THEORETICAL MODELS FOR ASTROPHYSICAL OBJECTS AND THE NEWMAN-JANIS ALGORITHM

ROSANGELA CANONICO, LUCA PARISI and GAETANO VILASI

Dipartimento di Fisica "E.R. Caianiello", Università di Salerno
I-84084 Fisciano and INFN, Sezione di Napoli, GC di Salerno, Italy

Abstract. We consider the problem of finding exact solutions of Einstein equations describing gravitational fields generated by isolated sources in order to provide theoretical models for astrophysical objects. To this aim, the Newman-Janis Algorithm is described along with the main results obtained through it and some ambiguities arising in dealing with the Newman-Janis Algorithm are reviewed. Many issues related to the introduction of a cosmological constant term are also pointed out and some detailed examples are discussed.

1. Introduction

General Relativity is a fundamental tool in the description of astrophysical system ranging from the solar system scales to the cosmological ones. Thus one would be able to describe, within this framework, compact isolated astrophysical objects in stationary rotation like neutron stars and white dwarfs. A description of these objects proves to be a very hard task requiring the use of both Quantum Mechanics and Relativity.

To give the idea of how complex is the physics involved in such kind of systems, let us shortly consider the internal structure of a neutron star. It consists of four main shells, whose composition changes dramatically with the radial distance from the center, because of the strong density gradient from the exterior to the interior of the star: *Outer crust*: a lattice of ionized nuclei and a degenerate relativistic gas of electrons (e). *Inner crust*: composed of rich nuclei in β-equilibrium, degenerate relativistic gas of e and a degenerate gas of neutrons (n) in the superfluid phase. *Outer core*: consists of superfluid of n, superconducting protons (p) and a mixture of degenerate e,p and possibly muons. *Inner core*: the last layer whose composition is largely unknown.

An accurate description of the internal structure of these systems and of the gravitational field outside them, can be gained only through the use of numerical methods with supercomputers. Nevertheless, many of the most important results concerning the physics of stars were already acquired in the first half of the twentieth century through the study of extremely simplified models.

For instance, the quantum statistic of identical particles, namely the Fermi-Dirac distribution, was applied for the first time just to the description of an astrophysical body, the White Dwarf Sirius B (Fowler, 1926). This model was non-relativistic, as noticed by Chandrasekhar who, through relativistic kinematic corrections, provided a better description leading to the discovery of its famous limiting mass in 1934. A major improvement in the field was then gained with the study of spherically symmetric body of isotropic matter in static gravitational equilibrium due to Tolman, Oppenheimer and Volkoff. It led to the first model of neutron star, an ideal mixture of nuclear particles in 1939. Since then, a plethora of interesting discoveries was made which aim to provide a complete description of them that seems to be a hopeless task. It is worth to stress the fact that most of the previous analytical results were obtained under the hypothesis of spherical symmetry (that is, non rotating bodies). The generalization to the (slowly or fast) rotating case is quite complicate and only a few approximate analytical results being available and it is usually approached via numerical techniques.

Here, we are mainly concerned with the problem of generating suitable exact solutions of Einstein equations, in order to provide simple models capable to describe the gravitational field generated by an isolated compact body in stationary rotation with extremely simple internal structure.

The vacuum gravitational field, generated by an isolated star in stationary rotation, should be *axial symmetric* and *asymptotically flat*. The prototype model for such a gravitational field is the well known Kerr metric. One may think that the job would be easily done just by extending the solution to the internal part of the star considering a suitably chosen, very simple, matter fluid fulfilling the strong and weak energy conditions. However, this is not at all an easy task and, as matter of fact, the internal Kerr solution is still unknown. The problem of finding possible Kerr sources is that, in order to obtain a physically sensible mass distribution, many restrictions must be imposed. E.g., the metric must be joined smoothly to the Kerr one on a reasonable surface for a rotating body and the hydrostatics pressure must vanish on such a surface. The star has to be a non-radiating source and, in the static limit, a reasonable Schwarzschild interior metric must be obtained. Finally, the energy conditions must hold.

In recent years, many papers appeared concerning the possibility of facing this problem through a combined employment of the Newman-Janis Algorithm (henceforth NJA), a generating technique which provides metrics of reduced symmetries

from symmetric ones, and the Darmois-Israel junction conditions which, through the continuity of the first fundamental form ($[h_{ab}] = 0$) and the second fundamental form ($[K_{ab}] = 0$), ensure the correct smooth joining of the interior and the exterior solutions at a chosen hypersurface Σ^1.

In this paper we focus on the NJA as a solution generating technique in General Relativity capable to provide physically meaningful solutions of the Einstein equation in the presence of matter. The idea is to employ the NJA in order to find sensible internal solutions for the gravitational field of isolated sources to be joined with the known external one. Some ambiguities arising in dealing with the NJA, related with the introduction of a cosmological constant term and the interpretation of matter sources, are pointed out and some detailed examples are discussed.

The paper is structured as follows. In Section 2, we provide a brief historical introduction to the NJA along with the main results obtained through it. In Section 3, we describe in some detail the NJA by considering the first two exact solutions obtained through it: a vacuum solution (Kerr) and an electro-vacuum solution (Kerr-Newman). In Section 4, some specific examples are presented. In particular, in subsection 4 the introduction of a cosmological constant term in the seed metric is considered along with some remarks on various metrics dubbed *rotating de Sitter*. In subection 4.1, further examples of the NJA are discussed, considering the Schwarzschild-de Sitter solution and the Reissner-Nordström-de Sitter solution as seed metrics. We show that in these cases the NJA does not provide the usual Kerr-de Sitter solution and Kerr-Newman-de Sitter as one might have naively expected. We also show that the new metrics do not allow for a simple interpretation of the required sources, it can not be neither a simple cosmological constant, nor a perfect fluid or an electromagnetic field.

2. The Newman-Janis Algorithm

After the discovery of the Kerr vacuum solution, describing the geometry of spacetime around a rotating massive body, Newman and Janis showed that this solution of the Einstein equations can be straightforwardly derived by performing a suitable complex coordinate transformation on the Schwarzschild vacuum solution [8]. In a following paper, Newman and Janis applied the same method to the non rotating charged **Reissner-Nordström solution**, to get the rotating charged Reissner-Nordström one, known as the **Kerr-Newman solution** [7].

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[1]If this condition is violated, then the spacetime is singular at hypersurface and a thin shell with surface stress tensor $S_{ab} = -\epsilon/8\pi([K_{ab}] - [K]h_{ab})$ is present at hypersurface. We can see that when $[K_{ab}] \neq 0$, only the Ricci part of the Riemann tensor acquires a singularity, and it is this part that can be associated with the matter. The remaining part of the Riemann tensor, the Weyl part, is smooth even when the extrinsic curvature is discontinuous.

In general, the NJA can be used to derive stationary and axially symmetric metrics from static spherically symmetric ones, thus the method turns out to be suitable for studying rotating systems in General Relativity. The NJA was used in the attempt to obtain new possible sources for Kerr metric. In [4] the algorithm was indeed applied to an interior spherically symmetric metric to describe an internal source model for the Kerr exterior solution. The resulting metric was then matched to the vacuum Kerr solution for the oblate spheroid. In [2] a further attempt to obtain new possible sources for Kerr metric was made. Firstly, the NJA was applied to a generic static spherically symmetric seed metric. The resulting class of metrics was written in Boyer-Lindquist type coordinates. Then, the joining of any two stationary and axially symmetric metrics belonging to this class was performed considering a suitable separating hypersurface determined by imposing the Darmois-Israel conditions, thus having a vanishing surface stress-energy tensor.

Subsequently, these results were considered as starting point to perform a generalization of the algorithm and to demonstrate why this method is successful. To do this, is necessary to remove some of the ambiguities present in the original derivation, as shown in [1], where it was also shown that the only perfect fluid space-time generated by applying the NJA to a static spherically symmetric seed metric, is the Kerr metric and that the Kerr-Newman metric is the most general algebraically special space-time which can be so obtained. The connection with [2] is in the fact that, while the NJA is successful in generating interior space-times, which match smoothly to the Kerr metric and are considered as perfect fluids in the non rotating limit, this is not the case when rotation is included.

In a recent paper [10], a simple constructive approach was presented to get a class of interior solutions that could act as physically reasonable Kerr sources starting from the incompressible perfect fluid Schwarzschild metric. Furthermore, the perturbative expansion at the first order in the parameter a (Slowly Rotating Limit) of the solutions previously obtained was discussed and some remarks on the energy conditions for these solutions were collected. Finally, the NJA was applied to the static, anisotropic, conformally flat solutions found by Stewart [9], leading to interior Kerr solutions with oblate spheroidal boundary surfaces.

The NJA was also used to obtain new metrics describing more general and complicate systems. This is the case of [6, 11], where a rotating radiating charge mass in a de Sitter cosmological background was studied. These new metrics may play an important role in order to incorporate relativistic aspect of Hawking's radiation in GR, as noticed in [5] where an application of NJA to generate rotating non-stationary metrics from spherically symmetric ones was considered. The seed metric in [5] was written in terms of the functions $M(u,r)$ and $e(u,r)$, where u and r are the coordinates of the space-time geometry, in particular the u-coordinate is

related to the retarded time in flat space-time. After the transformation of the metric through the NJA, these two functions depend on the three coordinates (u, r, ϑ). Then the Wang-Wu functions, which are an expansion of $M(u, r)$ in powers of r, were introduced in the rotating solution to generate new embedded rotating solutions like Kerr-Newman-de Sitter.

Finally, we stress the fact that the solutions found in [6] are quite different from those found in [5], as noticed by Ibohal himself. This is mainly due to the slightly different approaches followed by the authors and is an example of the ambiguities arising from the NJA already mentioned above. In both cases, the authors provide with a full description of the energy-momentum tensor required by these metrics in order for them to be solutions of the Einstein equations.

3. The Method

In this section we describe the NJA in details considering the two most famous results obtained through it [7,8]. This will give us the chance to set up the formalism and stress some ambiguities in the method that will be further discussed in the next section.

Following [8], we derive the Kerr solution from the Schwarzschild one through the NJA. Let's start by writing the Schwarzschild metric, considered as a static spherically symmetric seed metric, in advanced Eddington-Finkelstein coordinates

$$\mathrm{d}s^2 = \left(1 - \frac{2m}{r}\right) \mathrm{d}u^2 + 2\mathrm{d}u\mathrm{d}r - r^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2).$$

The contravariant metric components can be written as

$$g^{\mu\nu} = l^\mu n^\nu + l^\nu n^\mu - m^\mu \bar{m}^\nu - m^\nu \bar{m}^\mu \tag{1}$$

where $(l^\mu, n^\nu, m^\mu, \bar{m}^\nu)$ is the null tetrad defined by

$$l^\mu = \delta_1^\mu$$
$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - \frac{2m}{r}\right)\delta_1^\mu$$
$$m^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$
$$\bar{m}^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

This complex null tetrad system is the starting point for the derivation of Kerr space-time. Now, let the coordinate r takes complex values and $\bar{r}$ denotes its complex conjugate quantity

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - m\left[\frac{1}{r} + \frac{1}{\bar{r}}\right]\right)\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}\bar{r}}\left(\delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

Then it is possible to perform the following complex coordinate transformation on the null vectors

$$r' = r + ia\cos\vartheta$$

$$u' = u - ia\cos\vartheta$$

$$\vartheta' = \vartheta$$

$$\varphi' = \varphi$$

where a is a real parameter. By requiring that r' and u' are real (that is considering the transformations as a complex rotation of the ϑ, φ plane), one obtains the following new tetrad

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - \frac{2mr'}{r'^2 + a^2\cos^2\vartheta}\right)\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}(r' + ia\cos\vartheta)}\left(ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}(r' - ia\cos\vartheta)}\left(-ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

The contravariant components of a new metric can be defined from the above null vectors according to equation (1). This produces the promised Kerr solution in advanced null coordinates.

The same procedure can be used to get the Kerr-Newmann metric from the Reissner-Nordström one which, in advanced null coordinates, has the following form

$$\mathrm{d}s^2 = \left(1 - \frac{2m}{r} - \frac{e^2}{r^2}\right)\mathrm{d}u^2 + 2\mathrm{d}u\mathrm{d}r - r^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2).$$

For this space-time the null tetrad is

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - \frac{2m}{r} - \frac{e^2}{r^2}\right)\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

After the complexification of the radial coordinate r, the tetrad becomes

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - m\left[\frac{1}{r} + \frac{1}{\bar{r}}\right] - \frac{e^2}{r^2\bar{r}^2}\right)\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}\bar{r}}\left(\delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

Performing the complex coordinate transformation as above, one has

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - \frac{2mr'}{r'^2 + a^2\cos^2\vartheta}\right)\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}(r' + ia\cos\vartheta)}\left(ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}(r' - ia\cos\vartheta)}\left(-ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

Replacing in equation (1) allows to recover usual the Kerr-Newmann solution in advanced null coordinates.

The ambiguity underlining the NJA can be easily recognized by comparing the two outlined procedures. Indeed, the complex coordinate transformation introduced in [8], has no fundamental explanation or derivation. It should also be noted a certain arbitrariness since the complexification procedure of the terms $1/r$, $2m/r$, e^2/r^2. In [7], the authors just say that if the term e^2/r^2 is replaced by $\frac{1}{2}(1/r^2 + 1/\bar{r}^2)$, as expected, one does not obtain a solution of Einstein-Maxwell equations.

This ambiguity in the complexification of the r coordinate will be even more evident in the next section, where some examples and some discording results, previously available in literature, are discussed.

4. Examples

In this section we discuss the extension of the NJA to more general seed metrics. When applied to seed metrics other than the two considered above, the NJA does not provide their "standard" rotating generalization, as one might have naively expected. Moreover, due to the arbitrariness in the application of the method, i.e., the complexification of the r coordinate, the algorithm provides discording results.

Firstly, we consider the application of the NJA to a seed de Sitter metric. This is the simplest example allowing to discuss many interesting features of the NJA, when a cosmological constant term is introduced. It turns out that, in order for the new metric to be a solution of the Einstein equations, a suitable matter source is required, which does not allow for a simple interpretation in terms of cosmological constant or perfect fluid. Some comparison with similar results appearing in literature is drawn.

Then, we discuss the application of the NJA to two slightly more general seed metrics: Schwarzschild-de Sitter and Reissner-Nordström-de Sitter. The new metrics so obtained require suitable matter sources in order for them to be solutions of the Einstein equations. A comparison with similar results in literature is drawn.

4.1. Rotating de Sitter Metrics

According to the NJA, we are able to construct a *rotating de Sitter* metric by applying the algorithm to the de Sitter solution. We start by writing the de Sitter metric in terms of its null tetrad vectors

$$
\begin{aligned}
l^\mu &= \delta_1^\mu \\
n^\mu &= \delta_0^\mu - \frac{1}{2}\left(1 - \frac{\Lambda}{3}r^2\right)\delta_1^\mu \\
m^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right) \\
\bar{m}^\mu &= \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).
\end{aligned}
$$

The resulting tetrad can be written in the following way

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left(1 - \frac{\Lambda}{3}(r'^2 + a^2\cos^2\vartheta)\right)\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}(r' + ia\cos\vartheta)}\left(ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right)$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}(r' - ia\cos\vartheta)}\left(-ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right).$$

After the complex transformation, the metric becomes

$$ds^2 = \left[1 - \frac{1}{3}\Lambda\left(r^2 + a^2\cos^2\vartheta\right)\right]dt^2 + dtdr - (r^2 + a^2\cos^2\vartheta)d\vartheta^2$$

$$+ \left[\frac{1}{3}a\Lambda(r^2 + a^2\cos^2\vartheta)\sin^2\vartheta\right]dtd\varphi - a\sin^2\vartheta drd\varphi \qquad (2)$$

$$- \left[\frac{1}{3}a^2\Lambda(r^2 + a^2\cos^2\vartheta)\sin^4\vartheta + (r^2 + a^2)\sin^2\vartheta\right]d\varphi^2.$$

This is not a solution of the Einstein equations with cosmological constant Λ, as one could have expected recalling the fact that the well known Kerr-Newman-de Sitter solution (cf. the Appendix), after setting $q = m = 0$, is a solution of the Einstein equations with cosmological constant, involving the two remaining parameters a and Λ. This is the first indication that the actual Kerr-Newman-de Sitter solution can not be recovered through the NJA, as further discussed in the following.

The metric in equation (2) can be recovered as a particular case of metric presented in [6] by setting $Q(u) = M(u) = 0$, while it does not agree with the one dubbed **Rotating de Sitter solution** in [5]. This is due to the fact that in [5] the Wang-Wu functions are introduced in an intermediate step of the NJA and the involved coordinate r is dealt with in a different way in comparison with the complexification scheme discussed above and applied in our example and in [6].

4.2. Schwarzschild-de Sitter and Kerr-Newman-de Sitter

Let's consider the Schwarzschild-de Sitter solution as seed metric (cf. the Appendix in the case $a = q = 0$). Following the outlined procedure, that is, recasting the metric in advanced null coordinates, considering its representation in terms of

null tetrad vectors and applying the NJA, we get the new tetrad:

$$l^\mu = \delta_1^\mu$$

$$n^\mu = \delta_0^\mu - \frac{1}{2}\left[1 - \frac{\Lambda}{3}(r'^2 + a^2\cos^2\vartheta) - 2m\left(\frac{r'}{r'^2 + a^2\cos^2\vartheta}\right)\right]\delta_1^\mu$$

$$m^\mu = \frac{1}{\sqrt{2}(r' + ia\cos\vartheta)}\left[ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{\sin\vartheta}\delta_3^\mu\right]$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}(r' - ia\cos\vartheta)}\left[-ia\sin\vartheta(\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu - \frac{i}{\sin\vartheta}\delta_3^\mu\right].$$

Now, it is possible to write the new line element using the above tetrad. The result is strongly different from that expected for the analogy with the examples discussed in Section 3. The metric obtained from this tetrad is not a solution of the Einstein equations with cosmological constant while, as known, the actual Kerr-de Sitter metric (cf. the Appendix in the case $q = 0$) is indeed an exact solution.

The same reasoning about not intuitive results obtained through the NJA, can be carried on in the case of Reissner-Nordström-de Sitter seed metric (cf. the Appendix in the case $a = 0$). Indeed, it does not provide a solution of the Einstein equation with cosmological constant and a suitable electromagnetic field. In both the aforementioned cases, this can be easily observed by evaluating the corresponding Ricci scalars.

Notice that both the aforementioned metrics can be derived from the metric presented in [6] with $(Q(u) = 0,\ M(u) = \text{const.})$ and $(Q = \text{const.},\ M(u) = \text{const.})$ respectively. These metrics do not coincide with those presented in [5], as observed by the author himself. This is due to the modification of the NJA introduced by the author since he first applies the algorithm, then he makes use of the Wang-Wu function without applying the complexification procedure of the r involved coordinate. The author also provides a better interpretation of the sources in terms of non-perfect fluids, which, in turn, is quite different from the expected interpretation in terms of an electromagnetic field and a cosmological constant.

5. Conclusions

Motivated by the general problem of finding exact solutions of Einstein equations, describing the gravitational field generated by rotating isolated sources, we have discussed the Newman-Janis Algorithm. We have pointed out that, even in the standard examples, some ambiguities arise in the complexification procedure for the radial coordinate r. This ambiguity is even more evident when the NJA is carried out using the Wang-Wu functions rather then applying the algorithm on a specific explicit metric. We have considered various example in which the NJA

does not provide a solution belonging to the family of Kerr-Newman-de Sitter solutions as one may have expected.

Appendix: The Kerr-Newmann-de Sitter Metric

The most general form of the Kerr metric is [3]

$$
ds^2 = -\frac{\Delta_r}{\Xi^2 \Sigma}(dt - a\sin^2\vartheta d\varphi)^2 + \frac{\Delta_\vartheta \sin^2\vartheta}{\Xi^2 \Sigma}\left[adt - (r^2 + a^2)d\varphi\right]^2
$$
$$
+\frac{\Sigma}{\Delta_r}dr^2 + \frac{\Sigma}{\Delta_\vartheta}d\vartheta^2
$$

where

$$
\Delta_r = r^2 - 2Mr + a^2 - \frac{\Lambda r^2(r^2 + a^2)}{3} + q^2
$$
$$
\Delta_\vartheta = 1 + \frac{\Lambda a^2 \cos^2\vartheta}{3}
$$
$$
\Xi = 1 + \frac{\Lambda a^2}{3}
$$
$$
\Sigma = r^2 + a^2 \cos^2\vartheta
$$

for $q = 0$,   one obtains the Kerr-de Sitter metric
for $q = 0$, $\Lambda = 0$ one has the Kerr metric
for $a = 0$, $q = 0$ the result is the Schwarzschild-de Sitter metric
for $a = 0$, $q = 0, \Lambda = 0$ the Schwarzschild metric
for $a = 0$, $q = 0, M = 0$ the de Sitter metric is obtained
for $a = 0$,   one has the Newman-de Sitter metric.

Acknowledgements

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