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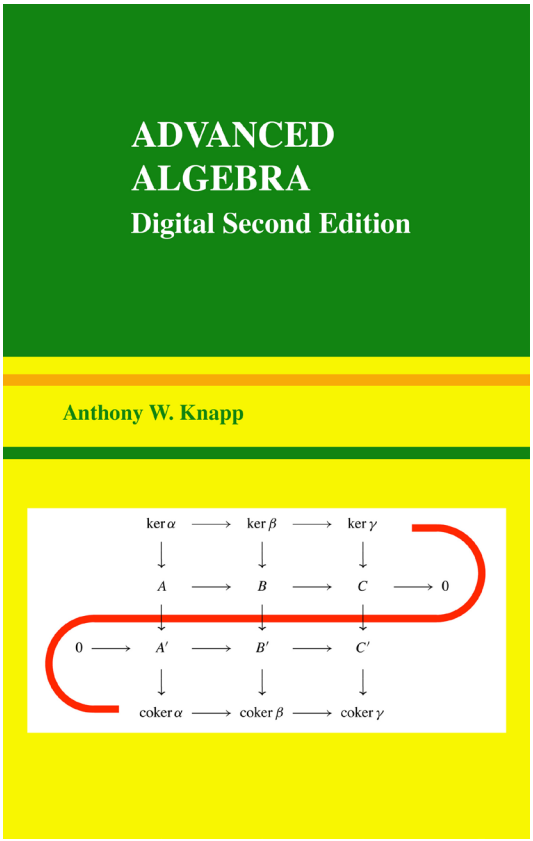

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INDEX OF NOTATION

This list indexes recurring symbols introduced in Chapters I through X (pages 1–648). For recurring symbols introduced in *Basic Algebra*, see the list of Notation and Terminology on pages xxiii–xxvi. Some of the latter notation has been repeated here for the reader's convenience.

In the list below, each piece of notation is regarded as having a key symbol. The first group consists of those items for which the key symbol is a fixed Latin letter, and the items are arranged roughly alphabetically by that key symbol. The next group consists of those items for which the key symbol is a Greek letter. The final group consists of those items for which the key symbol is a variable or a nonletter, and these are arranged by type. To locate an item below, first proceed on the assumption that the key symbol is a Latin or Greek letter; if the item does not appear to be in the list, then treat it as if its key symbol is a variable or a letter.

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