

formed in order to draw the consequences from the data displayed on the panel.

50. Table of Consequences.—But PORETSKY'S method can be illustrated, better than by geometrical and mechanical devices, by the construction of a table which will exhibit directly all the consequences and all the causes of a given equality. (This table is relative to this equality and each equality requires a different table). Each table comprises the 2^n classes that can be defined and distinguished in the universe of discourse of n terms. We know that an equality consists in the annulment of a certain number of these classes, viz., of those which have for constituents some of the constituents of its *logical zero* N . Let m be the number of these latter constituents, then the number of the sub-classes of N is 2^m which, therefore, is the number of classes of the universe which vanish in consequence of the equality considered. Arrange them in a column commencing with 0 and ending with N (the two extremes). On the other hand, given any class at all, any preceding class may be added to it without altering its value, since by hypothesis they are null (in the problem under consideration). Consequently, by the data of the problem, each class is equal to 2^m classes (including itself). Thus, the assemblage of the 2^n classes of discourse is divided into 2^{n-m} series of 2^m classes, each series being constituted by the sums of a certain class and of the 2^m classes of the first column (sub-classes of N). Hence we can arrange these 2^m sums in the following columns by making them correspond horizontally to the classes of the first column which gave rise to them. Let us take, for instance, the very simple equality $a = b$, which is equivalent to

$$ab' + a'b = 0.$$

The logical zero (N) in this case is $ab' + a'b$. It comprises two constituents and consequently four sub-classes: 0, ab' , $a'b$, and $ab' + a'b$. These will compose the first column. The other classes of discourse are ab , $a'b'$, $ab + a'b'$,

and those obtained by adding to each of them the four classes of the first column. In this way, the following table is obtained:

\circ	ab	$a'b'$	$ab + a'b'$	
ab'	a	b'	$a + b'$	
$a'b$	b	a'	$a' + b$	
$ab' + a'b$	$a + b$	$a' + b'$	$\mathbf{1}$	

By construction, each class of this table is the sum of those at the head of its row and of its column, and, by the data of the problem, it is equal to each of those in the same column. Thus we have 64 different consequences for any equality in the universe of discourse of 2 letters. They comprise 16 identities (obtained by equating each class to itself) and 16 forms of the given equality, obtained by equating the classes which correspond in each row to the classes which are known to be equal to them, namely

$$\begin{aligned} \circ &= ab' + a'b, & ab &= a + b, & a'b' &= a' + b', & ab + a'b' &= \mathbf{1} \\ a &= b, & b' &= a', & ab' &= a'b, & a + b' &= a' + b. \end{aligned}$$

Each of these 8 equalities counts for two, according as it is considered as a determination of one or the other of its members.

51. Table of Causes.—The same table may serve to represent all the causes of the same equality in accordance with the following theorem:

When the consequences of an equality $N = \circ$ are expressed in the form of determinations of any class U , the causes of this equality are deduced from the consequences of the *opposite* equality, $N = \mathbf{1}$, put in the same form, by changing U to U' in one of the two members.

For we know that the consequences of the equality $N = \circ$ have the form

$$U = (N' + X) U + NYU',$$

and that the causes of the same equality have the form

$$U = N'XU + (N + Y)U'.$$