SUBJECT INDEX TO THE PAGES OF THIS VOLUME.

Abelian functions, 236, 600, see **Inversion**; integrals, see **Integrals**; matrix, 669.

Abel's theorem, 207, ff.; statement of, 210, 214; proof of, 213; number of independent equations given by, 222 ff.; for radical functions, 377; for factorial functions, 397; for curves in space, 231; Abel's proof of, 219, 220; converse of, 222.

Abel's differential equations, 225, ff.

Addition equation for hyperelliptic theta functions, deduced algebraically, 331, ff.; for theta functions in general, 457—461, 472, 476, 481, 513, 521.

Adjoint polynomial (or curve), definition of, 121; number of terms in, 128; expression of rational function by, 127; see **Integrals, Sets, Lots.**

Argument and parameter, interchange of, 16, 185, 187, 189, 191, 194, 206.

Associated: Forms associated with fundamental integral functions, 62; integrals of second kind associated with integrals of the first kind, 193, 195, 198, 532; associated system of factorial functions, 397

Automorphic functions, simple case of, 352, ff.; connection with factorial functions, 439, ff.

Azygetic characteristics, 487, 497; transformation of, 542, 547; see **Characteristics.**

Bacharach's modification of Cayley's theorem for plane curves, 141.

Biquadratic, see **Göpel.**

Birational transformation of a Riemann surface: does not affect the theory, 3, 7; number of invariants in, 9, 144, 148, 150; of plane curves, 11; by ϕ-polynomials, 142—152; for hyperelliptic surface, 152, 85; when $p=1$, or 0, 153; of surface into itself, 653. See **Invariants**, and **Curves.**

Bitangents of a plane curve, 381—390; 644, 646.

Branch places, see **Places.**

Canonical equation for a Riemann surface, 83, 91, 103, 143, 145, 152; curve discussed by Klein, 159; integral of the third kind, 168, 185, 189, 194, 195.

Cayley's theorem for plane curves, 141.

Characteristics: of a theta function, number of odd and even, 251; expression of any half-integer characteristic by means of a fundamental system, 301, 487, 500, 502; Weirstrass's number notation for, 570, 337, 303; tables of half-integer characteristics for $p=2$, $p=3$, 303, 305; syzygetic, azygetic, 487; period characteristics and theta characteristics, 543, 564; of radical functions, 380, 564; Göpel groups and systems of, 489, 490, 494, ff.; general theory of, 486, ff.; transformation of, 536, 542, 547, 564, 568.

Coincidences of a correspondence, 645.

Column and row. See **Matrices.**

Column of periods, 571.

Complex multiplication of theta functions, 629, ff., 639, 660.

Composition of transformations of theta functions, 551.

Condition of dimensions, 49.

Conformal representation, 343, 356, 372.

Congruence, meanings of sign of, 236, 256, 261, 264, 487.

Constants, invariant in rational transformation, 9, 88, 144, 148, 150; in linear transformation of theta functions, 555—559; in any transformation of theta functions, 620, 622.

Contact curves, see **Curves**, and **Radical.**

Convergence of an automorphic series, 350; of transformed theta function, 538.
Coresidual sets of places on a Riemann surface, 135, ff., 213; are equivalent sets, 136; enter in statement of Abel's theorem, 210.
Correspondence of Riemann surfaces, 3, ff., 81, 639, 642, 647, 648, 649, 654, 662.
Covariant, see **Invariant.**
Cubic surface associated with a plane quartic curve, 382, 389.
Curves: as alternative interpretation of fundamental algebraic equation, 11; inflexions of a plane quartic in connection with the gap theorem, 36; generalisation, 40; inflexions and bitangents in connection with theory of correspondence, 644, 646; bitangents of a plane quartic curve, 384; adjoint curves, 121, 129; coresidual and equivalent sets upon, 134—136; transformation of, see **Birational, Invariants,** and **Constants**; correspondence of, see **Correspondence**; special sets upon, 146, ff.; contact curves, 381; general form of Plücker's equations for, 124; Weierstrass's canonical equation for, 93, 103; Cayley's theorem for, 141; curves in space, 157, 160, ff., 166, 664; Abel's theorem for, 231.
Cusps, 11, 114.

Deficiency of a Riemann surface, 7, 55, 60.
Defining relation for theta functions, 443.
Definition equation of theta functions of general order, 448.
Degenerate Abelian integrals, 657.
Dependence of the poles of a rational function, 27.
Differential equations of inversion problem, 225, ff.; of theta functions, see **Addenda** (p. xx).
Differentials of integrals of first kind, 25, 62, 67, 127, 169.
Dimension of an integral function, 48, ff., 55; condition of dimensions, 49.
Discriminant of a fundamental set of integral functions, 74, 101, 124.
Dissection of the Riemann surface, 26, 529, 253, 257, 569, 297, 550, 560.
Double points of a Riemann surface (or curve), 1, 2, 3, 11, 114; tangents of a plane curve, 644, 646.

Elementary integrals, see **Integrals.**
Equivalence, meanings of sign of, 236, 256, 261, 264, 487.
Equivalent sets of places on a Riemann surface, 134, ff., 136, 213.
Essential factor of the discriminant, 60, 74, 124.
Existence theorems, algebraically deducible, 78; references, 14.
Expression of any rational function, 77, 176, 212; of fundamental integral functions, 105, ff.; of half-integer characteristic by means of a fundamental system, 301, 487, 500, 502.

Factorial functions, 392, ff.; definition of, 396; which are everywhere finite, 399; expressed by factorial integrals, 403; expressed by fundamental factorial function, 413; with fewest poles, 406; used to express theta functions, 423, 426; connection with automorphic functions, 439, ff.
Factorial integrals, 398; which are everywhere finite, 399; fundamental, having only poles, 408; simplified form of that integral, 411; expression of factorial function by means of that integral, 412.
Function, automorphic, 352, ff., 439, ff.; **factorial,** see **Factorial**; **integral,** see **Rational,** and **Transcendental**; $\wp$ **function,** 292, 324, 333, 516; **prime,** 172, 177, 205, also 360, 363, 428; **radical,** 374, 390, 565; **rational,** see **Rational**; **Theta,** see **Theta functions,** and **Transformation**; ζ **function,** 287, 292, 320; see **Fundamental** rational.
Fundamental algebraical equation, 10, 113.
Fundamental rational function, Weierstrass's, 171, 175, 177, 178, ff., 182.
Fundamental set for the expression of rational integral functions, 48, ff., 55, 56, 57, 105, ff.
Fundamental system of theta characteristics, 301, 487, 500, 502.

Gap theorem, 32, 34, 93, 174.
Geometrical investigations, 113; see **Curves.**
Göpel biquadratic relation, 338—340; 465—468; see **Addenda** (p. xx).
Göpel group and system, see **Characteristics.**
Grade, of a polynomial, 120.
Group, Göpel, see **Characteristics.**

Hensel's determination of fundamental integral functions, 105, ff.

Homogeneous variables, 118, 441.

Homographic behaviour of differentials of integrals of first kind, 26.

Hyperelliptic surfaces, 80, ff., 152, 153, 373; see **Theta functions** and **Transformation.**

Independence of the poles of a rational function, 27; of the 2^{2p} theta functions with half-integer characteristics, 446, 447; See **Linearly.**

Index of a place on a Riemann surface, 122, 123, 124; at the infinite place of Weierstrass's canonical surface, 129.

Infinitesimal on a Riemann surface, 1, 2, 3.

Infinitesimal periods, 238, 573.

Infinities of rational function, 27, ff.; see **Residue.**

Infinity, the places at infinity on a Riemann surface, algebraic treatment of, 118.

Inflexions of a plane curve, 36, 40, 646.

Integrals, degenerate, 657; factorial, see **Factorial;** Riemann's, normal elementary, 15; all derivable from integral of third kind, 22; algebraic expression of, 65, ff., 127, 131, 163, 185, 189, 194; hyperelliptic, 195; formulae connecting with logarithmic differential coefficients of theta functions, 289, 290, 320.

Integral functions, see **Rational** and **Transcendental.**

Interchange of argument and parameter, 16, 185, 187, 189, 191, 194, 206; of period loops, see **Transformation.**

Invariants in birational transformation: the number p, 7; the $3p-3$ moduli, 9, 144, 148, 150; the ratios of ϕ-polynomials, 26, 153; the contact ϕ-polynomials, 281, 427; the g-places, 38, 653; for transformation of the dependent variable, 74, 124.

Inversion theorem, Jacobi's, 235, ff., 270; solution of, 239, 242, 244, 275; by radical functions, 390; in the hyperelliptic case, 317, 324.

Jacobi's inversion theorem, see **Inversion.**

Jacobian functions, their periods, are generalisation of theta functions, 579—588; their expression by theta functions, 588—594; there exists a homogeneous polynomial relation connecting any $p+2$ Jacobian functions of same periods and parameter, 594.

Klein, prime form, 360, 427, 430, 433.

Laurent's theorem, for p variables, 444.

Left side of period loop, 529.

Linearly independent ϕ-products of order μ, 154; columns of periods, 575; theta functions, 446, 447; Jacobian functions, 594.

Linear transformation, see **Transformation.**

Loops, period loops on a Riemann surface, 21, 529.

Lots, of sets of places on an algebraic curve, or Riemann surface, 135.

Matrices, 248, 283, 580, 666, 669.

Mittag-Leffler's theorem for uniform function on a Riemann surface, 202.

Moduli, of the algebraic equation, are $3p-3$ in number, 9, 144, 148, 150; for the hyperelliptic equation, 88.

Moduli of periodicity, see **Periods.**

Multiplication, complex, of theta functions, 629, ff.; by an integer, for theta functions, 527.

Multiply-periodic, 236; see **Inversion.**

Noether's (Kraus's) ϕ-curve in space, 156, 157.

Normal equation for a Riemann surface, 83, 91, 103, 143, 145, 152.

Normal integrals (Riemann's) see **Integrals.**

Number of independent products of μ ϕ-polynomials, 154; of odd and even theta functions, 251; of theta functions of general order, 452, 463; of Jacobian functions, 594.

Order of small quantity on a Riemann surface, 2; of a theta function, 448.

$\wp$ **Function,** 292, 324, 333, 516.

Parameter, interchange of argument and parameter, see **Interchange.**

Parameters, in the algebraic equation, see **Constants.**

Period loop, see **Loops.**

Period characteristics, see **Characteristics.**

Periodicity of a (1, 1) correspondence, 650.

Periods of Riemann's integrals, 16, 21; Riemann's and Weierstrass's relations for the periods of integrals of the first kind, and of associated integrals of the second kind, 197, 285, 581, 587;

rule for half-periods on a hyperelliptic surface, 297; for integrals of second kind, 323; of factorial integrals, 404; linear transformation of periods, 532; general transformation, 536, 538; general theory of systems of periods, 571, ff., 579, ff.; of degenerate integrals, 657.

Picard's theorem (Weierstrass's), 658.

Places, of a Riemann surface, 1, 2, 3; branch places, 7, 9, 46, 74, 122, 297, 569; where a rational function is infinite, to order less than $p+1$, 38, 41, 90, 653;

the places $m_1, \ldots, m_p$, 255; their geometrical interpretation, 265, 266; after linear transformation, 562; determination of, for a Riemann surface with assigned period loops, 567; for a hyperelliptic surface, 297, 563.

Plücker's equations, generalised form of, 123, 124; for curves in space, 166.

Poles, see **Infinities**.

Polynomial, grade of, 120; algebraic treatment of, 120; adjoint, 121, 128; ϕ-polynomials, 141; transformation of fundamental equation by ϕ-polynomials, 142, 154; expression of rational functions, and algebraic integrals by means of adjoint polynomials, 156; see **Curves**.

Positive direction of period loop, 529.

Primary and associated systems of factorial functions, 397.

Prime function (or form), see **Function**.

Product expression of uniform transcendental function with single essential singularity, 205.

Quartic. Double tangents of plane quartic curve, 381—390, 647.

Quotients of theta functions, 310, 311, 390, 426, 516.

Radical function, see **Function**.

Rational function, of order 1, only exists when $p=0, 8$; is an uniform function on the Riemann surface whose only infinities are poles, 27; infinities of, Riemann-Roch theorem, Weierstrass's gap theorem, 27, ff.; special, 25, 137; of order p, 38, 137; integral function, 47, ff., 55, 91, ff.; of the second order, 80, ff.; fundamental integral rational functions, algebraic determination of, 105, ff.; algebraic expression of, by adjoint polynomials, 125, ff., 156; Weierstrass's fundamental, 171, 175, 177, 178, ff., 182; expressed by Riemann's integrals, 24, 212; expressed by Weierstrass's function, 176.

Reciprocal sets of zeros of adjoint polynomials, 134.

Residual sets of places, 135.

Residue, fundamental residue theorem, 232, 189, 20.

Reversible transformation, see **Birational**.

Riemann-Roch theorem, 44, 133; for factorial functions, 405.

Riemann and Weierstrass's period relations, 197, 285, 581, 587.

Right side of period loop, 529.

Row and column, see **Matrices**

Schottky-Klein prime form and function, 360, 427, 430, 433.

Sequence, theorem of, 114, 161, 165.

Sequent sets of places, 135.

Sets of places on a Riemann surface or algebraic curve, 135. See **Special**.

Sign of equivalence and congruence, 236, 256, 261, 264, 487.

Special correspondences on a Riemann surface, 648.

Special rational functions, 25, 62, 137.

Special sets of zeros of adjoint polynomials, 134, 147.

Special transformation of a theta function, 629, ff., 639, 660.

Strength of assigned zeros, as determinators of a polynomial, 133.

Supplementary transformations of a theta function, 552.

System, Göpel, see **Characteristics**.

Syzygetic characteristics, 487, 542.

Tables of Characteristics, 303, 305.

Tangents, double, of a plane curve, by the principle of correspondence, 644, 646.

Theta functions:

Riemann's theta functions, 246, ff.; convergence of, 247; determination of, from periodicity, 444; period properties of, 249; number of odd and even, 251, 446; zeros of, 252, 255, 258, 567; identical vanishing of, 258, 271, 276, 303; hyperelliptic, 296, ff.; algebraic expression of quotients of, 310, 311,

390, 426; addition theorem for hyperelliptic, 332, 337; algebraic expression for hyperelliptic, 435; algebraic expression of first logarithmic derivatives of, 288, 290, 320; algebraic expression of second logarithmic derivatives of, 293, 324, 329, 333; solution of inversion problem by means of, 275, 324, 390, 426, ff.; Riemann's functions not the most general, 248, 628.

General theta function of first order, 283, 444; period relations, 285, 197, 581, 587; second logarithmic derivatives of, 516; addition theorems for, 457, 472, 481, 513, 521; Göpel relation for, in case $p=2$, see **Göpel**; expression of Jacobian functions by means of, 594.

Theta functions of second and higher order, 448; expression of, number of linearly independent, 452, 463; of order 2, of special kind, 509, 510; every $p+2$ theta functions of same order, periods, and characteristic, connected by a homogeneous polynomial relation, 453.

Transformation of theta functions, see **Transformation**; characteristics of theta functions, see **Characteristics**; complex multiplication of theta functions, 629, ff., 639, 660; theta functions expressed by factorial functions and simpler theta functions, 426; particular cases, 430, ff.; hyperelliptic case, 433.

Transcendental uniform function, 200; Mittag-Leffler's theorem for, 202; expressed in prime factors, 205; application of Laurent's theorem when the function is integral, 444.

Transformation

of the algebraic equation (or Riemann surface), 3, 143, 145, 151, 152, 654, 655; see **Birational**;

of theta functions, 535; linear transformation, 539; constants in, 554—559; for hyperelliptic case, 568; of second order, 603, 617; for any odd order, general theorem, 614; constants in, 620, 622; when coefficients not integers, 625; supplementary transformations, 552; composition of, 551; special transformations, 629, 630, 660;

of periods, 528, 534, 539, 551, 553, 555, 559, 568;

of characteristics, see **Characteristics**.

Uniform, see **Rational**, and **Transcendental**.

Vanishing of theta function, 253, 258, 271 ff., 276, 303.

Variables, homogeneous, 118, 429, 441

Weierstrass's gap theorem, 32, 34, 93, 174; special places which are the poles of rational functions of order less than $p+1$, 34, ff.; canonical surface (or equation), 90, ff., 93; fundamental rational function, 171, 175, 177, 178, 182, 189; period relations, 197, ff., 285, 581, 587; rule for characteristics of hyperelliptic theta functions, 569; theorem for degenerate integrals, 658.

Zeros, generalised zeros of a polynomial, 121; zeros of Riemann theta function, 252.

Zeta function, 287, 292, 320.

CAMBRIDGE: PRINTED BY J. AND C. F. CLAY, AT THE UNIVERSITY PRESS.