a^2)}{1 \pm \mathrm{e}^{-2\beta a^2}}. \tag{11}$$

The force between the spacetime "atoms" depends on the kind of particles we consider. Fermions will always give a repulsive force and bosons will give an attractive force when the distance is big enough ($\beta \gg a^{-2}$) and a repulsive one when the separation is very short $\beta \leq a^{-2}$

- for fermions

$$F = -\frac{\partial E}{\partial a} = \frac{2\hbar^2\beta^2 c^2 a}{w} \cdot \frac{\mathrm{e}^{-2\beta a^2}(-1 + 2\beta a^2 + \mathrm{e}^{-2\beta a^2})}{(1 - \mathrm{e}^{-2\beta a^2})^2} \tag{12}$$

- for bosons

$$F = -\frac{\partial E}{\partial a} = \frac{2\hbar^2\beta^2 c^2 a}{w} \cdot \frac{\mathrm{e}^{-2\beta a^2}(1 - 2\beta a^2 + \mathrm{e}^{-2\beta a^2})}{(1 - \mathrm{e}^{-2\beta a^2})^2}. \tag{13}$$

Both formulae above allow us to consider that the elasticity of spacetime fabric can emerges at large scale and constitutes a substitute to gravity. Unfortunately we do not know if at that scale quantum physics acts and so, we are lead to a more simple model. If we consider the effect of the quantum fluctuation at zero point energy level we find a geometric and even a topological force between cavity walls (simple borders of a geometric domain) at a distance d, which is the celebrated Casimir force [2], [8] (Hendrik Brugt, Gerhard Casimir, 1948)

$$F = \frac{\pi^2 \hbar c}{240} \cdot \frac{1}{d^3} \tag{14}$$

may be attractive or repulsive depending on the geometry. The Casimir force belongs to the quantum theory and thus the problem of dimensionality is not solved.

4. A Spacetime Simple, Foam Like Model

We have to introduce a simpler approach. In Fig. 2, it is represented how the "atoms" of a 1+1 dimensional spacetime fabric, evolve. The exterior "atoms" are faster.

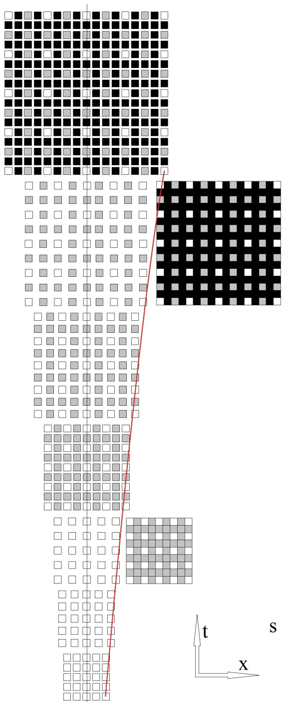

Figure 2. A simple expansion model.

Actually, the entire movement is accelerated. At every new stage, the "atoms"spread on and new entities emerge from nowhere (nothingness) filling the room which the expansion of the spacetime produces everywhere and every time. The time is passing in stages, labeled by natural numbers. The speed of expansion depends on these stages. If the stage is k then the speed is proportional to a simple geometric function (Fig. 3)

$$v = b \cdot \frac{1 + 2^k}{k} \tag{15}$$

in which b is constant. At a later stage the speed is bigger (Fig. 3).

Though, this representation is very poor, it allows us to conclude that the expansion is accelerated and the spacetime is granular and the "atoms" of spacetime are brought into existence continuously, everywhere. As we can see in Fig. 2, we have four initial atoms which spread on and between them it is created enough room for eight new atoms to emerge. The contribution at this creation is only a halved because of the vicinity of other eight initial atoms. So, we have only four offspring. This kind of evolution is very well studied in the frame of the Markov's chains. The method is called "the branching process". We consider$A_0 = 4$ the initial number of the atoms-parents. The energy carried by the offspring is one and is sheared in this way: $E_1 = n_1, ..., E_4 = n_4$. A little amount of energy remains to the parent $E_0 = n_0$. $n_1 + n_2 + n_3 + n_4 = 1$ represents the total emergent energy. Because of this mechanism it is possible to appear extinction and a certain number of offspring aborts. In order to compute the probability of extinction we take into account the generating function

$$\varphi(z) = n_0 + n_1 z + n_2 z^2 + n_3 z^3 + n_4 z^4 \, . \tag{16}$$

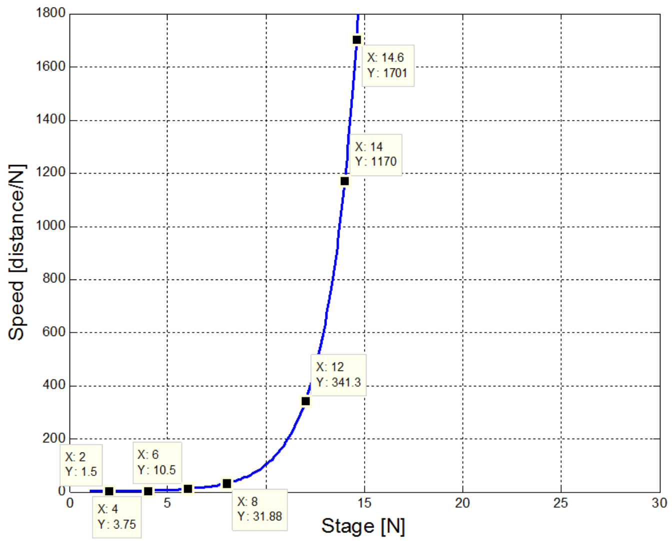

Figure 3. The speed of the spacetime "atoms" depends on the different stages of the expansion; the expansion is accelerated.

The smallest $z, z > 0$ which does obey

$$\varphi(z) = z \tag{17}$$

will give us the probability of extinction μ. The equation is: $\varphi(z) = n_0 + n_1 z + n_2 z^2 + n_3 z^3 + n_4 z^4 = z$ and the solutions are z_1, z_2, z_3, z_4. So, $\mu = z_1 \cdot z_2 \cdot z_3 \cdot z_4$. The mean number of offspring per atom is

$$l = 1 + n_0\,. \tag{18}$$

The mean size of the n^{th} generation is

$$N = l^n\,. \tag{19}$$

The iteration method used in this kind of Markov's chain gives us

$$\varphi_n^{''}(1) = \varphi^{''}(1) \sum_{k=n-1}^{2n-2} l^k. \tag{20}$$

The standard deviation, squared is

$$\sigma_n^2 = \varphi^{''}(1) \sum_{k=n-1}^{2n-2} l^k + l^n - l^{2n} = (4 - l) l^{n-1} (l^n - 1). \tag{21}$$

Because we have four parents, the probability of extinction will be μ^4, the mean size of n^{th} generation will be $4l^n$ and the standard deviation of the size of the n^{th} generation is $2\sigma_n$.

Once again, we conclude at the end of this paper, that the spacetime is elastic, the gravity is emergent and the gravity is nothing else then elasticity of the fabric of spacetime. Considering the fabric of spacetime being elastic, we are lead to this revolutionary conclusion that the spacetime has an inner structure and this means that the spacetime has a granular texture. The "atoms" of the spacetime fabric spread on in order to make room for new generations of "atoms". Between these peculiar "atoms", has to act a peculiar force which entitle us to consider an emergent elasticity. We do not know yet what kind of force acts into this structure and we do not know what kind of physics governs all these phenomena. Is frightening that we can not even imagine where the new generation of spacetime "atoms" comes from. These are tremendous questions waiting for tremendous answers.

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