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Inefficacy of temporary policy in Neumeyer=Yano's monetary model

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Abstract.

In this paper, we explore the effects of temporary policy in a dynamic general equilibrium by using Neumeyer and Yano's money-in-utility model. Even under dynamic interaction between consumers through markets, the impact of temporary policy both on the present and future economy is very small if the long-run interest rate level is low, which coincides with the intuitive explanation by the permanent income hypothesis.

§1. Introduction

The permanent income hypothesis proposed by Friedman [1958] implies that a temporary income change has a relatively small impact on consumers' consumption behavior compared with a permanent income change if the long-run interest rate is low. Yano [1984, 91, 98] formulates a temporary policy and proves a similar result using a dynamic general equilibrium theory, in which social interactions generated through markets are explicitly counted in. Although his frameworks hinge on a no restrictive assumption, they are real models without money.

The main purpose of this paper is to demonstrate the inefficacy of temporary policy by using a dynamic model with many consumers

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and with money in their utility functions. To that end, we use a closed-economy version of Neumeyer and Yano [1995]'s money-in-utility model. Neumeyer and Yano [1995] develop a two-country monetary model and explore how unanticipated temporary policy works across national borders. In this paper, we derive a general equilibrium path and conduct a comparative dynamic analysis highlighting the limit case, in which the long-run interest rate vanishes. We find that, in the near limit, a temporary policy that redistributes wealth among consumers has almost no impact on consumption and saving allocations of every consumer. Even with a complex dynamic interaction among any finite number of consumers through markets, the real effects of temporary fiscal and monetary policies can be ignored throughout the whole time horizon. Thus, this study is on the same line as Friedman and Yano, extending to a monetary dynamic general equilibrium theory.

The rest of this paper is organized as follows: First, we set up the model and define an equilibrium in Section 2. In Section 3, we derive the closed-form solution of an equilibrium path. In Section 4, we conduct comparative dynamics and establish the inefficacy theorem.

§2. Model

Think of an economy with an infinite time horizon and discrete time points $t = -1, 0, 1, \ldots$ We call the period between time $t-1$ and t the period t. There are H consumers and the government, and they transact a consumption good, money and bond at each period. The government imposes a lump-sum tax, τ_{ht}, that is in the consumption-good form on every consumer, and spends it, g_t, $t = 0, 1, \ldots$ The government provides money M_t^g and bond B_t^g to the consumers. Money brings liquidity service to the consumers, and a unit of bond generates an interest rate $1 + i_t$ from the period t to $t+1$.

Consumer h initially holds money $M_{h,-1}$ and bond $B_{h,-1}$ at $t = -1$, and gets a consumption good endowment y_{ht} for the period t. The stream $\{y_{ht}\}$ is supposed to be bounded over time. Every consumer has his own periodwise preference over his consumption $c_{ht} \in \mathbb{R}_+$ and real money balances $m_{ht} (= M_{ht}/p_t) \in \mathbb{R}_+$ that is represented by a utility function $u_h : \mathbb{R}_+^2 \to \mathbb{R}$. Every consumer at time t, $t = 0, 1, \ldots$, must be subject to the flow budget constraints:

$$(1) \quad M_{ht+s} + B_{ht+s} \\ \leq p_{t+s}(y_{ht+s} - \tau_{ht+s} - c_{ht+s}) + M_{ht+s-1} + (1 + i_{t+s-1})B_{ht+s-1},$$

$s = 0, 1, ...,$ and the no Ponzi-Game condition:

$$\text{(2)} \qquad \liminf_{T\to\infty} \left(\prod_{j=1}^{T} \frac{1}{1 + r_{t+j-1}} \right) \frac{A_{ht+T}}{p_{t+T}} \geq 0,$$

where $1 + r_t = (1 + i_t) p_t / p_{t+1}$ is the real interest rate from period t to $t + 1$ and $A_{ht} = M_{ht-1} + (1 + i_{t-1}) B_{ht-1}$ is the consumer h's financial asset at the beginning of the period t, $t = 0, 1, ...$ Following Neumeyer and Yano [1995], we integrate the flow budget constraints and the no Ponzi-Game condition at period t into the intertemporal budget constraints. (See (2).)

Consumer h's behavior is summarized as the following maximizing problem:

$$\text{(3)} \qquad \left\{ c^*_{ht+s}, M^*_{ht+s}, B^*_{ht+s} \right\}_{s=0}^{\infty}$$

$$\in \arg \max_{\{c_{ht+s}, M_{ht+s}, B_{ht+s}\}} \sum_{s=0}^{\infty} \beta^s u_h \left(c_{ht+s}, m_{ht+s} \right)$$

$$\text{s.t.} \sum_{s=0}^{\infty} \left(\prod_{j=1}^{s} \frac{1}{1 + r_{t+j-1}} \right) \left(c_{ht+s} + \delta_{t+s} m_{ht+s} \right) \leq w_{ht},$$

$$\text{where } w_{ht} = A_{ht}/p_t + \sum_{s=0}^{\infty} \left(\prod_{j=1}^{s} \frac{1}{1 + r_{t+j-1}} \right) \left(y_{ht+s} - \tau_{ht+s} \right),$$

$t = 0, 1, ...,$ where $\beta \in (0, 1)$ is a discount factor of the future utility and $\delta_t = i_t / (1 + i_t)$ is the opportunity cost of real money holding at the period t. To shorten notation we define an operator taking the discounted value $\widetilde{x}_t = \sum_{s=0}^{\infty} \left(\prod_{j=1}^{s} \frac{1}{1+r_{t+j-1}} \right) x_{t+s}$ for given t and $\{x_{t+s}\}_{s=0}^{\infty}$. Using this operator, we can rewrite $w_{ht} = A_{ht}/p_t + \widetilde{y}_{ht} - \widetilde{\tau}_{ht}$.

The government sets up a stream of its policy variables $\{M^g_{t+s}, B^g_{t+s}, g_{t+s}, \tau_{ht+s}; h \in \mathcal{H}\}$, $\mathcal{H} = \{1, ..., H\}$. The stream must be subject to the flow budget constraints

$$\text{(4)} \quad M^g_{t+s} + B^g_{t+s}$$

$$\geq p_{t+s} \left(g_{t+s} - \sum_{h=1}^{H} \tau_{ht+s} \right) + M^g_{t+s-1} + (1 + i_{t+s-1}) B^g_{t+s-1},$$

$s = 0, 1, ...$ and the no Ponzi-Game condition

$$\text{(5)} \qquad \limsup_{T\to\infty} \left(\prod_{j=1}^{T} \frac{1}{1 + r_{t+j-1}} \right) \frac{D^g_{t+T}}{p_{t+T}} \leq 0$$

for $t = 0, 1, ...$ in much the same way as for consumers. In this paper, the government is supposed to be subject to (4)-(5) with equality.

The stream of price and allocation of the economy is determined so that all markets are cleared:

(6) $$c_t^* + g_t = y_t; \quad M_t^* = M_t^g; \quad B_t^* = B_t^g; \quad \text{for every } t = 0, 1, ...$$

We denote by $x_t = \sum_{h=1}^{H} x_{ht}$ an aggregate variables. The prices in an equilibrium are distinguished with an asterisk. Since, by Walras's law, the bond market clearing condition is redundant for every $t = 0, 1, ...,$ this is ignored in the following analyses.

§3. Equilibrium path

This section derives a closed-form solution, which will be presented in Proposition 1. To this end, following Neumeyer and Yano [1995], we will specify the utility function of the consumer h as $u_h(c, m) = \log c + \gamma_h \log m$. Then, the optimal path is obtained as follows

(7) $$c_{ht}^* = \frac{1}{1+\gamma_h}(1-\beta)\, w_{ht}; \qquad \delta_t m_{ht}^* = \frac{\gamma_h}{1+\gamma_h}(1-\beta)\, w_{ht};$$

for $h \in \mathcal{H}$ and $t = 0, 1, ...$

As was seen in (7), to get the consumption path we need to obtain the dynamics of the real wealth $\{w_{ht}\}$, which are determined as follows:

Lemma 1. *It holds that* $w_{ht+1} = \beta(1 + r_t)\, w_{ht}$ *for every* $h \in \mathcal{H}$ *and* $t = 0, 1, ...$

Proof. By the flow budget constraint, it holds that

(8) $$c_{ht} + \delta_t m_{ht} \leq y_{ht} - \tau_{ht} + \frac{A_{ht}}{p_t} - \frac{1}{1+r_t}\frac{A_{ht+1}}{p_{t+1}}.$$

Since (7) holds, the left-hand-side of (8) is equal to $LHS = (1-\beta)\, w_{ht}$. By the definition of the real wealth $w_{ht} = \frac{A_{ht}}{p_t} + \widetilde{y}_{ht} - \widetilde{\tau}_{ht}$, the right-hand-side of (8) can be rewritten as $RHS = w_{ht} - \frac{1}{1+r_t} w_{ht+1}$. Hence, $w_{ht+1} = \beta(1 + r_t)\, w_{ht}$. Q.E.D.

Thus, by (7) and Lemma 1, $c_{ht+1}^* = \beta(1 + r_t)\, c_{ht}^*$ and $\delta_{t+1} m_{ht+1}^* = \beta(1 + r_t)\, \delta_t m_{ht}^*$ for every $h \in \mathcal{H}$ and $t = 0, 1, ...$ The real interest rate, which is crucial to the equilibrium path as well as the real wealth, is determined in an equilibrium by the disposable products and the discount factor β; it holds that $1 + r_t^* = \frac{1}{\beta}\frac{y_{t+1}-g_{t+1}}{y_t-g_t}$ for every $t = 0, 1, ...$ since $\frac{y_{t+1}-g_{t+1}}{y_t-g_t} = \frac{c_{t+1}^*}{c_t^*}$ in an equilibrium.

The nominal variables — the price level, the opportunity cost of the real money balances and the nominal interest rate — must satisfy the following relationships in an equilibrium; $\frac{p^*_{t+1}}{p^*_t} = \frac{M^g_{t+1}}{M^g_t}\frac{\delta^*_{t+1}}{\delta^*_t}\frac{y_t - g_t}{y_{t+1} - g_{t+1}}$ and $1 + i^*_t = \frac{1}{\beta}\frac{M^g_{t+1}}{M^g_t}\frac{\delta^*_{t+1}}{\delta^*_t}$ for every $t = 0, 1, ...$, which can be easily ascertained.

Now, we denote the growth rate of money supply by $\mu_t = \left(M^g_{t+1}/M^g_t\right) - 1$ and make the following assumptions:

- **Assumption.**
  A. $y_{ht} = y_h,\ g_t = g,\ \tau_{ht} = \tau_h,\ \mu_t = \mu,\ \gamma_h = \gamma,$ for every $h \in \mathcal{H}$ and $t = 0, 1, ...$
  B. $\beta < 1 + \mu$.
  C. $\lim_{T\to\infty}\left(\prod_{j=1}^{T}\frac{1}{1+i^*_{t+j-1}}\right) = 0$.

By Assumption A, $1 + r_t = \beta^{-1}$ for any t. Without Assumption B, no equilibrium may exist in money-in-utility models. For this point, see Brock [1974]. Assumption C excludes hyperdeflationary paths[1].

Further, we can obtain the equilibrium level of the real money balance m^*_{ht} (Lemma 2).

Lemma 2. *It holds that* $m^*_t = \frac{\gamma(1-\beta)}{1+\gamma} w^*_t \frac{1+\mu}{1+\mu-\beta}$ *for every* $t = 0, 1, ...$

Proof. First, note that $\frac{1}{1+\mu_t}\frac{1}{1+r_t} = \frac{1}{1+i_t}\frac{m^g_t}{m^g_{t+1}}$. Let $t = 0, 1, ...$ By using (7), we can obtain, by induction, that

$$m^*_t - \left(\prod_{j=1}^{T}\frac{1}{1+\mu_{t+j-1}}\frac{1}{1+r_{t+j-1}}\right) m^*_{t+T}$$
$$= \gamma\sum_{s=0}^{T-1}\left(\prod_{j=1}^{s}\frac{1}{1+\mu_{t+j-1}}\frac{1}{1+r_{t+j-1}}\right) c^*_{t+s}$$

for every T=1, 2, ... In an equilibrium, the limit of both sides of the above expression as $T \to \infty$ can be obtained as follows. Since $\mu_t = \mu$ and $\beta < 1 + \mu$,

RHS

$$= \gamma\sum_{s=0}^{T-1}\left(\prod_{j=1}^{s}\frac{1}{1+\mu_{t+j-1}}\frac{1}{1+r^*_{t+j-1}}\right)\beta^s\left(\prod_{j=1}^{s}\left(1 + r^*_{t+j-1}\right)\right) c^*_t$$
$$= \gamma c^*_t \prod_{s=0}^{T-1}\left(\frac{\beta}{1+\mu}\right)^s \longrightarrow \gamma c^*_t \frac{1+\mu}{1+\mu-\beta} \text{ as } T \to \infty.$$

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[1]Brock [1974] proved that a hyperdeflationary path may be an equilibrium that is self-fulfilling in a money-in-the-utility-function model.

Since $\lim_{T\to\infty}\left(\prod_{j=1}^{T}\frac{1}{1+i^*_{t+j-1}}\right)=0$, it holds that

$$LHS = m^*_t - \left(\prod_{j=1}^{T}\frac{1}{1+i^*_{t+j-1}}\frac{m^*_{t+j-1}}{m^*_{t+j}}\right)m^*_{t+T}$$
$$= m^*_t - m^*_t\left(\prod_{j=1}^{T}\frac{1}{1+i^*_{t+j-1}}\right) \longrightarrow m^*_t \text{ as } T\to\infty.$$

Thus, $m^*_t = \gamma c^*_t \frac{1+\mu}{1+\mu-\beta} = \frac{\gamma(1-\beta)}{1+\gamma} w^*_{ht} \frac{1+\mu}{1+\mu-\beta}$. Q.E.D.

As a direct consequence, the equilibrium price of the real money $\delta^*_t = \frac{1+\mu-\beta}{1+\mu}$ is obtained. Now, we can derive an equilibrium path. Before that though, we normalize the equilibrium initial price to be constant $p^*_0 = p_0$.[2]

Proposition 1. *An equilibrium path is given as below*

$$(i)\ p^*_t = (1+\mu)^t p_0; \quad (ii)\ 1+r^*_t = \beta^{-1}; \quad (iii)\ 1+i^*_t = \frac{1}{\beta}(1+\mu);$$
$$(iv)\ w^*_{ht} = \frac{A_{h0}}{p_0} + \frac{y_h-\tau_h}{1-\beta}; \quad (v)\ c^*_{ht} = \frac{1-\beta}{1+\gamma}\frac{A_{h0}}{p_0} + \frac{y_h-\tau_h}{1+\gamma};$$
$$(vi)\ m^*_{ht} = \frac{\gamma}{1+\gamma}\frac{1+\mu}{1+\mu-\beta}\left[(1-\beta)\frac{A_{h0}}{p_0} + y_h - \tau_h\right];$$
$$(vii)\ M^*_{ht} = \frac{\gamma}{1+\gamma}\frac{(1+\mu)^{t+1}}{1+\mu-\beta}\left[(1-\beta)A_{h0} + p_0(y_h-\tau_h)\right];$$

for every $t = 0, 1, \ldots$

The proof of the Proposition 1 is so easy that we omit it here. Proposition 1 implies that, as was recognized by Bewley [1982] and Yano [1984, 91, 98], even a stationary path depends on the initial conditions A_{h0} of the model in dynamic general equilibrium models with many consumers. A similar result robustly holds for the present monetary model. Correspondingly, Proposition 1 indicates that both temporary fiscal and monetary policies that redistribute wealth among consumers have prolonged periods of impact on the economy. (The precise definition for the terminology "temporary policy" in this paper is given in the next section.)

[2]Previous studies have often adopted the normalization $\sum_{h=1}^{H}(1/\lambda_h) = 1$ (See, e.g. Bewley [1982] or Yano [1998]), where λ_h is the Lagrange multiplier of consumer h's maximizing problem.

§4. Inefficacy of temporary policy

This study highlights the near limit as $\beta \to 1$, which directly implies that the economies with low long-run interest rate level are considered. To conduct meaningful comparative dynamics, we need to suppose $\mu \geq 0$ since Assumption B: $\beta < 1 + \mu$ is essential for the analyses given in the previous section. In addition, it is necessary that national bond issuance level and the fiscal deficit $g - \tau$ be adequately low in the low interest rate case.

Temporary policy in this paper is thought to be conducted in a lump-sum fashion at the initial time point $t = -1$. That consists of fiscal policy $(\tau_{h,-1};\ h \in \mathcal{H})$ and monetary policy $(M_{h,-1};\ h \in \mathcal{H})$ satisfying $\sum_{h=1}^{H} \tau_{h,-1} = 0$ and $\sum_{h=1}^{H} M_{h,-1} = 0$, i.e. they purely redistribute the physical and financial assets among consumers.

Now, we are in an appropriate position to proceed to the main analysis in this paper. By Proposition 1, it holds in the limit as $\beta \to 1$ that

$$1 + r_t^* \to 1; \qquad 1 + i_t^* \to 1 + \mu; \qquad c_{ht}^* \to \frac{y_h - \tau_h}{1+\gamma};$$

$$m_{ht}^* \to \frac{\gamma}{1+\gamma}\frac{1+\mu}{\mu}(y_h - \tau_h)\,; \qquad M_{ht}^* \to \frac{\gamma}{1+\gamma}\frac{(1+\mu)^{t+1}}{\mu}p_0(y_h - \tau_h);$$

for every $t = 0, 1, \ldots$ This, in comparison with Proposition 1, demonstrates that the dependence of the allocation path on the initial conditions A_{h0} will disappear for the vanishing discount rate $\beta \to 1$. In other words, temporary fiscal policy $(\tau_{h,-1};\ h \in \mathcal{H})$ and monetary policy $(M_{h,-1};\ h \in \mathcal{H})$ have almost no impact on current and future allocations if the long-run interest rate is very low.

We formally summarize the result of the comparative dynamic analysis as follows:

Theorem 1. *(Inefficacy of Temporary Policy; Friedman=Yano) Temporary policies have almost no impact both on current and future consumption allocations in a dynamic general equilibrium if the interest rate level is low.*

As already discussed in the introduction, the result is on the same line as Friedman and Yano. However, my analyses here take the two following aspects into account. First, price effect generated through markets, which is ignored in the arguments concerning the standard permanent income hypothesis, is considered. Second, money that helps consumers to transact, which is not counted in by the existing works of Yano [1984, 91, 98], is also considered.

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