PREFACE.

IT may perhaps be fairly stated that no better guide can be found to the analytical developments of Pure Mathematics during the last seventy years than a study of the problems presented by the subject whereof this volume treats. This book is published in the hope that it may be found worthy to form the basis for such study. It is also hoped that the book may be serviceable to those who use it for a first introduction to the subject. And an endeavour has been made to point out what are conceived to be the most artistic ways of formally developing the theory regarded as complete.

The matter is arranged primarily with a view to obtaining perfectly general, and not merely illustrative, theorems, in an order in which they can be immediately utilised for the subsequent theory; particular results, however interesting, or important in special applications, which are not an integral portion of the continuous argument of the book, are introduced only so far as they appeared necessary to explain the general results, mainly in the examples, or are postponed, or are excluded altogether. The sequence and scope of ideas to which this has led will be clear from an examination of the table of Contents.

The methods of Riemann, as far as they are explained in books on the general theory of functions, are provisionally regarded as fundamental; but precise references are given for all results assumed, and great pains have been taken, in the theory of algebraic functions and their integrals, and in the analytic theory of theta functions, to provide for alternative developments of the theory. If it is desired to dispense with Riemann's existence theorems, the theory of algebraic functions may be founded either on the arithmetical ideas introduced by Kronecker and by Dedekind and Weber; or on the quasi-geometrical ideas associated with the theory of adjoint polynomials; while in any case it does not appear to be convenient to avoid reference to either class of ideas. It is believed that, save for some points in the periodicity of Abelian integrals, all that is necessary to the former elementary development will be found in Chapters IV. and VII., in connection with which the reader may consult the recent paper of Hensel, *Acta Mathematica*, XVIII. (1894), and also the papers of Kronecker and of

Dedekind and Weber, *Crelle's Journal*, XCI., XCII. (1882). And it is hoped that what is necessary for the development of the theory from the elementary geometrical point of view will be understood from Chapter VI., in connection with which the reader may consult the *Abel'sche Functionen* of Clebsch and Gordan (Leipzig, 1866) and the paper of Noether, *Mathematische Annalen*, VII. (1873). In the theory of Riemann's theta functions, the formulae which are given relatively to the ζ and $\wp$ functions, and the general formulae given near the end of Chapter XIV., will provide sufficient indications of how the theta functions can be algebraically defined; the reader may consult Noether, *Mathematische Annalen*, XXXVII. (1890), and Klein and Burkhardt, *ibid.* XXXII.—XXXVI. In Chapters XV., XVII., and XIX., and in Chapters XVIII. and XX., are given the beginnings of that analytical theory of theta functions from which, in conjunction with the general theory of functions of several independent variables, so much is to be hoped; the latter theory is however excluded from this volume.

To the reader who does not desire to follow the development of this volume consecutively through, the following course may perhaps be suggested; Chapters I., II., III. (in part), IV., VI. (to § 98), VIII., IX., X., XI. (in part), XVIII. (in part), XII., XV. (in part); it is also possible to begin with the analytical theory of theta functions, reading in order Chapters XV., XVI., XVII., XIX., XX.

The footnotes throughout the volume are intended to contain the mention of all authorities used in its preparation; occasionally the hazardous plan of adding to the lists of references during the passage of the sheets through the press, has been adopted; for references omitted, and for references improperly placed, only mistake can be pleaded. Complete lists of papers are given in the valuable report of Brill and Noether, "Die Entwicklung der Theorie der algebraischen Functionen in älterer und neuerer Zeit," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Dritter Band, 1892—3 (Berlin, Reimer, 1894); this report unfortunately appeared only after the first seventeen chapters of this volume, with the exception of Chapter XI., and parts of VII., were in manuscript; its plan is somewhat different from that of this volume, and it will be of advantage to the reader to consult it. Other books which have appeared during the progress of this volume, too late to effect large modifications, have not been consulted. The examples throughout the volume are intended to serve several different purposes; to provide practice in the ideas involved in the general theory; to suggest the steps of alternative developments without interrupting the line of reasoning in the text; and to place important consequences which are not utilised, if at all, till much subsequently, in their proper connection.

For my first interest in the subject of this volume, I desire to acknowledge my obligations to the generous help given to me during Göttingen vacations,

on two occasions, by Professor Felix Klein. In the preparation of the book I have been largely indebted to his printed publications; the reader is recommended to consult also his lithographed lectures, especially the one dealing with Riemann surfaces. In the final revision of the sheets in their passage through the press, I have received help from several friends. Mr A. E. H. Love, Fellow and Lecturer of St John's College, has read the proofs of the volume; in the removal of obscurities of expression and in the correction of press, his untiring assistance has been of great value to me. Mr J. Harkness, Professor of Mathematics at Bryn Mawr College, Pennsylvania, has read the proofs from Chapter XV. onwards; many faults, undetected by Mr Love or myself, have yielded to his perusal; and I have been greatly helped by his sympathy in the subject-matter of the volume. To both these friends I am under obligations not easy to discharge. My gratitude is also due to Professor Forsyth for the generous interest he has taken in the book from its commencement. While, it should be added, the task carried through by the Staff of the University Press deserves more than the usual word of acknowledgment.

This book has a somewhat ambitious aim; and it has been written under the constant pressure of other work. It cannot but be that many defects will be found in it. But the author hopes it will be sufficient to shew that the subject offers for exploration a country of which the vastness is equalled by the fascination.

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